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Human-Capital Formation: The Importance of Endogenous Longevity

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Abstract

We present a theory of human capital, with its two most essential components, health capital and, what we term, skill capital, endogenously determined within the model. Using the theory, and a calibrated version of it, we uncover and highlight an important economic mechanism driving human-capital formation, socio-economic and health disparities, human-capital based economic growth, and causal relations among the stocks of wealth, skill and health, namely whether individuals can influence their own length of life (endogenous longevity). Without the ability of individuals to influence their longevity, the effects of health, skill and wealth on later-life skill and health are muted. Any additional health, skill or wealth is not used for additional investment, but essentially consumed. These findings have important implications for the modeling of, and our understanding of, human-capital formation, disparities in human capital and health, and human-capital based economic growth.

Keywords: health investment; education; human capital; health capital; dynamic optimal control; longevity

JEL Codes: D91, I10, I12, J00, J24

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1 Introduction

The United States' 20th Century was characterized by unprecedented increases in income, longevity, health, and educational attainment. Real per-capita income in 2000 was five to six times its level in 1900 (Goldin and Katz, 2009), life expectancy at birth increased from 46 to 74 years for white men (Centers for Disease Control and Prevention; cdc.gov), and years of schooling rose from 7 to 13 years (Bleakley et al., 2014). Similarly impressive advances took place in other developed and increasingly also in developing nations (Deaton, 2013). Are these trends in wealth, education, health and longevity related? And do they reflect causal mechanisms?

Longevity gains do not always translate into gains in education or, related, in economic growth (Acemoglu and Johnson, 2007; Hazan, 2009; Cervellati and Sunde, 2011). Likewise, and despite a very strong association between them, whether education causes health and longevity is widely debated. An effect of education on health exists in some contexts but not in others and seems to depend on age, gender, the returns to education, and the quality and type of education (Lochner, 2011; Galama et al., 2018; Kaestner et al., 2020). Finally, an essential feature of human capital is that "skills beget skills" through self-productivity and dynamic complementarity (e.g., Cunha and Heckman, 2007a), but the extent to which these mechanisms operate appears to be context-specific (e.g., Almond and Mazumder, 2013; Malamud et al., 2016; Rossin-Slater and Wüst, 2016; Almond et al., 2018). What explains the substantial heterogeneity in the causal effects of, e.g., education on health, gains in life expectancy on education, and more generally in the productivity of, and dynamic complementarities in, human capital?

In this paper, we uncover and highlight an important economic mechanism by which causal relations and complementarities among the stocks of wealth, skill and health may arise, namely whether individuals can influence their own longevity. If individuals can use additional resources – in terms of wealth, skill or health – to extend life, then these resources lead to additional investment in skill and health. On the other hand, if life cannot be extended, the additional resources are mostly spent on consumption and leisure, and only minimally on skill and health. As a result, causal effects are weak (fade out).

We derive the above mechanism from a life-cycle theory of human capital and longevity. In the theory, human capital consists of two stocks, skill and health, both endogenously determined. Individuals invest in skill through years of education and on-the-job training, and they invest in health through medical care and health behaviors. We provide evidence from this theory, and a calibrated version of the theory, that

1. *Endogenous* gains in longevity are a *necessary* condition for
    (a) persistent[1] *causal* relations between wealth, skill and health and the investments in them.

[1]By "persistent" we technically mean that causal effects between wealth, skill and health, are not muted. As we show in this paper, causal effects may exist in models with exogenous longevity but they are dampened/die out.

(b) persistent *self-productivity* and *dynamic complementarity* in skill and health.

2. However, *endogenous* gains in longevity are not a *sufficient* condition for *causal* effects among wealth, skill and health, and, related, for *self-productivity* and *dynamic complementarity* in skill and health.

   Institutions, human biology, medical technology, the quality and availability of schooling, and the returns to health and skill investment, to name a few, also need to be conducive to skill formation and health.

These findings, as will be discussed later, are distinct from those obtained in the extensive (modern) Ben-Porath literature where relations between wealth, skill and health arise from variation in exogenous (fixed) longevity.

To illustrate our main findings, consider, for example, the effect of an exogenous variation in health at time t', $\delta H_{t'} = \delta H(t = t') > 0$, on later-life skill $\theta(t)$ (for any $t > t'$), i.e. $\partial\theta(t)/\partial H_{t'}$. The effect of greater health $\delta H_{t'}$ on the optimal path of skill $\theta(t)$ can be broken down into variation for fixed longevity T and variation due to the resulting change in endogenous longevity T

$$\frac{\partial\theta(t)}{\partial H_{t'}} = \left.\frac{\partial\theta(t)}{\partial H_{t'}}\right|_T + \left.\frac{\partial\theta(t)}{\partial T}\right|_{H_{t'}} \frac{\partial T}{\partial H_{t'}}. \tag{1}$$

We demonstrate in section 3 that, if longevity T is fixed, the comparative dynamic effect $\partial\theta(t)/\partial H_{t'}$ (left-hand side [LHS] of equation [1]) is small (compared to the case where T is free) and fades out.

This is intuitive. The returns to human-capital investments (skill or health) are the product of the returns per period (e.g., earnings, utility) and the duration over which these returns are reaped (longevity, Becker, 1964). First, for fixed T, the duration is unchanged. Thus, the Ben-Porath (1967) mechanism, whereby longevity gains drive additional investment in skill, $\partial\theta(t)/\partial T|_{H_{t'}} \partial T/\partial H_{t'}$, is shut down and no additional investment in skill is made through the pathway of increased longevity. Second, at high absolute *levels of utility* (i.e., developed world, rich individuals) the returns to additional periods are substantial, adding *levels of utility* for each additional period when life can be extended. By contrast, when life cannot be extended, only *marginal improvements in utility* can be made over the fixed number of periods, and these are small at high levels of utility as a result of diminishing returns (Hall and Jones, 2007). Thus the returns to human capital, and the causal effect of endowed health on skill, are comparatively small in a developed-world setting when length of life is fixed.

With small effects of greater health $\delta H_{t'}$ on skill $\theta(t)$ $(t > t')$ for fixed T, the term $\partial\theta(t)/\partial H_{t'}|_T$ (first term on the right-hand side [RHS] of equation [1]) is small for all $t \in [t', T]$. A large causal effect of health $\delta H_{t'}$ on later-life skill $\theta(t)$ then necessarily requires endogenous life extension $\partial T/\partial H_{t'} > 0$ (second term on the RHS of equation [1]). Thus, our first finding is that *endogenous* gains in longevity $\partial T/\partial H_{t'} > 0$ are a *necessary* condition for a persistent *causal* effect of greater health on skill formation.

Similar reasoning applies to the effects of greater wealth and greater skill, on later-life skill,[2] and to the effects of greater wealth, skill and health on later-life health[3] (see earlier statement 1a).

To understand statement 1b, consider the definitions of self-productivity and dynamic complementarity.

Self-productivity: Cunha and Heckman (2007a) define self-productivity as "*the skills produced at one stage augment the skills attained at later stages. It embodies the idea that skill acquired in one period persist into future periods. It also embodies the idea that skills are self-reinforcing and cross fertilizing.*" Self-productivity arises when

$$\frac{\partial g(t)}{\partial g(t')} > 0, \qquad t' < t, \tag{2}$$

where $g(t)$ denotes skill $\theta(t)$ or health $H(t)$. Self-productivity can also be across stocks of human capital, where, e.g., skills $\theta(t')$ produced at one stage t' augment health $H(t)$ at later stages $(t > t')$.

Dynamic complementarity: Cunha and Heckman (2007a,b) define dynamic complementarity as "*Skills produced at one stage rais[ing] the productivity of investment at later stages.*" It arises when

$$\frac{\partial^2 g(t)}{\partial g(t'')\partial I_g(t')} > 0, \qquad t'' \leq t' < t, \tag{3}$$

where $I_g(t')$ denotes investment in skill $I_\theta(t')$ or in health $I_H(t')$, and $g(t'') = \{\theta(t''), H(t'')\}$. As is the case for self-productivity, dynamic complementarity can also be across the stocks, where, e.g., skills $\theta(t'')$ produced at one stage t'' raise investment in health $I_H(t')$ at later stages $(t'' \leq t' < t)$. Dynamic complementarity encompasses two distinct yet related concepts: (i) investments are more productive when the stock of skills is higher (i.e., a property of the production function); and (ii) individuals optimally choose to invest more when their stock of skills is higher (i.e., an endogenous response). In our life-cycle theory, we are mainly interested in (ii). That is, do individuals choose to invest more in their skill (health) when the stock of skill (health) is higher? Therefore, in the remainder, we will refer to dynamic complementarity as the *endogenous response*, and will

[2]The effect of greater wealth $\delta A_{t'}$ on later-life skill $\theta(t)$ is similar to that of greater health $\delta H_{t'}$ as both wealth and health are resources that can be drawn from. The "own" effect of greater skill $\delta\theta_{t'}$ on later-life skill $\theta(t)$ also operates as a resource but has an additional effect, derived from what Cunha and Heckman (2007b, p. 15) call a "carry-over" effect of non-depreciated human capital (i.e., starting out with more of the stock to begin with, the stock will also be higher at later times).

[3]Some differences exist. In the health-capital literature (Grossman, 1972a,b; Ehrlich and Chuma, 1990; Dalgaard and Strulik, 2014; Galama and Van Kippersluis, 2019), life is no longer sustainable below a minimum health level $H_{\min}$ and death is defined by the first time $H_{\min}$ is reached. Thus, any health gain $\delta H_{t'}$ has to be depleted in order to reach the minimum health level $H_{\min}$ at the fixed (unchanged) time of death T. Thus effects on health are more muted for health than for skill as the end value for skill $\theta(T)$ is free, whereas for health $H(T)$ it is fixed.

explicitly state it when we instead refer to dynamic complementarity *as a property of the production function.*

Given definitions (2) and (3), our first finding, that endogenous longevity gains are a necessary condition for persistent causal relations (statement 1a), naturally extends to both self-productivity and dynamic complementarity of skill and health (statement 1b). First, self-productivity $\partial\theta(t)/\partial H_{t'}$ $(t \geq t')$ is equivalent to the causal effect of (endowed) stocks on the life-cycle evolution of the stocks (see equation 1). Second, as we show in section 3, taking the derivative of equation (1) with respect to investment, dynamic complementarity $\partial^2\theta(t)/\partial H_{t'}\partial I_\theta(t) > 0$ $(t \geq t')$ effects are muted too.

The second finding is that endogenous gains in longevity $\partial T/\partial H_{t'} > 0$, are, however, not a *sufficient* condition for a *causal* effect of health on skill formation, or related, for self-productivity and dynamic complementarity effects. This stems from the simple observation that this also requires the term $[\partial\theta(t)/\partial T|_{H_{t'}}]\,\partial T/\partial H_{t'}$ (second term on the RHS of equation 1) to be large (and positive). Since the effect of longevity on skill $\partial\theta(t)/\partial T|_{H_{t'}}$ is influenced by institutions, biology, medical technology, the quality of schooling, the returns to skill in the labor market, etc. (see section 2), these need to be conducive to skill-capital formation (statement 2).

We stress the *causal* nature of these results because models that feature an exogenous (fixed) duration of life T can produce *associations* between wealth $A(t)$, skill $\theta(t)$ and health $H(t)$, since each independently responds to variation in exogenous longevity (between people and countries). This is because in such models, a longer life $\delta T > 0$ *causally* leads individuals to invest more in skill and in health. Greater earnings, associated with greater levels of skill and health, in turn enable people to amass greater wealth. This produces *associations* that result from differences in lifespan. However, these associations *do not* stem from causal effects of skill, health, and wealth, on later-life skill, health, and wealth.[4] They stem from exogenous variation in longevity.

Our findings are obtained from a simple, yet comprehensive, theory of joint investments in skill, health and longevity.[5] The theory responds to a call for action by Michael Grossman (2000) that *" . . . Currently, we still lack comprehensive theoretical models in which the stocks of health and knowledge are determined simultaneously . . . The rich empirical literature treating interactions between schooling and health underscores the potential payoffs to this undertaking . . . ".* Relying on an extended version of the assumption of Ben-Porath neutrality, we analyze the theory analytically and obtain novel predictions that can be tested. We next corroborate our analytical findings by formulating,

[4]For proof, see sections 3 and 4.

[5]Our model is most closely related to two other models, in the narrow sense that they model skill, health and longevity endogenously. Becker (2007) develops a simple two-period model of joint decisions regarding health, education and longevity, where longevity is modelled as the endogenous probability to make it to the second period. Strulik (2018) presents calibrated simulations of a multi-period model of schooling, health and longevity, developed independently around the same time as ours. These two models differ from ours in formulation, methods employed, and research questions investigated. Most significantly, these two papers do *not* contrast the exogenous (fixed longevity) and endogenous (free longevity) cases, and thereby do not uncover the importance of endogenous longevity to human-capital formation.

calibrating, and simulating a more general model that drops each of the three assumptions that underlie Ben-Porath neutrality, and find that the results continue to hold.

The central thesis advanced in this paper – that *endogenous* longevity gains drive causal effects of wealth, skill and health on later-life skill and health – is related to the work by Heckman (1976) and Hall and Jones (2007). Heckman (1976) observed that, under Ben-Porath neutrality, wealth does not *cause* greater investments in skill, but is used to finance additional consumption in a model with *fixed* longevity. We go beyond Heckman (1976) in showing that without the ability to extend life, the causal effects of endowed wealth, skill, and health on later-life wealth, skill, education, and health are all zero or small. Importantly, we also demonstrate, by calibrating a more general model, that this is not an artifact of Ben-Porath neutrality, but instead is driven by the assumption of exogenous, fixed longevity. Hall and Jones (2007) were the first to recognize that in a model with *endogenous* longevity, additional wealth would be spend on investments in longevity rather than per-period consumption. Yet, Heckman (1976) and Hall and Jones (2007) do not contrast the endogenous to the exogenous longevity case and they do not model health- and human-capital formation jointly.[6] This paper models (i) endogenous longevity, (ii) contrasts the full life-cycle responses of the fixed with the free longevity case, and (iii) models both skill and health. This is the first paper, to our knowledge, that does all three.

Our results suggests that the ability to extend life is a fundamental driver of economic behavior: of human-capital formation, of socio-economic and health disparities, of human-capital based economic growth, and of fundamental processes, such as self-productivity and dynamic complementarity, that are thought to be essential characteristics of why skills beget skills. The ability to extend life enables a potentially important virtuous cycle. Investments in health (medical care, health behaviors) make a longer life possible. A longer horizon, in turn, is a crucial determinant of investments in skills (Becker, 1964). Better health and better skills, in turn, raise income, which makes possible the additional spending on health and skills. However, the returns to these additional investments are only meaningful when they make a longer life possible, completing the cycle. This finding has both theoretical and empirical implications.

Theoretically, our results suggest there are substantial gains to incorporating endogenous longevity in models of human capital. The above described virtuous cycle is generally missing from economic models of health and skill. This is because extant models of human-capital formation (i.e., the skill-capital literature and, with a handful of exceptions, the health-capital literature) assume exogenous longevity. For example,

[6] Another closely related literature studies self-productivity and dynamic complementarity in skill. This literature typically employs a model with a fixed, given number of periods, with 1 or 2 childhood phases and some adult phases (e.g., Becker and Tomes, 1976; Cunha and Heckman, 2007a; Cunha et al., 2010; Currie and Almond, 2011; Caucutt and Lochner, 2020). Its focus is on modelling and estimating the technology of skill production, and on whether investments in different periods are complements or substitutes. However, this literature too abstracts from the important role of longevity (the number of periods), and the extent to which individuals can influence their longevity. As our theory highlights, self-productivity and dynamic complementarity effects are muted if individuals cannot influence their longevity.

while health-capital theory (Grossman, 1972a,b) recognizes the role of education as a productivity-enhancing factor in health investment, it treats both education and longevity as exogenous.[7] At the same time, human-capital theory (Schultz, 1961; Becker, 1964; Ben-Porath, 1967; Mincer, 1974) considers investments in skills that are potentially multidimensional (e.g., Cunha and Heckman, 2007a; Acemoglu et al., 2012), but does not model decisions regarding health and longevity. Thus, there is no intrinsic mechanism in models of skill formation, such as declining health, as to why life is finite, and therefore a fixed and exogenous length-of-life is naturally assumed (e.g., Ben-Porath, 1967).[8]

The absence of endogenous longevity decisions also applies to the burgeoning modern Ben-Porath literature that studies the effect of longevity gains on human capital, fertility and economic growth. Some distinguish between exogenous longevity gains in childhood and adulthood (Soares, 2005; Cervellati and Sunde, 2013), and some model both schooling and retirement endogenously (e.g., Boucekkine et al., 2002; Hazan, 2009; Cervellati and Sunde, 2013). Nevertheless, a defining feature of these models is their focus on *exogenous* longevity gains, deriving from medical technological advancements (Boucekkine et al., 2002; Kalemli-Ozcan et al., 2000; Chakraborty, 2004; Soares, 2005; Hazan, 2009; Cervellati and Sunde, 2013), income per capita in the economy (Hazan and Zoabi, 2006), or from human-capital investments by parents in their children (Cervellati and Sunde, 2005).

While exogenous (fixed) longevity models can generate *associations* between wealth, skill and health (by assuming differences between individuals in exogenous longevity), they have difficulty producing *causal* effects of wealth, skill and health, on later-life wealth, skill, health and longevity. This is because in models with exogenous longevity, agents do not use additional resources (e.g., greater health, skill, wealth) to make additional investments in their health or their skills. In essence, the processes of health- and skill-capital formation are stalled. Extant models of human-capital formation therefore have to rely on differences in exogenous (fixed) longevity between individuals to explain the existence of strong socio-economic and health inequalities. After all, if variation in early-life stocks does not cause substantially higher investments in health and skill, this leaves variation in exogenous longevity as the only driver of differences between individuals. Hence, these models have to assume inequality in longevity in order to produce inequality in wealth, health and skill. By contrast, with endogenous longevity, even small differences in endowed wealth, health or skill, lead to more substantial differences between individuals over the life cycle. Thus, our theory *produces*, as opposed to *assumes*, inequality.

Empirical evidence has convincingly demonstrated that early-life health and wealth

[7] Ehrlich and Chuma (1990) were the first to include endogenous longevity in the Grossman model. Galama and Van Kippersluis (2019) have extended the model further by including health behaviors and unhealthy working conditions. A series of papers by Strulik and co-authors models endogenous longevity in a health-deficit framework (Dalgaard and Strulik, 2014; Strulik, 2018). To the best of our knowledge, Fonseca et al. (2020) are the first to develop a fully estimated structural model of endogenous health and longevity. Still, these models treat skill / education as being determined outside of the model.

[8] Gilleskie et al. (2017) and Hai and Heckman (2019) present structurally estimated life-cycle models of this kind, which enable quantifying pathways through which health, education, wages and wealth are related, but treating longevity as exogenously given.

affect skill formation (e.g., Akee et al., 2010; Currie and Almond, 2011; Dahl and Lochner, 2012; Almond et al., 2018), that education affects health at least in certain contexts (e.g., Galama et al., 2018; Heckman et al., 2018; Kaestner et al., 2020), and that skill and health are essential to human-capital based economic development (e.g., Lucas, 1989; Arora, 2001; Barro, 2001; Weil, 2007). Moreover, skill formation is generally believed to exhibit self-productivity and dynamic complementarity, processes that are considered fundamental to explaining why, particularly early in life, skill begets skill (Cunha and Heckman, 2007a). As we demonstrate, modeling endogenous longevity is crucial for these processes to operate. Even in economic models with explicit self-productivity and/or dynamic complementarity in the *production function*, this only translates in self-productivity and dynamic complementarity when individuals can influence their own longevity (endogenous longevity). In sum, economic models of human capital ought to consider endogenous longevity to do justice to these empirical facts.

Empirically, the concept of endogenous longevity provides potential new explanations for economic phenomena, suggesting a fruitful agenda for empirical research aimed at testing them. We list a few potential cases.

1. Our findings may provide an explanation as to why generally causal estimates of the effects of wealth, skill and health, on later-life wealth, skill and health are found to be small compared to associations between them (e.g., Conti et al., 2010; Clark and Royer, 2013; Bijwaard et al., 2015; Galama et al., 2018). In situations where it is difficult for individuals to influence their own longevity, *causal* relations between wealth, skill, and health, are expected to be weak because of limited returns to investments in skill and in health. This may be the case for a developing nation (where there may be lack of access to basic medical care), for a nation with a high disease burden (where gains from tackling a certain disease may be limited due to the existence of other major diseases in the environment), for the developed world if it were faced with diminishing ability of technology to further extend life,[9] or for individuals faced with incurable life-shortening diseases, such as Huntington's disease (Oster et al., 2013). In contrast to potentially weak *causal* relations, *associations* between wealth, skill and health, would still be strong if their exists variation between individuals in (exogenous) longevity (i.e., the Ben-Porath mechanism whereby longevity increases the returns to investments in human capital).

2. Related, a novel testable implication of the theory is that causal effects of endowments on health and skill should be stronger in settings where individuals are able to influence their own longevity. A robust test of this prediction requires a situation in which there are simultaneously shocks to (i) endowments and (ii) the possibility to influence one's length of life. This setting is admittedly rare,

[9]For example, gains in longevity appear to have resulted from a *"process of progressive "rectangularization" of the survival function, which led to sizable reductions in mortality at intermediate (working) ages, but left survival rates at young (schooling) ages and the life-span essentially unchanged..."* (Cervellati and Sunde, 2013).

yet somewhat akin to existing tests for dynamic complementarity, which require independent variation in both endowments and investments. Creative empirical designs have successfully been employed to test the theoretical predictions for dynamic complementarity (e.g., Adhvaryu et al., 2018; Johnson and Jackson, 2019; Rossin-Slater and Wust, 2020), and similarly creative endeavors may be able to test ours.

3. Our findings may provide an explanation as to why generally causal estimates of the effects of wealth, skill and health on later-life skill and health exhibit substantial heterogeneity depending on the context (e.g., Almond et al., 2018). As the 2^{nd} term on the RHS of equation 1 illustrates, even if individuals have substantial ability to extend their lives ($\partial T/\partial H_{t'}$ large), the term $\partial \theta(t)/\partial T|_{H_{t'}}$ needs to be large too (again, using the example of the effect of health on later-life skill). If skill-capital investment is relatively unproductive (e.g., low quality teachers, children infected with worms, or malaria), if the cost of skill-capital investment is high (e.g., high tuition, long distance to schools, crops that need to be harvested), or if the institutional environment generates only limited demand for skill (e.g., poor infrastructure, corruption, limited technological capabilities, etc.), then the effects of health on later-life skill is predicted to be modest. Similar reasoning applies to effects on later-life health and wealth. If returns, however, are high then effects may be large, as long as individuals can influence their own longevity (endogenous longevity). Thus, empirical analyses of human capital need to carefully consider differences between groups, settings (e.g., countries) and time periods, in a) the extent to which resources can be used by individuals to postpone death (statements 1a, 1b), and b) the extent to which the returns to skill and health from life extension are conducive to skill and health formation (statement 2).

4. Our findings speak to the academic and policy discussion regarding longevity gains and economic growth (Acemoglu et al., 2012; Bloom et al., 2018). Longevity gains, from improvements in modern-day medicine, are mostly concentrated among the elderly in developed countries (e.g., Catillon et al., 2018). It has been argued that these gains therefore do not directly contribute to productivity and human-capital investments, and hence economic growth (e.g., Breyer et al., 2010; Eggleston and Fuchs, 2012). Our theory suggests, however, that such longevity gains may still serve a useful economic purpose. As our discussion suggests, the substantial returns to life extension may well provide the incentives for investments in human and financial capital when young to ensure a long life and high utility levels when old.

The remainder of the paper is organized as follows. In section 2 we present our analytical model, first-order and transversality conditions, analytical solutions, and discuss life-cycle trajectories. In section 3 we analyze heterogeneity in these trajectories by employing comparative dynamic analyses, and develop predictions. In section 4 we corroborate our predictions in a more general, calibrated model. We conclude in section 5.

2 Model formulation and solutions

If God had meant there to be more than two factors of production, He would have made it easier for us to draw three-dimensional diagrams.

Robert Solow (1995)

2.1 Model

Our theory merges the human-capital and health-capital literatures. In order to distinguish health from the traditional notion of human capital, we employ in the remainder the term "skill capital" to refer to traditional human capital and "health capital" to refer to health. Investments in health capital consist of, e.g., medical expenditures and physical exercise, while investments in skill capital consist of, e.g., expenditures on education and (on-the-job) training.

To gain insight into the characteristics of our theory we will resort to comparative dynamic analyses, which allow analyzing variation in the life-cycle profiles with respect to the three types of resources an individual possesses, financial capital (wealth), skill capital, and health capital, as well as with respect to other model parameters of interest.

Because of the multi-dimensionality of our problem, following Ben-Porath (1967) and Heckman (1976), we make some convenient assumptions to arrive at a tractable theory that permits derivation of analytical expressions for the comparative dynamic results. The simple model retains the essential characteristics of what one may desire from a more general theory. Indeed, in section 4 we confirm that the main predictions hold also in a more general model that we calibrate using empirical data. There we also discuss at an intuitive level as to why these results hold more generally.

Individuals maximize the lifetime utility function

$$\mathbb{U} = \max_{X_C, L, X_\theta, \tau_\theta, X_H, \tau_H, T} \left\{ \int_0^T U[.] e^{-\beta t} dt \right\}, \tag{4}$$

where time $t = 0$ corresponds to the mandatory schooling age (around 16 to 18 years for most developed nations), T denotes total lifetime (endogenous), β is a subjective discount factor and individuals derive utility $U(t)$ from consumption goods and services $X_C(t)$, leisure time $L(t)$, skill $\theta(t)$ and health $H(t)$.

Individuals maximize a constant relative risk aversion (CRRA) lifetime utility function

$$U(t) = \frac{1}{1-\rho} \left(X_C(t)^\zeta \left\{ L(t)[\theta(t) + H(t)] \right\}^{1-\zeta} \right)^{1-\rho}, \tag{5}$$

with ζ the "share" of consumption and $1-\zeta$ the "share" of leisure in utility, and $1/\rho$ the elasticity of substitution. Consumption $X_C(t)$ and "effective" leisure time $L(t)[\theta(t)+H(t)]$ are complements in utility if $\rho < 1$ and substitutes in utility for $\rho > 1$. Leisure time $L(t)$ is multiplied by $\theta(t) + H(t)$, reflecting the notion that human capital (consisting of the sum of skill and health capital, $\theta(t)+H(t)$) augments the agent's consumption time (Heckman, 1976).[10] The utility function is increasing in each of its arguments and strictly concave.

The objective function (4) is maximized subject to the following dynamic constraints for skill capital $\theta(t)$ and health capital $H(t)$:

$$\frac{\partial \theta}{\partial t} = F_\theta\,[.] = f_\theta\,[X_\theta(t), \tau_\theta(t), \theta(t), H(t)] - d_\theta(t)\theta(t), \tag{6}$$

$$\frac{\partial H}{\partial t} = F_H\,[.] = f_H\,[X_H(t), \tau_H(t), \theta(t), H(t)] - d_H(t)H(t). \tag{7}$$

Skill capital $\theta(t)$ (equation 6) and health capital $H(t)$ (equation 7) can be improved through investments in, respectively, skill capital $f_\theta[.]$ and health $f_H[.]$, and deteriorate at the biological deterioration rates $d_\theta(t)$ and $d_H(t)$. Goods and services $X_\theta(t)$, $X_H(t)$, purchased in the market and own time inputs $\tau_\theta(t)$, $\tau_H(t)$, are used in the production of skill capital $F_\theta[.]$ and health capital $F_H[.]$. The skill-capital $F_\theta[.]$ and health-capital $F_H[.]$ production processes are assumed to be increasing and strictly concave in the investment inputs $X_\theta(t)$, $\tau_\theta(t)$, and $X_H(t)$, $\tau_H(t)$, respectively.[11]

Earnings consist of the product of the wage rate $w(t)$ and the fraction of time available for work

$$Y[\theta(t), H(t)] = w[\theta(t), H(t)]\,[1 - \tau_\theta(t) - \tau_H(t) - L(t)]\,, \tag{8}$$

where the wage rate

$$w[\theta(t), H(t)] = [\theta(t) + H(t)]\,, \tag{9}$$

equals the sum of the stocks of human capital and health, and, last, the production functions of skill and of health are of a Cobb-Douglas form,

$$f_\theta[\tau_\theta(t), X_\theta(t), \theta(t), H(t)] = \psi_\theta(t)\,\{\tau_\theta(t)\,[\theta(t) + H(t)]\}^{\alpha_\theta}\,X_\theta^{\beta_\theta} \tag{10}$$

$$f_H[\tau_H(t), X_H(t), \theta(t), H(t)] = \psi_H(t)\,\{\tau_H(t)\,[\theta(t) + H(t)]\}^{\alpha_H}\,X_H^{\beta_H}, \tag{11}$$

where $\psi_\theta(t)$ and $\psi_H(t)$ denote the technologies of production of skill investments and health investments, respectively. The technologies of production can be considered as being determined by technology as well as biology.

The intertemporal budget constraint for assets $A(t)$ is given by

$$\frac{\partial A}{\partial t} = rA(t) + Y[t, \theta(t), H(t)] - p_C(t)X_C(t) - p_\theta(t)X_\theta(t) - p_H(t)X_H(t). \tag{12}$$

[10] Heckman (1976) motivates the assumption with the notion that those with higher stocks of human capital are more efficient users of their time.

[11] Concavity implies $\partial^2 F_\theta/\partial X_\theta^2 < 0$, $\partial^2 F_\theta/\partial \tau_\theta^2 < 0$, $\partial^2 F_H/\partial X_H^2 < 0$, $\partial^2 F_H/\partial \tau_H^2 < 0$, $\left(\partial^2 F_\theta/\partial X_\theta^2\right)\left(\partial^2 F_\theta/\partial \tau_\theta^2\right) > \left(\partial^2 F_\theta/\partial X_\theta \partial \tau_\theta\right)^2$ and $\left(\partial^2 F_H/\partial X_H^2\right)\left(\partial^2 F_H/\partial \tau_H^2\right) > \left(\partial^2 F_H/\partial X_H \partial \tau_H\right)^2$. The assumption of diminishing returns to investment (concavity) addresses the "bang-bang" nature of the solution for investment that results from the common assumption in the health-capital literature of constant returns to scale (see for a discussion Ehrlich and Chuma, 1990; Galama and Van Kippersluis, 2013; Galama, 2015).

Assets $A(t)$ (equation 12) provide a return r (the rate of return on capital) and increase with income $Y[\theta(t), H(t)]$. Assets decrease with expenditures on investment and consumption goods and services $X_\theta(t)$, $X_H(t)$ and $X_C(t)$, at prices $p_\theta(t)$, $p_H(t)$ and $p_C(t)$.

Thus, we have the following optimal control problem: the objective function (4) is maximized with respect to the control functions $X_C(t)$, $X_\theta(t)$, $X_H(t)$, $L(t)$, $\tau_\theta(t)$, $\tau_H(t)$, and T, subject to the constraints (6) to (12), and the following initial and end conditions: $H(0) = H_0$, $H(T) = H_T$, $\theta(0) = \theta_0$, $A(0) = A_0$, $A(T) = A_T$, and $\theta(T) \geq 0$ (not fixed). Length of life T (Grossman, 1972a,b) is determined by a minimum health level below which an individual dies: $H_T \equiv H_{\min}$.

The Hamiltonian (Seierstad and Sydsaeter, 1986; Caputo, 2005) of this problem is:

$$\Im \quad = \quad U[.]e^{-\beta t} + q_\theta(t)\frac{\partial \theta}{\partial t} + q_H(t)\frac{\partial H}{\partial t} + q_A(t)\frac{\partial A}{\partial t}, \tag{13}$$

where $q_\theta(t)$, $q_H(t)$, and $q_A(t)$ are the co-state variables associated with, respectively, the dynamic equations (6) for skill capital $\theta(t)$, (7) for health $H(t)$, and (12) for assets $A(t)$.[12]

The co-state variables $q_\theta(t)$, $q_H(t)$, and $q_A(t)$ find a natural economic interpretation in the following standard result from Pontryagin

$$q_Z(t) = \frac{\partial}{\partial Z(t)} \int_t^{T^*} U(*)e^{-\beta s} ds, \tag{14}$$

(e.g., Caputo, 2005, eq. 21, p. 86) with $Z(t) = \{\theta(t), H(t), A(t)\}$, and where T^* denotes optimal length of life and $U(*)$ denotes the maximized utility function (i.e., along the optimal paths for the controls, state functions, and for the optimal length of life). Thus, for example, $q_\theta(t)$ represents the marginal value of remaining lifetime utility (from t onward) derived from additional skill capital $\theta(t)$. We refer to the co-state functions as the "marginal value of skill", the "marginal value of health", and the "marginal value of wealth" (these are also often referred to in the literature as shadow prices).

Since skill capital $\theta(T)$ is non-negative but not fixed, the individual chooses it to have no value at the end of life, $q_\theta(T) = 0$ (Caputo, 2005). However, health capital $H(T)$ and assets $A(T)$ are constrained to their values $H_{\min}$ and A_T, respectively, and as a result their marginal values are positive at the end of life: $q_H(T) \geq 0$ and $q_A(T) \geq 0$.

The transversality condition for the optimal length of life T, follows from the dynamic envelope theorem (e.g., Caputo, 2005, p. 293):

$$\frac{\partial}{\partial T} \int_0^T \Im(t) dt \quad = \quad \Im(T) = 0, \tag{15}$$

[12] In specifying the Hamiltonian we have left out the multiplier $\lambda_{H_{\min}}(t)$ associated with the pure state constraint that $H(t) > H_{\min}$ for $t < T$ ($\lambda_{H_{\min}}(t) = 0$ if $H(t) > H_{\min}$ and $\lambda_{H_{\min}}(t) > 0$ if $H(t) = H_{\min}$). In practice, employing the condition entails restricting solutions to those where the constraint is not imposing/binding (i.e. $\lambda_{H_{\min}}(s) = 0$, $\forall s \in [t, T)$). The implication for the calibrated simulations is that we rule out solutions for which health initially falls below $H_{\min}$, then returns to $H_{\min}$ at $t = T$.

where $\Im(T)$ is the marginal value of life extension T (e.g., Caputo, 2005, Theorem 9.1, p. 232), and the age at which life extension no longer has value defines the optimal length of life T^*.

2.2 First-order conditions and interpretation

In this section we present and discuss the first-order and transversality conditions of the optimal-control problem discussed above. The first-order conditions determine the optimal solutions of the controls skill-capital investment, health-capital investment, consumption, and leisure time. Appendix A.1 provides detailed derivations.

Consumption and leisure: The first-order conditions for consumption and leisure are

$$\frac{1}{q_A(t)}\frac{\partial U}{\partial X_C} = p_C(t)e^{\beta t}, \tag{16}$$

$$\frac{1}{q_A(t)}\frac{\partial U}{\partial L} = w[\theta(t), H(t)]e^{\beta t}. \tag{17}$$

Consumption $X_C(t)$ and leisure time $L(t)$ increase with current wealth $A(t)$ under the standard assumption of diminishing returns to wealth $\partial q_A(t)/\partial A(t) < 0$ and with permanent income (the marginal value of wealth $q_A(t)$ decreases with permanent income).[13] Consumption and leisure decrease with their respective costs: the price of goods and services $p_C(t)$ (for consumption) and the opportunity cost of time $w[\theta(t), H(t)]$ (for leisure). The analytical solutions for consumption and leisure are given by (60) and (61), respectively.

Skill-capital investment: The first-order condition for purchases of skill-capital goods / services and own-time investments $X_\theta(t)$ and τ_θ are given by

$$q_{\theta/a}(t) = \pi_\theta(t), \tag{19}$$

which equates the marginal benefit of skill-capital investment, given by the ratio of the marginal value of skill capital to the marginal value of wealth $q_{\theta/a}(t) \equiv q_\theta(t)/q_A(t)$, to the marginal monetary cost of skill-capital investment $\pi_\theta(t)$. From hereon we refer to $q_{\theta/a}(t)$ as the *relative marginal value of skill*. It reflects the competition between the value of

[13] A natural and frequently made assumption is that financial capital (wealth) $A(t)$, skill capital $\theta(t)$, and health capital $H(t)$, increase remaining lifetime utility, but at a diminishing rate

$$\frac{\partial q_Z(t)}{\partial Z(t)} = \frac{\partial^2}{\partial Z(t)^2}\int_t^{T^*} U(*)e^{-\beta s}ds < 0, \tag{18}$$

with $Z(t) = \{\theta(t), H(t), A(t)\}$ (see 14). In practice, calibrated models exhibit this feature (e.g., Galama and Van Kippersluis, 2019). Further, if the opposite were true, wealthy individuals would consume and invest less. While theoretically such solutions are feasible, in practice they are generally not encountered.

having more skill $q_\theta(t)$, or of having more wealth $q_A(t)$, in terms of remaining life-time utility. Its solution is given by

$$q_{\theta/a}(t) = \int_t^T e^{-\int_t^s [d_\theta[x]+r]dx} \left(\frac{\partial Y}{\partial \theta}\right) ds, \tag{20}$$

and represents the lifetime production benefit of skill $\partial Y/\partial\theta$, discounted at the rate $d_\theta(t) + r$ (note that $\partial Y/\partial\theta = 1$).[14]

The marginal cost of skill-capital investment is defined as

$$\pi_\theta(t) \equiv \frac{p_\theta(t)}{\partial f_\theta/\partial X_\theta} = \frac{w[\theta(t), H(t)]}{\partial f_\theta/\partial \tau_\theta}. \tag{21}$$

It increases with the price of investment goods and services $p_\theta(t)$, and the opportunity cost of not working $w[\theta(t), H(t)]$, and decreases in the efficiency of the use of investment inputs in the skill-production process, $\partial f_\theta/\partial X_\theta$ and $\partial f_\theta/\partial \tau_\theta$.

In sum, the decision to invest in skill today (19) weighs the current monetary price and current opportunity cost of time (see 21) with its future benefits (from t to T): the discounted value of increased earnings (see 20). The analytical expressions for goods and services $X_\theta(t)$ and time inputs $\tau_\theta(t)$ devoted to skill are given by (41) and (42).

Health-capital investment Analogous to skill-capital investment, the first-order condition for health-capital investment is given by

$$q_{h/a}(t) \quad = \quad \pi_H(t), \tag{22}$$

where the *relative marginal value of health* $q_{h/a}(t)$ equals the ratio of the marginal value of health to the marginal value of wealth $q_{h/a}(t) \equiv q_H(t)/q_A(t)$, and $\pi_H(t)$ represents the marginal monetary cost of health-capital investment.

The relative marginal value of health is given by

$$\begin{aligned} q_{h/a}(t) = q_{h/a}(T) e^{-\int_t^T [d_H(x)+r]dx} \\ + \int_t^T e^{-\int_t^s [d_H[x]+r]dx} \left(\frac{\partial Y}{\partial H}\right) ds, \end{aligned} \tag{23}$$

and the marginal cost of health investment is defined as

$$\pi_H(t) \equiv \frac{p_H(t)}{\partial f_H/\partial X_H} = \frac{w[\theta(t), H(t)]}{\partial f_H/\partial \tau_H}. \tag{24}$$

[14] Note that skill capital also raises the efficiency of skill-capital production $\partial f_\theta/\partial\theta > 0$, and the efficiency of health-capital production $\partial f_H/\partial\theta > 0$, but due to the Ben-Porath "neutrality" functional-form assumptions the benefits in terms of self-production are exactly compensated by the increased market-production (opportunity cost of time) induced by higher skill.

Like skill capital, the benefits of health capital consist of the lifetime production benefit of health, $\partial Y/\partial H$ (note that $\partial Y/\partial H = 1$). Unlike skill capital, health at the end of life T is constrained to its end value $H(T) = H_{\min}$, the minimum level of health below which life is no longer sustainable (see footnote 12; Ehrlich and Chuma, 1990). Therefore, in contrast to skill, the relative marginal value of health $q_{h/a}(t)$ does not converge to zero at time T (compare equations 20 and 23).

The marginal cost $\pi_H(t)$ of health investment (see 24) increases with the price of goods and services in the market $p_H(t)$ and the opportunity cost of time $w[\theta(t), H(t)]$. It decreases in the efficiency of the use of investment inputs in the health production process, $\partial f_H/\partial X_H$ and $\partial f_H/\partial \tau_H$.

In sum, similar to investment in skill, the decision to invest in health today (22) weighs the current monetary price and current opportunity cost of time (see 24) with its future benefits (from t to T), consisting of increased earnings and a longer life (see 15 and 23). The analytical expressions for goods and services $X_H(t)$ and time inputs $\tau_H(t)$ devoted to health are given by (43) and (44).

2.3 Dynamics and differences between skill and health

Skill and health are considered to be the most critical components of human capital (Schultz, 1961; Grossman, 2000), and while they share the defining characteristic of human capital that investing in them makes individuals more productive, there are several important differences between them. In particular, skill capital and health capital have different initial and terminal conditions. Individuals begin life with limited skills and end life with various degrees of cognitive and mental fitness. This notion is captured in the skill-capital literature by an initial low level of skill θ_0 and an end value $\theta(T)$ that is, apart from being non-negative, unconstrained. Because there is no restriction on the terminal value of the stock of skill $\theta(T)$, it is chosen such that skill no longer has value at the end of life, $q_\theta(T) = 0$ (see Heckman, 1976; Caputo, 2005). Thus the relative marginal value of skill capital $q_{\theta/a}(t)$, and therefore investment in skill, decreases over the life-cycle and approaches zero at the end of life (see 20).

In contrast, most people start adult life with a healthy body, and for natural causes of death the terminal state of health is universally frail. The notion that health cannot be sustained below a certain minimum level is captured in the health-capital literature by the condition that death T occurs when health reaches the minimum level needed to sustain life $H(T) = H_{\min}$ (Grossman, 1972a,b). Health eventually deteriorates and because the terminal stock is restricted to $H_{\min}$, it cannot be chosen to have no value, $q_H(T) \geq 0$. This is captured by the extra term in $q_{h/a}(T)$ in the expression for the relative marginal value of health (23), which is not present in the expression for the relative marginal value of skill (20).

Hence, even with almost identical formulations (see sections 2.1 and 2.2), health and skill have distinct life-cycle profiles as a result of their different initial and end conditions. Skill capital is found to increase, at least initially (e.g., Becker, 1964; Ben-Porath, 1967), while health capital is found to decrease with age (e.g., Grossman, 1972a,b). Skill-capital investment is thus characterized by a production process that enables improvements in the stock of skill, while health-capital investment is characterized by a production process that (eventually) cannot prevent declining health, no matter how much one invests in it (a dismal fact of life). Further, Becker (1964) observes that investments in skill capital should decrease with age as the remaining period over which benefits can be accrued decreases, while investments in health tend to increase with age (e.g., Zweifel et al., 1999; Serdula et al., 2004; Podor and Halliday, 2012; Halliday et al., 2019).[15] This suggests that the relative marginal value of health $q_{h/a}(t)$ increases with age, while the relative marginal value of skill $q_{\theta/a}(t)$ decreases with age. Skill is valued early in life while health is valued later in life.

In a more general, less restrictive, theory (i.e., one that does not utilize the Ben-Porath neutrality assumption), one could imagine additional differences between the two components of human capital. As in Grossman (1972a,b), only health could have a consumption benefit $\partial U/\partial H > 0$ (where direct utility is derived from health), as opposed to a formulation where human capital also influences the marginal utility of leisure. Further, earnings might be better represented by a multiplicative process (as opposed to an additive sum of health and skill, as in 8) where skill mainly determines the wage rate per period, and health mainly determines the period over which the wage return is received (both within a day by reducing sick time, but also over the lifetime by lengthening working life). Finally, one could incorporate a separate schooling period that would be mainly, or exclusively, devoted to skill investment. In this way, there would be a clear conceptual distinction between "schooling" as a period of life devoted to the formation of skills, and "skill" as a stock that is produced and deteriorates over the life-cycle, two distinct concepts that are often treated as identical. See Galama and van Kippersluis (2015) and Galama et al. (2018) for sketches of a more general theory that is conceptually richer but does not permit the derivation of analytical solutions.

Our purpose here is to use a simple model to obtain sharp predictions that follow transparently from analytical solutions. In section 4 we also present a calibrated model, which permits assessing the robustness of our results to our admittedly restrictive functional-form assumptions ensuring Ben-Porath neutrality.

3 Model predictions

In this section we present comparative dynamic analyses, exploring heterogeneity in the life-cycle profiles of health and skill with respect to small variations in endowments. We start with an analysis of endowed wealth, health, and skill.

[15] Investments in health consist of, e.g., medical care, physical exercise, a healthy diet and a healthy lifestyle. Not all such components of health investment necessarily increase with age. For example, the lifecycle profile of exercise is relatively flat (Podor and Halliday, 2012). But medical expenditures (e.g., Zweifel et al., 1999) and intake of fruit and vegetables do increase with age (Serdula et al., 2004; Pearson et al., 2005), and smoking rates drop with age (DHHS, 2020).

Consider a generic control, state, or co-state function $g(t)$, and a generic variation δZ_0 in an initial condition or model parameter. The effect of the variation δZ_0 on the optimal path of $g(t)$ can be broken down into variation for fixed longevity T and variation due to the resulting change in the horizon T

$$\frac{\partial g(t)}{\partial Z_0} = \left.\frac{\partial g(t)}{\partial Z_0}\right|_T + \left.\frac{\partial g(t)}{\partial T}\right|_{Z_0} \frac{\partial T}{\partial Z_0}. \tag{25}$$

The comparative dynamic effects of a small perturbation in initial health δH_0, initial skill $\delta\theta_0$, and initial wealth δA_0, are summarized in Table 1. The Table is divided into 9 areas (cells), reflecting health, skill and wealth domains and variations with respect to initial health, skill and wealth (i.e., a 3 by 3 matrix). The diagonal reflects 'own effects' (e.g., the effect of variation in health on health over the life cycle), and the off-diagonal cells reflect 'cross effects' (e.g., the effect of variation in health on skill over the life cycle). We distinguish between two cases, one in which length of life is fixed (exogenous), and one in which length of life can be freely chosen (endogenous). Detailed derivations are provided in Appendix section A.2.[16]

Table 1: Comparative dynamic effects of initial health H_0, initial skill θ_0, and initial wealth A_0, on the state and co-state functions, control functions and the parameter T.

	δH_0		$\delta\theta_0$		δA_0	
Function	T fixed	T free	T fixed	T free	T fixed	T free
	1		2		3	
T	n/a	> 0	n/a	> 0	n/a	> 0
$H(t)$	> 0	> 0	0	> 0	0	> 0
$X_H(t)$	< 0	+/-	0	> 0	0	> 0
$\tau_H(t)\,[\theta(t) + H(t)]$	< 0	+/-	0	> 0	0	> 0
	4		5		6	
$\theta(t)$	0	> 0	> 0	> 0	0	> 0
$X_\theta(t)$	0	> 0	0	> 0	0	> 0
$\tau_\theta(t)\,[\theta(t) + H(t)]$	0	> 0	0	> 0	0	> 0
	7		8		9	
$A(t)$	+/-	+/-	+/-	+/-	≥ 0	+/-
$X_C(t)$	> 0	$> 0^\dagger$	> 0	$> 0^\dagger$	> 0	$> 0^\dagger$
$L(t)\,[\theta(t) + H(t)]$	> 0	$> 0^\dagger$	> 0	$> 0^\dagger$	> 0	$> 0^\dagger$

Notes: 0 is used to denote 'not affected', $+/-$ is used to denote that the sign is 'undetermined', n/a stands for 'not applicable', and † is used to denote that 'the sign holds under the plausible assumption that the wealth effect dominates the effect of life extension'. This is consistent with the empirical finding (Imbens et al., 2001; Juster et al., 2006; Brown et al., 2010) that additional wealth leads to higher consumption, even though the horizon over which consumption takes place is extended (see section A.2 for further detail).

[16]See equations (74), (75), and (76) for initial wealth A_0, equations (77) to (81) for initial skill θ_0, and equations (83) to (87) for initial health H_0.

Consider first the last column. Absent ability to increase the horizon over which benefits can be accrued (fixed length of life T), additional wealth δA_0 does not lead to more health investment and skill investment, leaving skill and health unchanged (cells 3 and 6 of Table 1, for T fixed, equation 74). The additional wealth is solely used to finance additional consumption and leisure (cell 9, for T fixed).

Both skill capital and health capital are forms of wealth, in the sense that they increase wages and therefore lifetime wealth (reducing the marginal value of initial wealth $q_A(0)$). Thus a positive variation in health δH_0 and skill $\delta \theta_0$ operates in a manner similar to a positive variation in wealth δA_0 (see cells 1, 4, and 7 for δH_0 and cells 2, 5 and 8 for $\delta \theta_0$ in Table 1, for T fixed, equations 78 and 83). There are however a few differences: greater endowed health δH_0 leads to greater health (cell 1, for T fixed; except at $t = T$ when it equals $H_{\min}$ by construction), and greater endowed skill $\delta \theta_0$ leads to greater skill (cell 5, for T fixed). These positive effects of initial health on health and initial skill on skill reflect nothing more than the fact that individuals started out with higher initial stocks to begin with (own effects). Importantly, also for additional health δH_0 (cells 1 and 4) and skill $\delta \theta_0$ (cells 2 and 5) there are no additional investments made, and for additional health, health investments $X_H(t), \tau_H(t)$ (cell 1, for T fixed) are even reduced (necessary to ensure death occurs at the same [fixed] age of death). Here too, the additional health and skill are solely used to finance additional consumption and leisure (cells 7 and 8, for T fixed). Thus, absent ability to extend life T, the causal effect of endowments on skill and health are absent or small.

The lack of an effect of endowed wealth on skill (and in our case also health) in the Ben-Porath model has been noted before (Heckman, 1976).[17] It arises because length of life (fixed in the traditional human-capital literature) is a crucial determinant of the return to investments. The intuition is straightforward for health investment. For fixed length of life, any additional health needs to be "dissipated" by reducing health investment in order for health to reach $H_{\min}$ at the unchanged (fixed) age of death $t = T$ (explaining the reduction in health investment). The intuition for skill investment is that for fixed length of life only marginal gains in lifetime utility can be made as opposed to absolute gains in lifetime utility from life extension. Thus, without the ability to extend life, the gains from skill investment are relatively small (see the next paragraph for more detail). The response to additional resources is therefore muted. As a result, there is no causal effect of endowments on health or skill for exogenous T, and $\partial g(t)/\partial Z_0|_T$, the first term on the RHS of (25), is generally small for variation δZ_0 in any model parameter of interest.

Now consider the case where T is free. Table 1 shows that positive variations in endowments, in the form of health, skill or wealth, lead to a longer life span (cells 1, 2 and 3). For variation in initial wealth A_0, the intuition is as follows. At high values of wealth (and hence consumption and leisure), additional consumption or leisure per

[17] Levhari and Weiss (1974) also make the observation in a simple two-period human-capital model with uncertainty but do not explicitly derive the result. Graham (1981) suggests the lack of an effect is due to the fact that in the Ben-Porath model individuals maximize lifetime earnings, and not utility. We find, however, that it also holds for a model in which individuals maximize lifetime utility.

period yields only limited utility due to diminishing marginal utility of consumption and leisure. In contrast, investments in health extend life, increasing the period over which one can enjoy the full (as opposed to marginal) utility benefits of leisure and consumption (Hall and Jones, 2007). As a result, with sufficient wealth one starts caring more about health as it extends life. A similar reasoning can be applied to variations in initial skill and initial health, as both skill and health increase earnings and having greater skill and health relaxes the dynamic constraints (equations 6, 7 and 12). Wealthy, skilled, and healthy individuals invest more in health and live longer because the returns to health investments (in terms of life-time utility) are large.

Taken together, we obtain the following novel predictions:

Prediction 1a: Endogenous gains in longevity are a *necessary* condition for persistent *causal* relations between wealth, skill and health and the investments in them.

Gains in longevity play a powerful role in generating causal effects of endowments on the human capital stocks of skill and health. Consider again equation (25) and note that we can restart the problem at any time t', taking $H(t') = H_{t'}$, $\theta(t') = \theta_{t'}$, and $A(t') = A_{t'}$, as the new initial conditions. In this way one arrives at the following more general result

$$\frac{\partial g(t)}{\partial X_{t'}} = \left.\frac{\partial g(t)}{\partial X_{t'}}\right|_T + \left.\frac{\partial g(t)}{\partial T}\right|_{X_{t'}} \frac{\partial T}{\partial X_{t'}}. \tag{26}$$

Formally, prediction 1a states that *causal* effects of health, skill and wealth on later-life skill, and health, are small (i.e., fade out) when longevity can *not* be influenced by an individual's decisions / behavior. Technically, this is when $\partial T/\partial X_{t'}$ is small, where $X_{t'} = \{H_{t'}, \theta_{t'}, A_{t'}\}$. This suggests, that causal effects of health, skill and wealth on later-life skill, and health only occur in contexts / settings where longevity gains are at least in part the result of individual-decision making (e.g., through health behaviors, medical care, etc.) This is when $\partial T/\partial X_{t'}$ is substantial, or at least non-negligible.

Equation (26) illustrates this. We previously obtained the result that the first term on the RHS $\partial g(t)/\partial X_{t'}|_T$ is generally small for $g(t) = \{\theta(t), H(t)\}$, and for variation $\delta X_{t'}$ in any model parameter of interest: additions (or reductions) in resources do not change investment in skill and health much for fixed (exogenous) T. Thus, the size of the effect of $X_{t'}$ on $g(t)$ depends on the sign and size of $\partial g(t)/\partial T|_{X_{t'}}$ and increases with the degree of life extension $\partial T/\partial X_{t'}$.

If resources, biology, medical technology, institutional, environmental and/or other factors, do not allow for individuals to influence their length of life ($\partial T/\partial X_{t'}$ small), then the effect closely resembles that of the exogenous T case. As in the exogenous T case, there is no, or a muted, causal effect of endowments on skill and health investments (small $\partial g(t)/\partial X_{t'}$). In contrast, if additional resources afford considerable life extension

($\partial T/\partial X_{t'}$ large), the horizon over which the benefits of skill investment and health investments can be reaped is larger, and utility from leisure and consumption can be enjoyed with additional years of life. These benefits of life extension (adding levels of utility per additional period, as opposed to marginal benefits during existing periods) can substantially raise investment in skill and in health, thereby improving skill and health. Thus, endogenous longevity gains are a *necessary* condition for a *causal* effect of health, skill and wealth, on human-capital investments and thereby on later-life skill, health and longevity.

Prediction 1b: Endogenous gains in longevity are a necessary condition for persistent *self-productivity* and *dynamic complementarity* in skill and health.

Table 1 presents the comparative dynamics in terms of endowments, i.e., the initial levels of the stocks H_0 and θ_0. Yet, as mentioned before, we can restart the problem at any time t', taking $H(t')$ and $\theta(t')$ as the new initial conditions. Thus the comparative dynamic results have greater validity, and can shed light on two defining features of human-capital investments: self-productivity, where skills produced at one stage augment skills at later stages, and dynamic complementarity, where skills produced at one stage raise investment at later stages. Focusing once more on the example of skill, self-productivity can be self-reinforcing $\partial\theta(t)/\partial\theta(t') > 0$, and/or cross-fertilizing, $\partial\theta(t)/\partial H(t') > 0$, with $t' < t$. Dynamic complementarity too can be self-reinforcing, $\partial^2\theta(t)/\partial\theta(t'')\partial X_\theta(t') > 0$, $\partial^2\theta(t)/\partial\theta(t'')\partial\tau_\theta(t') > 0$, and/or cross-fertilizing, $\partial^2\theta(t)/\partial H(t'')\partial X_\theta(t') > 0$, $\partial^2\theta(t)/\partial H(t'')\partial\tau_\theta(t') > 0$ with $t'' \leq t' < t$.

Self-productivity: Table 1 and Appendix A.3 show that for exogenous longevity T there exists self-reinforcing self-productivity of health and skill, $\partial H(t)/\partial H(t') > 0$ (cell 1, for T fixed) and $\partial\theta(t)/\partial\theta(t') > 0$ (cell 5, for T fixed), but this purely derives from what Cunha and Heckman (2007b, p. 15) call a "carry-over" effect of non-depreciated human capital (see 88 and 89). Positive additions are added to the stock and depreciate over time, but do not enhance production. Further, there is no cross-fertilizing self-productivity $\partial H(t)/\partial\theta(t')|_T = \partial\theta(t)/\partial H(t')|_T = 0$ (cells 2 and 4, for T fixed). Hence, in a world with exogenous longevity T there is no self-productivity in terms of enhanced investment (only depreciation of the stock), and cross-fertilizing self-productivity does not exist. In sharp contrast, when longevity is endogenous, self-reinforcing (cells 1 and 5, for T free) and cross-fertilizing self-productivity (cells 2 and 4, for T free) all are apparent.

Dynamic complementarity: Likewise, for exogenous longevity, both self-reinforcing and cross-fertilizing dynamic complementarity do not exist for skill capital $\partial^2\theta(t)/\partial\theta(t')\partial I_\theta(t)\big|_T = \partial^2\theta(t)/\partial H(t')\partial I_\theta(t)\big|_T = 0$, where I_θ stands for both goods X_θ and time τ_θ investments in skill (see 95 and 96). For health capital, cross-fertilizing dynamic complementarity does not exist either, $\partial^2 H(t)/\partial\theta(t')\partial I_H(t)\big|_T = 0$, where I_H stands for both goods X_H and time τ_H investments in health (see 97). However, self-reinforcing dynamic complementarity for health depends on the parameter $\gamma_H = \alpha_H + \beta_H$ being smaller or larger than 0.5 (see 98). The Grossman model (Grossman, 1972b) assumes constant returns to scale in health investment (i.e. $\gamma_H = 1$). Recent studies estimate the parameter to be in the range 0.73 to 0.77 (Hugonnier et al., 2013, 2020), suggesting decreasing returns to scale (i.e. $\gamma_H < 1$). For constant returns, or for decreasing returns to scale in the range estimated in the literature, dynamic complementarity for health does not exist for exogenous longevity and in fact is even reversed: health investments are lower for higher initial levels of the stock. The intuition is that, for a fixed duration of life, a higher level of health requires reductions in health investment, such that one reaches the same minimum level of health over the same duration of life T. In sharp contrast, dynamic complementarity is present when individuals can extend their lives (see 81 and 87). Thus, endogenous longevity is a *necessary* condition for both self-productivity and dynamic complementarity in human-capital formation. Their presence is essential for the existence of human capability-formation processes by which health begets health and skill and by which skill begets health and skill (Cunha and Heckman, 2007a; Caucutt and Lochner, 2020).

Whereas these results depend on the assumption of Ben-Porath neutrality, necessary to arrive at analytically tractable solutions, they do apply more generally. We demonstrate this in section 4 through simulations of a more general model that drops each of three assumptions that underlie Ben-Porath neutrality and which we calibrated using empirical data to reflect reasonably realistic behavior.

Prediction 2: *Endogenous* gains in longevity are not a *sufficient* condition for *causal* effects among wealth, skill and health, and, related, for *self-productivity* and *dynamic complementarity* in skill and health.

Equation (26) illustrates the crucial role endogenous longevity gains play in investments in skill and health. However, endogenous longevity is not a sufficient condition for causal effects among wealth, skill and health, or (related) for self-productivity and dynamic complementarity. The size of the effect of $X_{t'}$ on $g(t)$ depends on the sign and size of $\partial g(t)/\partial T|_{X_{t'}}$ and increases with the degree of life extension $\partial T/\partial X_{t'}$ (second term in 26). But, even when endogenous gains in longevity are substantial, $\partial T/\partial X_{t'} > 0$, we still require $\partial g(t)/\partial T|_{X_{t'}}$ to be non-negligible for $g(t) = \{\theta(t), H(t)\}$ (see 73). Thus, while life may be extended, this does not necessarily imply that the returns to skill and health $g(t) = \{\theta(t), H(t)\}$, derived from additional longevity $\partial g(t)/\partial T|_{X_{t'}}$ are sufficiently large. This depends on institutions, labor-markets, biology, medical technology, the quality of schooling, the returns to skill investment, etc.

To gain intuition for this prediction, consider the effect of life expectancy T on skill capital $\theta(t)$ (73 in Appendix A.2), reproduced here for convenience

$$\frac{\partial \theta(t)}{\partial T}\bigg|_{Z_0} = \frac{\gamma_\theta}{1-\gamma_\theta}\int_0^t \mu_\theta(s) q_{\theta/a}(s)^{\frac{2\gamma_\theta-1}{1-\gamma_\theta}} \frac{\partial q_{\theta/a}(s)}{\partial T}\bigg|_{Z_0} e^{-\int_s^t d_\theta(x)dx} ds.$$

The effect of longevity on skill has attracted much attention in both the theoretical and empirical literatures (e.g., Ben-Porath, 1967; Hazan, 2009; Jayachandran and Lleras-Muney, 2009; Fortson, 2011; Oster et al., 2013).[18] The expression shows that it is reinforced by a higher productivity of skill-capital investment $\mu_\theta(t)$ and by a higher relative marginal value of skill $q_{\theta/a}(t)$. The generalized productivity factor $\mu_\theta(t)$ (see 45) increases in the efficiency of skill production $\psi_\theta(t)$ (see 10) and decreases in the price $p_\theta(t)$ of skill-capital investment goods and services $X_\theta(t)$. The relative marginal value of skill $q_{\theta/a}(t)$ captures the remaining life-time utility returns to skill, and depends on the resources at the individual's disposal, technology, institutions and markets (see equation 20). For example, if the demand for skilled labor is high (high earnings $\partial Y/\partial\theta$ for the skilled, existence of high-tech sectors, etc), then investing in skill has value. An implication is that responses to longevity gains are predicted to be small if the returns to education are small (e.g., in a society where an extractive elite controls the nation's wealth; Deaton, 2013), and they would be large in societies where skill investment is productive and affordable, and the institutional environment is favorable.

Comparison with exogenous longevity gains: Consider a hypothetical world, where gains in longevity are outside of the control of individuals, i.e., exogenous (e.g., clean water technologies, introduction of antibiotics).[19] Would the existence of exogenous trends in longevity change our predictions (1a), (1b) and (2)?

First, if the agent anticipates such trends, then the (rational) agent would incorporate this information in her forecast. There would not be any adjustments made in response to the longevity gains as the information is (at least roughly) already incorporated in the forecast. We are back to the world described before and represented by equations (1) and (26). The Ben-Porath mechanism would be operational, and as progressive cohorts live longer lives, their educational investments, and thereby earnings and wealth, grow over time. However, the relations between wealth, skill and health would reflect associations, operating through the causal effect of longevity. They would not reflect causal effects among wealth, skill and health. Thus our conclusions remain unchanged: endogenous longevity is a necessary condition for causal relations among wealth, skill and health.

Second, what if the longevity gains are unanticipated? In the context of a model with endogenous longevity, an exogenous shock to longevity could be represented by variation in, e.g., the productivity of health investment $\mu_H(s)$ at some time s, as by definition we

[18]The effect of longevity on skill $\partial\theta(t)/\partial T$ is quite general. As (25) shows, for $g(t) = \theta(t)$ and since $\partial g(t)/\partial Z_0|_T$ is small, $\partial\theta(t)/\partial T$ encompasses the effect of any model parameter Z_0 on skill $\theta(t)$, provided that Z_0 affects longevity T.

[19]Improvements in life expectancy in the first half of the 20^{th} century were arguably of this kind (Cutler and Miller, 2005; Catillon et al., 2018).

cannot change longevity directly since it is an endogenous variable in the model. Assume a purely exogenous and instantaneous medical improvement that happens at time s (e.g., development of a new vaccine), represented by a jump in the efficiency of health production $\delta\mu_H(t)$ at time s (a jump to a higher level), and consider once again the effect of exogenous variation in health $\delta H_{t'}$ at time t' on later-life skill $\theta(t)$ ($t' < t$) as in equation (1):

$$\begin{aligned} & \frac{1}{\delta\mu_H(s)}\left[\frac{\partial\theta[t;\mu_H(s)+\delta\mu_H(s)]}{\partial H_{t'}} - \frac{\partial\theta[t;\mu_H(s)]}{\partial H_{t'}}\right] = \frac{\partial^2\theta(t)}{\partial\mu_H(s)\partial H_{t'}} = \frac{\partial[Equation\ (1)]}{\partial\mu_H(s)} \\ = \; & \left.\frac{\partial^2\theta(t)}{\partial\mu_H(s)\partial H_{t'}}\right|_T + \left.\frac{\partial^2\theta(t)}{\partial\mu_H(s)\partial T}\right|_{H_{t'}}\frac{\partial T}{\partial H_{t'}} + \left.\frac{\partial\theta(t)}{\partial T}\right|_{H_{t'}}\frac{\partial^2 T}{\partial\mu_H(s)\partial H_{t'}}. \end{aligned} \tag{27}$$

Once more, and for the same reasons as discussed before, the first term on the second line of (27) is small for fixed T as forcing individuals to die at a fixed age entails dissipation of effects (see prediction 1a). For endowed health $H_{t'}$ to causally affect skill, endogenous gains in longevity $\partial T/\partial H_{t'}$, $\partial^2 T/\partial\mu_H(s)\partial H_{t'}$, are a necessity (the remaining terms on the RHS of equation 27). Our conclusions thus remain the same. Endogenous gains in longevity are a necessary condition for a causal effect of greater health on skill. Similar reasoning holds for the effects of wealth, skill and health on later-life wealth, skill and health, and for other exogenous changes that may affect longevity besides the one, used in the example, that operates through the efficiency of health production $\mu_H(s)$.

4 Generalized and calibrated model

The convenient "Ben-Porath neutrality" assumptions of our stylized model (section 2) enabled the derivation of analytic solutions, thereby greatly facilitating comparative dynamic analyses and generating predictions regarding the effects of endowments on human-capital outcomes. However, these assumptions come at the cost of some realism. To ensure that model predictions are not artifacts of the assumptions made, we discuss in this section a more general theory, which we calibrate with empirical data. The aim is not to estimate a structural model, but more modestly to verify our main predictions in a model that better matches the data moments, i.e., reflects reasonably realistic behavior, and that makes less stringent assumptions. In what follows, we discuss the key assumptions of the analytical model, introduce a more general model inspired by Scholz and Seshadri (2016) and Galama and Van Kippersluis (2019), discuss the calibration, and demonstrate that our main predictions also hold for more realistic models of skill- and health-capital formation.

4.1 Towards an extended theory

Arguably the most important assumption of our model is the so-called "Ben-Porath neutrality" assumption: (i) all time inputs (leisure, investments) are entered multiplicatively with the stocks (referred to as "effective time"), (ii) the two stocks of human capital are entered linearly and additively, and (iii) both the utility and production functions combine "effective" time inputs with goods in a Cobb-Douglas manner. These assumptions conveniently ensure that the marginal value of skill $q_\theta(t)$ and of health $q_H(t)$ are no longer functions of the stock of skill and health, and as a result we can investigate the model analytically. This special case is referred to as Ben-Porath neutrality and each of the three sub-assumptions need to hold to ensure it.

As a result of these assumptions, health and skill are effectively modelled as being perfect substitutes. They enter the utility function and the budget constraint in identical ways, they operate additively in producing earnings and "effective" time inputs, and their production functions are of a Cobb-Douglas form. To some degree, we believe this is a strength. The only difference between the two stocks of human capital is that length-of-life is determined by health, not by skill. We follow the Grossman model for which this is the first moment at which health reaches its minimum level $H_{\min}$, below which life is no longer sustainable. Health and skill are therefore not perfectly substitutable since health determines length of life and skill does not. Their end conditions also differ: it is unconstrained for skill and constrained to $H_{\min}$ for health. This "simple" model already creates distinct life-cycle profiles for the stocks of skill and health (see section 2.3).

Still, these assumptions are simplifications that bear little resemblance to reality. Health and skill are not perfect substitutes. For example, earnings are more likely to be multiplicative in skill and health as skill arguably increases wages and health promotes time spent working (e.g., Currie and Madrian, 1999). Also, the assumption that the rate at which health depreciates increases with the stock of health – those with higher levels of the stock experience a larger health decline, all else equal – has been criticized (McFadden, 2005; Dalgaard and Strulik, 2014). For example, it implies that health shocks early in life die out as one ages, while empirical evidence suggests the opposite (Almond and Currie, 2011; Dalgaard et al., 2021). In a more general model, therefore, the marginal values of skill and health would be functions of the stocks of skill and health. Would that affect our main findings?

Next, we discuss the calibrated model in which we relax all three sub-assumptions.

4.2 Model formulation and calibration

To facilitate the calibration, we resort to discrete time. Skill capital is typically not observed, and is often modelled as a linear function of years of education (e.g., Boucekkine et al., 2002; Hazan, 2009). We follow the literature and use years of education S as a measure of skill capital in the calibrated model. Individuals maximize the life-time utility function

$$\sum_{t=0}^{S-1}\Big[\frac{U(C_t,H_t)}{(1+\beta)^t}-p^S\Big]+\sum_{t=S}^{T-1}\frac{U(C_t,H_t)}{(1+\beta)^t}, \tag{28}$$

where $t = 0$ represents the minimum-school leaving age 16 (as in Card, 2001; Hai and Heckman, 2019), individuals live for T (endogenous) periods, S is the (endogenous) number of years of schooling individuals choose beyond what is compulsory, β is a discount factor, and individuals derive utility $U(C_t, H_t)$ from consumption C_t and from health H_t. During schooling years, individuals face a psychic cost of schooling p^S (Boucekkine et al., 2002; Heckman et al., 2006; Sanchez-Romero et al., 2016).

We follow Scholz and Seshadri (2016) in specifying the utility function as:

$$U[C_t, H_t] = \frac{1}{1-\rho}\left[\lambda C_t^{\zeta} + (1-\lambda)H_t^{\zeta}\right]^{\frac{1-\rho}{\zeta}} + B, \tag{29}$$

where ρ is the coefficient of relative risk aversion, λ is a measure of the relative importance of consumption versus health in utility, $1/(1-\zeta)$ is the elasticity of substitution between consumption and health, and B is a constant to ensure utility is positive (Hall and Jones, 2007). The chosen utility function is a departure from assumption (iii) and ensures that for plausible parameter values, consumption and health are complements in utility, in line with empirical evidence (e.g., Finkelstein et al., 2013).

The objective function (28) is maximized subject to the dynamic constraints:

$$\begin{aligned}
H_{t+1} - H_t &= \mu_I(t,S)m_t^{\alpha} - d_t H_t^{\nu}, & 0 < t < T-1 \quad (30)\\
A_{t+1} - A_t &= rA_t - p_C C_t - p_m m_t + c_t^S, & 0 < t < S-1 \quad (31)\\
A_{t+1} - A_t &= rA_t + \gamma_w w_t(S, t-S)H(t)^{\gamma_H} - p_C C_t - p_m m_t. & S < t < T-1 \quad (32)
\end{aligned}$$

Health production (30) increases with health investment m_t at a diminishing rate, $0 < \alpha < 1$ (a departure from assumption ii). Further, the model does not contain time inputs (a departure from assumptions i and iii). The efficiency of health investment $\mu_I(t,S)$ is a function of years of schooling S, as in Grossman (1972b). We assume a simple linear form

$$\begin{aligned}
\mu_I(t,S) &= \kappa t, & 0 < t < S-1 \quad (33)\\
\mu_I(t,S) &= \kappa S, & S < t < T-1 \quad (34)
\end{aligned}$$

capturing the notion that skill (knowledge) accumulates gradually during schooling. Health decline is modelled flexibly to capture various possible relationships between the deterioration rate and health. For example, $\nu = 1$ represents the Grossman (1972b) case, while $\nu < 0$ is akin to the health-deficit model of Dalgaard and Strulik (2014), in which the health of unhealthy individuals deteriorates faster (addressing the critique that the rate of health deterioration is greater for the healthy). Health deterioration also depends in a flexible way on age t (as in Cropper, 1981; Wagstaff, 1986; Pelgrin and St-Amour, 2016; Galama and Van Kippersluis, 2019; Halliday et al., 2019; Fonseca et al., 2020; Lleras-Muney and Moreau, 2020), with

$$d_t = at + bt^2. \tag{35}$$

The asset equations (31) and (32) are similar to those of our stylized "Ben-Porath" model, with two exceptions. First, c_t^S represents the net balance of parental transfers and tuition fees. Parents pay the costs of living $p_C C_t$ (consumption) during schooling years when the child is still at home, set at \$12,358 (based on PSID consumption levels, following Andreski et al., 2014, of those aged 15-19 in 1999). In college, students also spend \$4,000 dollars on tuition (the average yearly college tuition fee; Ma and Baum, 2016). Second, we follow Scholz and Seshadri (2016) in specifying earnings as a function of health

$$Y_t = \gamma_w w_t H_t^{\gamma_H}, \tag{36}$$

with wages w_t given by a standard Mincer equation (see Appendix A.5). This represents a departure from assumption ii. Initial and end conditions H_0, H_T, A_0 and A_T are given, and life cannot be sustained below a minimum health level $H_{\min}$.

The first-order conditions and model solutions are presented in Appendix A.4. Details on the calibration of the model are presented in Appendix A.5. The calibration exercise is based on U.S. males around the year 2000, and the resulting life-cycle profiles for health and health investment are shown in Figure 1.

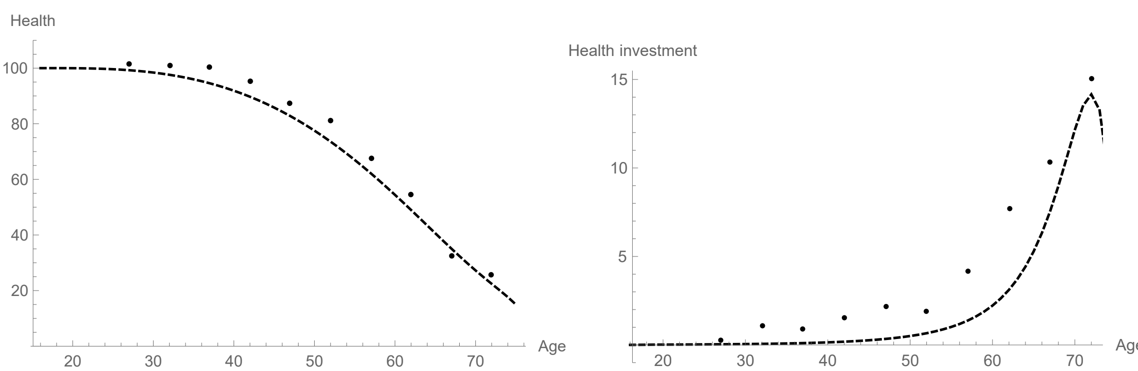

Figure 1: *Health and health investment, data moments (dots) and fitted profile (dashed lines).*

Our calibrated model reproduces the life-cycle profiles and targeted means of health and health investment quite well (see Figure 1). Health declines slowly until middle-age, after which it drops more rapidly, in line with the health moments. Health investment is low up to ages 55-60, after which it increases rapidly, in line with the health investment moments.[20] Even though not calibrated, the evolution of assets and consumption over the life cycle is reasonable (see Figure 12 in Appendix A.5). Individuals incur some debt early in life (tuition fees, mortgages), then build up assets, which peak around the age of retirement and are subsequently drawn down. As in any life-cycle model, consumption is predicted to be smoother than what most empirical data show, but the average level of consumption is targeted reasonably well (\$25,206 average over the 1999 and 2001 PSID

[20]In the final years of life, health investment levels off a bit while our empirical moments do not. We reran our calibration including data moments in the final year of life to better match the data moments in that period. The calibration accurately matched the increasing expenditures in the final years of life, but did less well in middle age, and the overall model fit was inferior. Reassuringly, none of our conclusions are affected (results are available upon request).

versus \$27,483 in our calibration) and our model reproduces a reduction in consumption towards the end-of-life.

Figure 2 shows the indirect utility function for different values of S and T, in three dimensions (left-panel) and in a two-dimensional contour plot (right-panel).[21] The indirect utility function produces an optimum at $S = 13$ and $T = 75$, exactly matching the schooling duration and life expectancy of the average U.S. male high-school graduate around 2000. Moreover, while the model is calibrated upon the average high-school graduate only, our 'local' optima in longevity for different years of schooling reproduces the empirically observed gradient between years of schooling and life expectancy (e.g., Sasson, 2016): $(S = 11, T = 72)$; $(S = 13, T = 75)$; and $(S = 16, T = 76)$.[22] Overall, the calibrated model captures some general features of real-world behavior. We will use it to re-examine the predictions from our stylized theory.

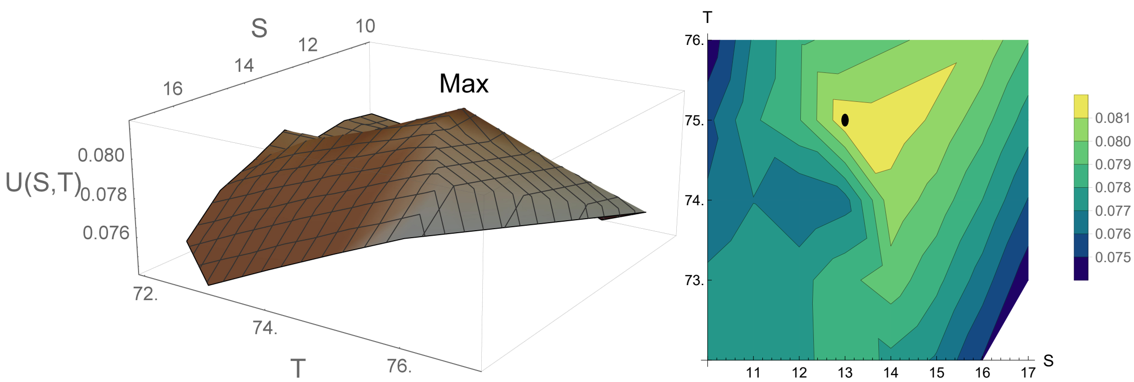

Figure 2: *Indirect utility function over S and T in 3D (left-panel) and in a contour 2D format (right-panel). The optimum in the contour plot is indicated by the black circle.*

4.3 Simulations and predictions

To test whether the predictions of the stylized Ben-Porath model in Table 1 also hold in our more general calibrated model, we conduct simulations for each of the health, skill and wealth domains and each of the variations in initial health, skill and wealth (the nine cells of Table 1). We find that the results of the calibrated simulations and the comparative dynamics of the stylized Ben-Porath model are qualitatively the same, suggesting our results are not an artefact of Ben-Porath neutrality.

[21]Not all model solutions were attainable. For example, in the bottom-right corner, for years of schooling of $S = 17$, a potential solution with a life expectancy of $T = 72$ did not converge.

[22]By local optima we mean the value of life expectancy T that provides the highest indirect utility when holding schooling constant at a given level S (i.e., the highest indirect utility for each value on the x-axis in Figure 2).

4.3.1 Variation in δH_0 and life-cycle choices

We start by considering the first column of Table 1, i.e., life cycle responses to variation in initial health δH_0.

$\boldsymbol{\partial S(t)/\partial H_0}$ (cell 4 of Table 1): A key prediction of our stylized theoretical model is that endogenous longevity gains are a necessary condition for the causal effect of endowments on human capital investments. Using the example from the introduction of the effect of health on skill capital, and translating the prediction to fit our calibrated model by using years of schooling S as our measure of skill $\theta(t)$, we have:

$$\left.\frac{\partial S}{\partial H_0}\right|_T \sim 0$$
$$\frac{\partial S}{\partial H_0} = \left.\frac{\partial S}{\partial H_0}\right|_T + \left.\frac{\partial S}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} > 0. \tag{37}$$

We test the first part of this prediction by holding life expectancy fixed at $T = 75$ years and simulating responses in years of schooling S to variations in initial health H_0 of 5% and 20% (i.e., changing H_0 from 100 to 105 and to 120, respectively). As Figure 3 shows, even though lifetime utility increases for a 5% and 20% increase in initial health H_0, the optimal level of schooling S remains at $S = 13$ years. Thus, $\partial S/\partial H_0|_T \sim 0$. In a world of exogenously given longevity, years of schooling S (a measure of skill formation) does not respond to variations in health H_0 (cell 4 of Table 1 for T fixed).

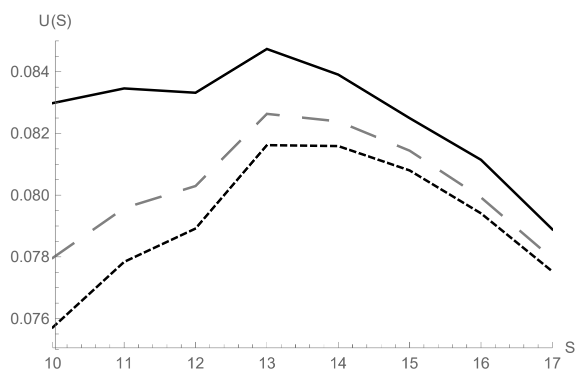

Figure 3: *Indirect utility function $U(S)$ as a function of years of schooling S, for a fixed life expectancy of $T = 75$ years. $H_0 = 100$ is the baseline case (short dash), and $H_0 = 105$ (long dash) and $H_0 = 120$ (solid), represent increases in initial health H_0 of 5 and 20 percent, respectively.*

We test the second part of the prediction by simulating responses in years of schooling S to variations in initial health H_0, but now allowing individuals to optimally choose both S and T simultaneously. Figure 4 presents a contour plot of the indirect utility function for the baseline case ($H_0 = 100$; left panel) and for variation in H_0 of 20% ($H_0 = 120$; right panel).[23] For a marginal increase in initial health by 5% (i.e., from $H_0 = 100$ to $H_0 = 105$;

[23]Note that the grid is not always filled entirely, since certain model solutions are not feasible. For

not shown), the optimum in lifetime utility switches from the point $(S = 13, T = 75)$ to $(S = 13, T = 77)$, i.e., years of schooling S is unchanged but longevity T increases. With a more substantial variation in initial health of 20% (i.e., from $H_0 = 100$ to $H_0 = 120$), the optimum in lifetime utility switches from the point $(S = 13, T = 75)$ to $(S = 14, T = 79)$ (right-panel of Figure 4), i.e., both years of schooling S and longevity T increase.

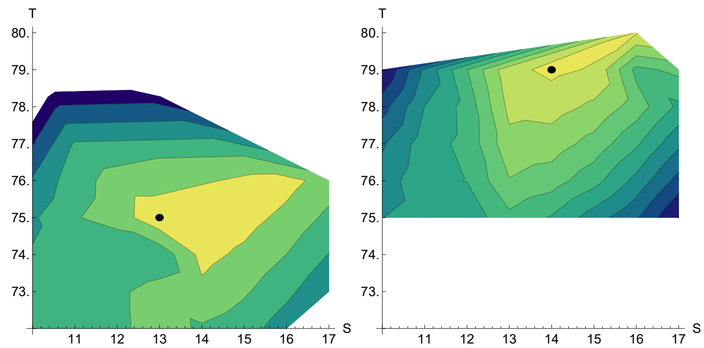

Figure 4: *Left: Contour plot of the indirect utility function $U(S, T)$ by years of schooling S and life expectancy T for the baseline model ($H_0 = 100$). Right: Contour plot of $U(S, T)$ for $H_0 = 120$. Lighter colors indicate higher values for the indirect utility function, and the maximum is indicated by the black circle.*

Now compare this with Table 1, with years of schooling S as our measure of skill $\theta(t)$. In our stylized Ben-Porath model, skill and skill investment are also unchanged for fixed T in response to variation in initial health and they also increase for free T, i.e., when longevity can be optimally chosen (cell 4 in Table 1).

$\boldsymbol{\partial H(t)/\partial H_0}$ **and** $\boldsymbol{\partial A(t)/\partial H_0}$ (cells 1 and 7 of Table 1): Now consider the effect of initial health H_0 on later-life health $H(t)$ (top-left panel of Figure 5). For fixed (exogenous) length of life, higher initial health ($H_0 = 120$; long-dashed trajectory) leads to higher health over the life cycle (compare with the short-dashed trajectory, $H_0 = 100$) for no other reason than that the individual had more health to begin with ("own" effect). In fact, healthier individuals invest less in their health (compare the short-dashed with the long-dashed trajectory in the top-right panel of Figure 5) as they are mechanically forced to die at the same age of death T when longevity is fixed (i.e., health needs to decline at a faster rate by reducing health investments). What is not spend on health is used for additional consumption (bottom-left panel of Figure 5): discounted lifetime consumption increases by $\sim$ \$34,000, which is financed by lower lifetime health investment of $\sim$ \$18,000 and by accumulating less wealth (i.e., higher health generates earnings, leading to a reduced need to save) compared to someone with lower initial health (compare dashed with long-dashed trajectories in the bottom-left and bottom-right panels of Figure 5).

Thus, when length of life cannot be influenced, responses are muted: schooling does not increase (see earlier), health investment and wealth accumulation are even reduced,

example, with a substantially higher initial health of H_0=120, solutions where one lives shorter than 75 years are not feasible.

and what is not invested in health, or saved, is consumed (a utility enhancing but non-productive use of the additional health resource). By contrast, when length of life can be extended, health and health investment, borrowing (early in life) and wealth accumulation (later in life), and consumption all substantially increase with respect to the fixed length of life case (compare the solid trajectories with the long-dashed trajectories in the four panels). These results are qualitatively the same as those in Table 1 (cells 1 and 7).

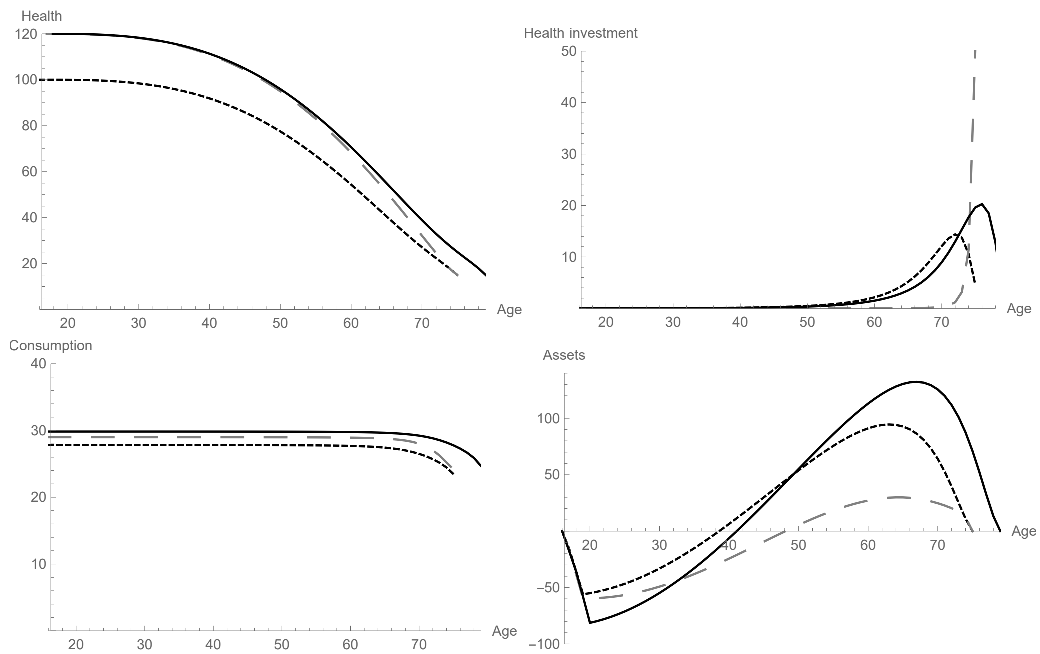

Figure 5: *Top-left: Health (top-left), health investment (top-right), consumption (bottom-left) and assets (bottom-right) over the life-cycle for* $H_0 = 100$ *(baseline model; short-dashed line),* $H_0 = 120$ *(T fixed; long-dashed line) and* $H_0 = 120$ *(T free; solid line).*

4.3.2 Variation in $\delta\theta_0$ and life-cycle choices

The literature employs years of schooling as a proxy for the accumulated skills or experience derived from schooling. In other words, we can think of S_0 as an initial stock of accumulated skills θ_0 from compulsory schooling. In the baseline run we start the model at the compulsory schooling age of 16 (or 10 years of schooling) and model years of schooling $S = S_0 + s^*$, with $S_0 = 10$ and s^* the choice of how many years to stay in school beyond what is compulsory. We can then model variation in its initial endowment S_0 by simply assuming that individuals complete their compulsory schooling with different levels of the stock of skills, say with the "equivalent" of an extra year of schooling, i.e. $S_0 = 11$, instead of 10. The most intuitive interpretation of an extra year of schooling at age 16 is a year of pre-school (i.e., starting school at age 5 rather than age 6), but it can also be interpreted more broadly as individuals having different endowments of skills at the age of 16 (i.e., reflecting differences in cognitive and non-cognitive capabilities between individuals with the same actual number of years in school).

$\partial S(t)/\partial S_0$ (cell 5 of Table 1): Figure 6 shows that the equivalent of an extra year of compulsory schooling S_0 translates into exactly the equivalent of one extra year S of schooling when longevity is fixed (moving from $S = 13$ to $S = 14$), but not more. Thus, in line with Table 1 $\partial S/\partial S_0|_T > 0$ (cell 5, for T fixed). This increase purely derives from a "carry-over" effect (Cunha and Heckman, 2007b), reflecting nothing more than the fact that the individual started with one more year of schooling to begin with ('own effect').

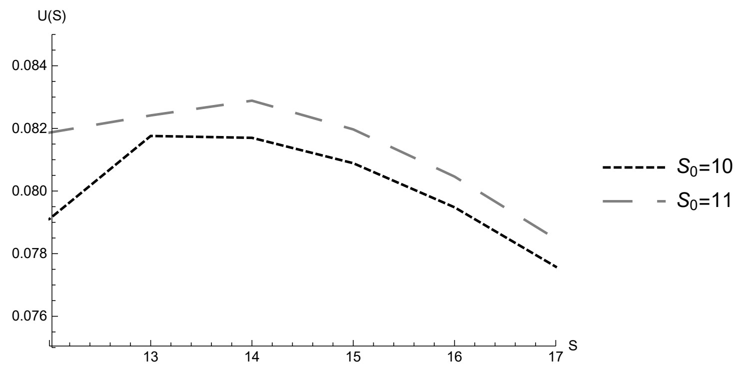

Figure 6: *Indirect utility function $U(S)$ as a function of years of schooling S, for a fixed life expectancy of $T = 75$ years. $S_0 = 10$ is the baseline case (short dash), and $S_0 = 11$ (long dash) represents an additional year of initial schooling (i.e., pre-school).*

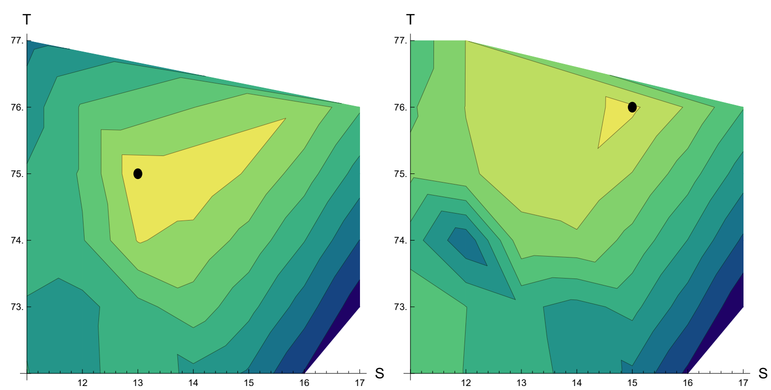

Figure 7: *Left: Contour plot of the indirect utility function $U(S,T)$ by years of schooling S and life expectancy T for the baseline model ($S_0 = 10$). Right: Contour plot of $U(S,T)$ for $S_0 = 11$. Lighter colors indicate higher values for the indirect utility function, and the maximum is indicated by the black circle.*

Next, we allow individuals to optimally choose both S and T simultaneously. Figure 7 presents a contour plot of the indirect utility function for the baseline case ($S_0 = 10$; left panel), and for $S_0 = 11$ (right panel). The optimum in lifetime utility switches from the point $(S = 13, T = 75)$ to $(S = 15, T = 76)$, i.e., equivalent years of schooling

S increases by two years and longevity T increases by one year. Thus in our calibrated model $\partial S/\partial S_0 > 0$, in line with Table 1 (cell 5, T free), and the response is larger when longevity is endogenous (T free) than when it is exogenous (T fixed).

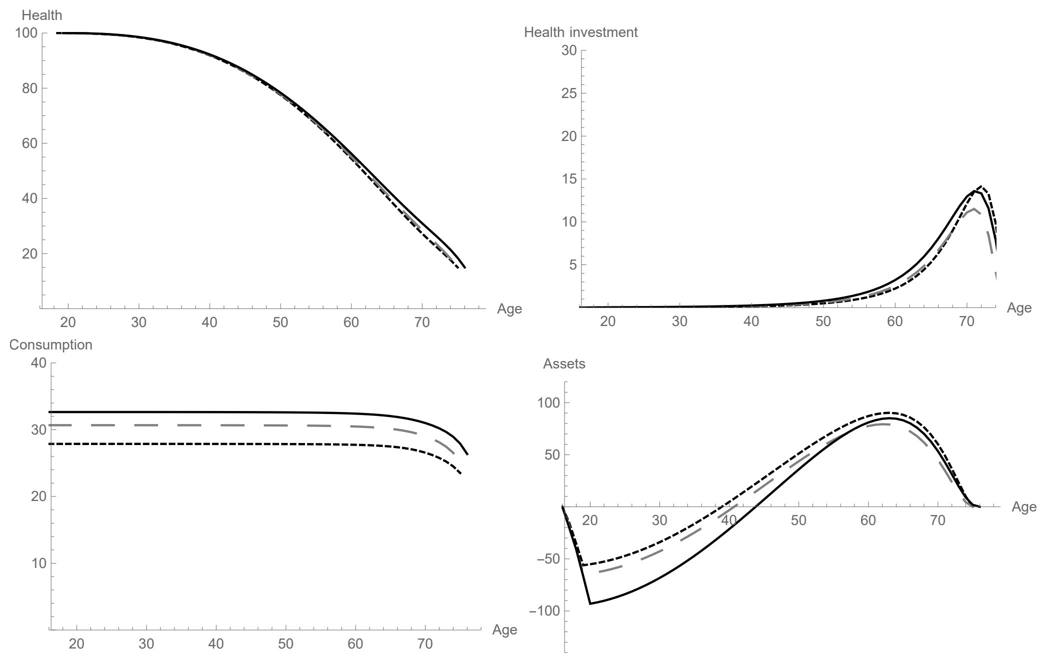

Figure 8: *Top-left: Health (top-left), health investment (top-right), consumption (bottom-left) and assets (bottom-right) over the life-cycle for* $S_0 = 10$ *(baseline model; short-dashed line),* $S_0 = 11$ *(T fixed; long-dashed line) and* $S_0 = 11$ *(T free; solid line).*

$\boldsymbol{\partial H(t)/\partial S_0}$ **and** $\boldsymbol{\partial A(t)/\partial S_0}$ (cells 2 and 8 of Table 1): Now consider the effect of initial years of schooling S_0 on other life-cycle choices. For fixed (exogenous) length of life, additional years of schooling ($S_0 = 11$; long-dashed trajectory) are only modestly associated with health and health investment (the short- and long-dashed trajectories are essentially overlapping; top-left and top-right panels in Figure 8). The only observable changes are in consumption (bottom-left panel) and the wealth trajectory (bottom-right panel), which reflects an increase in consumption and a reduced need to save, as a result of the higher earnings associated with the equivalent of an extra year of schooling. By contrast, when length of life can be extended, schooling and length of life (see earlier), health, health investment, and consumption all increase compared to the fixed length of life case (contrast the solid with the long-dashed trajectories in the top-left, top-right and bottom-left panels). When the individual's choices are less constrained (endogenous longevity), the individual borrows more aggressively early in life to finance a higher-level of consumption and health investment throughout life. These results are qualitatively the same as those in Table 1 (cells 2 and 8).

Variation in δA_0 and life-cycle choices: $\boldsymbol{\partial S(t)/\partial A_0}$ (cell 6 of Table 1): As Figure 9 shows, for exogenous longevity, increasing initial wealth from A_0=\$0 to \$5,000 and \$20,000, respectively, increases lifetime utility, but the optimal level of schooling S remains the same. Thus, $\partial S/\partial A_0|_T \sim 0$. However, when longevity is endogenous, the optimum in lifetime utility switches from $(S = 13, T = 75)$ for $A_0 = \$0$ to $(S = 16, T = 78)$ for $A_0 = \$20,000$ (right-panel of Figure 10). Thus $\partial S/\partial A_0 > 0$ when life can be extended (cf. cell 6 of Table 1).

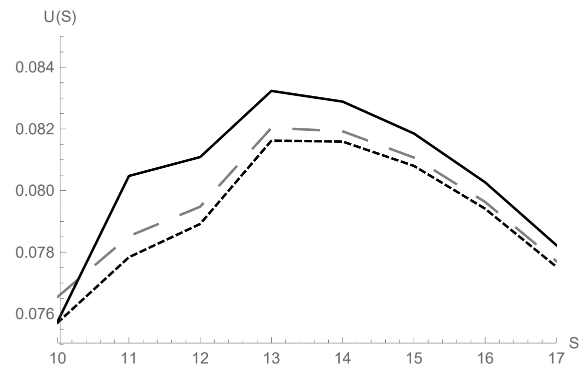

Figure 9: *Indirect utility function $U(S)$ for different values of A_0 holding length of life fixed at $T = 75$. A_0 =\$0 is the baseline case (short dash), and A_0 =\$5,000 (long dash) and A_0 =\$20,000 (solid), represent increases in initial wealth A_0 by \$5,000 and \$20,000, respectively.*

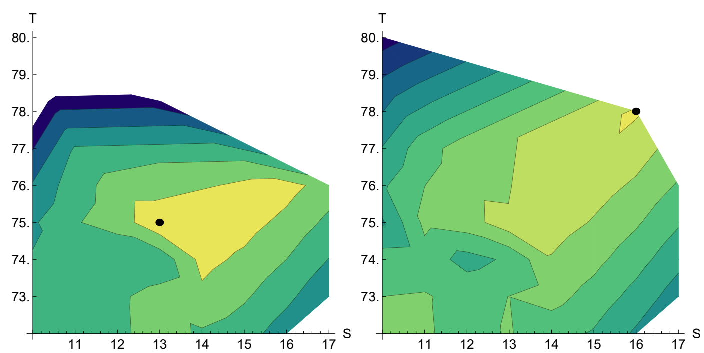

Figure 10: *Left: Contour plot of the indirect utility function $U(S,T)$ by years of schooling S and life expectancy T for the baseline model ($A_0 = \$0$). Right: Contour plot of $U(S,T)$ for $A_0 = \$20,000$. Lighter colors indicate higher values for the indirect utility function, and the maximum is indicated by the black circle.*

$\boldsymbol{\partial H(t)/\partial A_0}$ **and** $\boldsymbol{\partial A(t)/\partial A_0}$ (cells 3 and 9 of Table 1): The effect of initial wealth A_0 on health $H(t)$ (top-left panel of Figure 11) has been the subject of a large body of empirical work (e.g., O'Donnell et al., 2015; Cesarini et al., 2016). For fixed length of life, higher initial wealth (A_0 = \$20,000; long-dashed trajectory) is not associated with any change in health or in health investment (the short- and long-dashed trajectories are essentially overlapping; top-left and top-right panels). The only clearly observable change is in the wealth trajectory (bottom-right panel), a change that reflects nothing more than the fact that the individual started with more wealth to begin with ("own" effect). Visible is also a marginal increase in consumption (bottom-left panel). Interestingly, the increase in discounted lifetime consumption is \$20,482, almost exactly matching the increase in initial wealth of \$20,000, suggesting the increase in the wealth endowment was simply consumed (a utility enhancing but not productive use of the additional wealth). Thus, when length of life cannot be influenced, responses are muted: schooling does not increase (see earlier), health and health investment are essentially the same, and the increase in the wealth endowment is entirely spent on consumption. By contrast, when length of life can be extended, schooling, health, health investment, and consumption all substantially increase compared to the fixed length of life case (contrast the solid with the long-dashed trajectories in the top-left, top-right and bottom-left panels). Interestingly, when the individual's choices are less constrained (endogenous longevity), life is extended and the individual borrows more aggressively early in life to finance a higher-level of consumption and health investment throughout life (bottom-right panel). These results are qualitatively the same as those in Table 1 (cells 3 and 9).

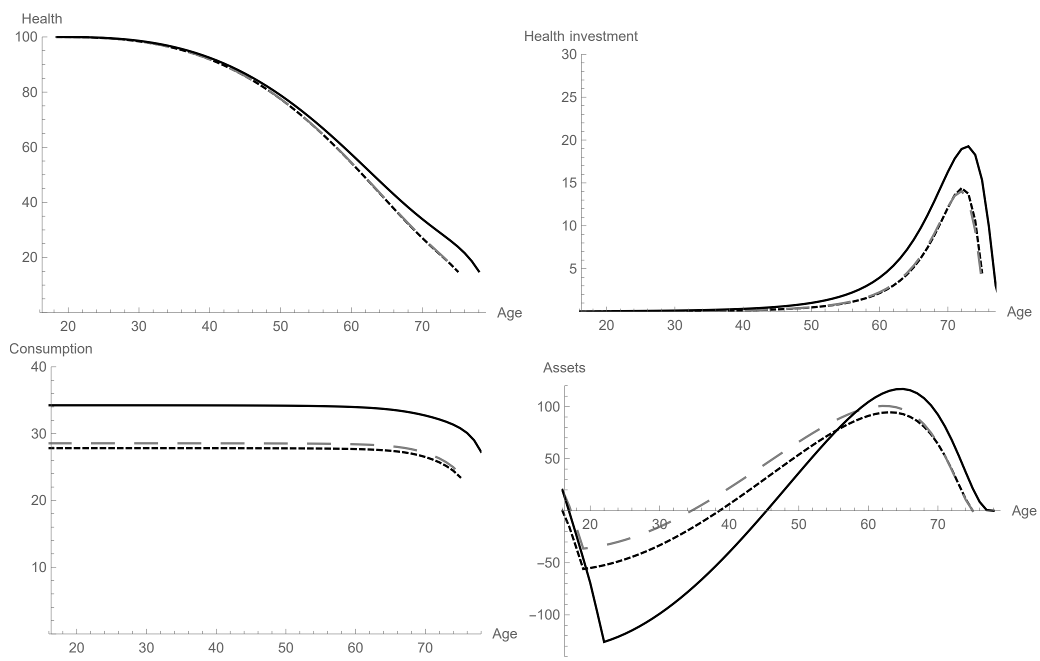

Figure 11: *Top-left: Health (top-left), health investment (top-right), consumption (bottom-left) and assets (bottom-right) over the life-cycle for* $A_0 = \$0$ *(baseline model; short-dashed line),* $A_0 =$ *\$20,000 (T fixed; long-dashed line) and* $A_0 =$ *\$20,000 (T free; solid line).*

Summary and conclusion: For exogenous (fixed) longevity, responses to variation in wealth, skill (schooling), or health, on investments in skill (schooling) and health are small / muted (statements 1a and 1b), except that variations in the own stock affect the own stock for the simple mechanical reason that one starts out with more (or less) of it due to the variation. By contrast, for endogenous (free) longevity, responses are substantial, as long as conditions for, and returns to, skill- and health-capital investments are conducive to skill- and health-capital formation (statement 2). In our calibration exercise, we also find evidence consistent with dynamic complementarity: when individuals have control over their length-of-life (endogenous longevity), investments increase in response to higher levels of the stocks. Hence, the ability of individuals to influence their longevity is a crucial driver of dynamic complementarity (without it, it is absent / muted). In short, the predictions of the stylized Ben-Porath model also hold in our more general model, with the nuance that in the stylized Ben-Porath model some responses are identical to zero, whereas in a more realistic model these are small / muted.

These predictions are not limited to variation in endowments, but also apply to other model parameters. For example, in Appendix A.6 we show that the results apply equally to variations in the price of medical care p_m, where a reduction in the price of medical care only boosts health and health investment when life can be extended, and, if life cannot be extended, any savings from reduced medical spending are simply consumed.

The calibrated model further suggests that the monetary value of endogenous longevity is substantial. Consider two individuals in the calibrated model who each experience an increase in initial health of 20%, but where the longevity of individual 1 is fixed at $T = 75$ while individual 2 is free to choose her longevity optimally. Based on an analysis of compensating variation, individual 1 would require \$93,000 (or about 3 times annual earnings) in order to make her indifferent. Likewise, if two individuals receive an increase in initial wealth of \$20,000, an individual who cannot influence her longevity would have to be compensated with an additional \$28,000 (i.e., 140% of the initial amount and corresponding to roughly one year of earnings in mid life) in order to be equally well-off as an individual who can choose her longevity optimally.

5 Discussion

We developed a theory of joint investments in skill, health, and longevity, and obtained a key new insight from it, namely that the ability to extend life (endogenous longevity) acts as a fundamental driver of human-capital formation: enabling (i) causal effects of past stocks of wealth, skill and health, on later-life wealth, skill and health, and (ii) persistent self-productivity and dynamic complementarity. Without the ability of individuals to extend their lives (exogenous longevity), such causal effects and self-productivity and dynamic complementarity are absent, or muted.

Since our main focus is on modeling endogenous longevity, for simplicity and clarity we assume a 'rectangular survival function' as is common in the health (Grossman, 1972b,a) and human-capital (Ben-Porath, 1967) literatures. As illustrated by Boucekkine et al. (2002); Soares (2005); Cervellati and Sunde (2013), in practice it matters whether longevity gains derive from the young or the old. While our rectangular survival function is somewhat crude, it does enable us to distinguish clearly and transparently between exogenous and endogenous models of length-of-life T.

Structural modelling of investments in the two stocks of human capital as well as longevity would unite theory and empirical analysis and have potentially large payoffs. Such a structural model could be employed to study the joint distribution of human capital, wealth accumulation and longevity and potentially explain the inter-dependencies in the trends of income, education and health over the 20^{th} Century. It would also provide a framework to determine optimal spending on education and health, two very large items on the government balance sheet. This paper serves as a building block towards that aim.

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A Appendix

A.1 First-order (necessary) conditions

The first-order conditions are obtained by taking the derivative of the Hamiltonian

$$\Im = U\{X_C(t), L(t)[\theta(t)+H(t)]\}e^{-\beta t} + q_\theta(t)\frac{\partial \theta}{\partial t} + q_H(t)\frac{\partial H}{\partial t} + q_A(t)\frac{\partial A}{\partial t}, \tag{38}$$

with respect to the controls (not shown). Start with the first-order condition for the optimal expenditures on skill capital goods, $X_\theta(t)$, and for time inputs, $\tau_\theta(t)$, and divide the two resulting expressions by one another to obtain the relation

$$\tau_\theta(t)\,[\theta(t)+H(t)] = \frac{\alpha_\theta}{\beta_\theta} p_\theta(t) X_\theta(t). \tag{39}$$

Similarly for health investment one obtains the relation

$$\tau_H(t)\,[\theta(t)+H(t)] = \frac{\alpha_H}{\beta_H} p_H(t) X_H(t). \tag{40}$$

Now insert these relations back into the first-order condition for $X_\theta(t)$, $\tau_\theta(t)$, $X_H(t)$, and $\tau_H(t)$, to obtain the analytical solutions:

$$\begin{aligned}
X_\theta(t) &= \frac{\beta_\theta \mu_\theta(t)}{p_\theta(t)} q_{\theta/a}(t)^{\frac{1}{1-\gamma_\theta}}, &(41)\\
\tau_\theta(t)\,[\theta(t)+H(t)] &= \alpha_\theta \mu_\theta(t) q_{\theta/a}(t)^{\frac{1}{1-\gamma_\theta}}, &(42)\\
X_H(t) &= \frac{\beta_H \mu_H(t)}{p_H(t)} q_{h/a}(t)^{\frac{1}{1-\gamma_H}}, &(43)\\
\tau_H(t)\,[\theta(t)+H(t)] &= \alpha_H \mu_H(t) q_{h/a}(t)^{\frac{1}{1-\gamma_H}}, &(44)
\end{aligned}$$

where $\gamma_\theta = \alpha_\theta + \beta_\theta < 1$, $\gamma_H = \alpha_H + \beta_H < 1$ (diminishing returns to scale), and the functions $\mu_\theta(t)$ and $\mu_H(t)$ are generalized productivity factors

$$\begin{aligned}
\mu_\theta(t) &\equiv \left[\frac{\alpha_\theta^{\alpha_\theta}\beta_\theta^{\beta_\theta}\psi_\theta(t)}{p_\theta(t)^{\beta_\theta}}\right]^{\frac{1}{1-\gamma_\theta}}, &(45)\\
\mu_H(t) &\equiv \left[\frac{\alpha_H^{\alpha_H}\beta_H^{\beta_H}\psi_H(t)}{p_H(t)^{\beta_H}}\right]^{\frac{1}{1-\gamma_H}}. &(46)
\end{aligned}$$

The co-state equations for $q_\theta(t)$, $q_H(t)$, and $q_A(t)$ follow from the usual conditions $\partial q_\theta/\partial t = -\partial\Im/\partial\theta$, $\partial q_H/\partial t = -\partial\Im/\partial H$, and $\partial q_A/\partial t = -\partial\Im/\partial A$, and using (41) to (44), we obtain

$$\begin{aligned}
\frac{\partial q_\theta}{\partial t} &= q_\theta(t) d_\theta(t) - q_A(t), &(47)\\
\frac{\partial q_H}{\partial t} &= q_H(t) d_H(t) - q_A(t), &(48)\\
\frac{\partial q_A}{\partial t} &= -r\, q_A(t). &(49)
\end{aligned}$$

The convenient choices made for the functional forms, referred to as "Ben-Porath neutrality" ensure that the relative marginal value of skill capital $q_{\theta/a}(t)$, and in our case also of health capital $q_{h/a}(t)$, are independent of the capital stocks (see 47 and 48). The system of equations for (the relative marginal value of) skill capital, and (the relative marginal value of) health capital reduces to the following system:

$$\begin{aligned} \frac{\partial q_{\theta/a}}{\partial t} &= q_{\theta/a}(t)\left[d_\theta(t)+r\right]-1, & (50)\\ \frac{\partial \theta}{\partial t} &= \mu_\theta(t) q_{\theta/a}(t)^{\frac{\gamma_\theta}{1-\gamma_\theta}} - d_\theta(t)\theta(t), & (51)\\ \frac{\partial q_{h/a}}{\partial t} &= q_{h/a}(t)\left[d_H(t)+r\right]-1, & (52)\\ \frac{\partial H}{\partial t} &= \mu_H(t) q_{h/a}(t)^{\frac{\gamma_H}{1-\gamma_H}} - d_H(t)H(t). & (53) \end{aligned}$$

Using the dynamic relation for skill- (6) and health-capital formation (7), the Ben-Porath production functions (10) and (11), and the solutions for the controls (41) to (44), one obtains analytical expressions for the relative marginal value of skill capital $q_{\theta/a}(t)$, skill capital $\theta(t)$, the relative marginal value of health capital $q_{h/a}(t)$, and health capital $H(t)$:

$$\begin{aligned} q_{\theta/a}(t) &= \int_t^T e^{-\int_t^s [d_\theta(x)+r]dx} ds, & (54)\\ \theta(t) &= \theta_0 e^{-\int_0^t d_\theta(x)dx} + \int_0^t \mu_\theta(s) q_{\theta/a}(s)^{\frac{\gamma_\theta}{1-\gamma_\theta}} e^{-\int_s^t d_\theta(x)dx} ds, & (55) \end{aligned}$$

$$\begin{aligned} q_{h/a}(t) &= q_{h/a}(0) e^{\int_0^t [d_H(x)+r]dx} - \int_0^t e^{\int_s^t [d_H(x)+r]dx} ds, & (56)\\ H(t) &= H_0 e^{-\int_0^t d_H(x)dx} + \int_0^t \mu_H(s) q_{h/a}(s)^{\frac{\gamma_H}{1-\gamma_H}} e^{-\int_s^t d_H(x)dx} ds, & (57) \end{aligned}$$

where we have used $q_{\theta/a}(T) = 0$, and the solution for the marginal value of assets $q_A(t) = q_A(0)e^{-rt}$, see (49).

Using the dynamic relation for assets (12), (8), and (41) to (61), we obtain

$$\begin{aligned} A(t)e^{-rt} &= A_0 + \int_0^t e^{-rs}\left[\theta(s)+H(s)\right]ds\\ &- \gamma_\theta \int_0^t \mu_\theta(s) q_{\theta/a}(s)^{\frac{1}{1-\gamma_\theta}} e^{-rs} ds\\ &- \gamma_H \int_0^t \mu_H(s) q_{h/a}(s)^{\frac{1}{1-\gamma_H}} e^{-rs} ds\\ &- q_A(0)^{-1/\rho}\Lambda \int_0^t p_C(s)^{-\zeta(1-\rho)/\rho} e^{-\frac{(\beta-r(1-\rho)r)}{\rho}s} ds. & (58) \end{aligned}$$

Finally, the analytical solutions for consumption $X_C(t)$ and leisure $L(t)\,[\theta(t)+H(t)]$ are obtained by dividing the two first-order conditions, leading to the relation

$$L(t)\,[\theta(t)+H(t)] = \frac{(1-\zeta)}{\zeta} p_C(t) X_C(t). \tag{59}$$

Inserting this relation back into the first-order conditions for consumption $X_C(t)$ and leisure $L(t)$, leads to the analytical solutions

$$\begin{aligned} X_C(t) &= \zeta \Lambda q_A(0)^{-1/\rho} p_C(t)^{-(1-\zeta+\zeta/\rho)} e^{-\frac{\beta-r}{\rho}t}, &(60)\\ L(t)[\theta(t)+H(t)] &= (1-\zeta)\Lambda q_A(0)^{-1/\rho} p_C(t)^{-\zeta(1-\rho)/\rho} e^{-\frac{\beta-r}{\rho}t}, &(61)\end{aligned}$$

where

$$\Lambda \equiv \left[\zeta^{\zeta}(1-\zeta)^{1-\zeta}\right]^{\frac{1-\rho}{\rho}}. \tag{62}$$

The analytical solutions for the controls, state variables, and co-state variables (41) to (61), are functions of the marginal value of initial wealth $q_A(0)$, the initial relative marginal value of skill-capital $q_{\theta/a}(0)$, and the initial relative marginal value of health-capital $q_{h/a}(0)$. These in turn are determined by initial, end, and transversality conditions.

From (58), and the initial, $A(0) = A_0$, and end condition, $A(T) = A_T$, follows a condition for $q_A(0)$

$$\begin{aligned} A_T e^{-rT} &= A_0 + \int_0^T e^{-rs}\left[\theta(s)+H(s)\right] ds \\ &- \gamma_\theta \int_0^T \mu_\theta(s) q_{\theta/a}(s)^{\frac{1}{1-\gamma_\theta}} e^{-rs} ds \\ &- \gamma_H \int_0^T \mu_H(s) q_{h/a}(s)^{\frac{1}{1-\gamma_H}} e^{-rs} ds \\ &- q_A(0)^{-1/\rho}\Lambda \int_0^T p_C(s)^{-\zeta(1-\rho)/\rho} e^{-\frac{(\beta-r(1-\rho))}{\rho}s} ds. &(63)\end{aligned}$$

From (57) and the initial, $H(0) = H_0$, and end condition, $H(T) = H_{\min}$, follows a condition for $q_{h/a}(0)$

$$H_{\min} e^{\int_0^T d_H(x)dx} = H_0 + \int_0^T \mu_H(s) q_{h/a}(s)^{\frac{\gamma_H}{1-\gamma_H}} e^{\int_0^s d_H(x)dx} ds. \tag{64}$$

The condition for $q_{\theta/a}(0)$ follows from the transversality condition $q_\theta(t) = 0$ ($\theta(t)$ free) and is obtained from (54)

$$q_{\theta/a}(0) = \int_0^T e^{-\int_0^s [d_\theta(x)+r]dx} ds. \tag{65}$$

The remaining endogenous parameters and functions in the above three conditions (63), (64), and (65), are T, which is determined by (15), $q_{\theta/a}(t)$, which is determined by (54), $\theta(t)$, which is determined by (55), $q_{h/a}(t)$, which is determined by (56), and $H(t)$, which is determined by (57).

A.2 Comparative dynamics

Consider a generic control, state, or co-state function $g(t)$ and a generic variation δZ_0 in an initial condition or model parameter. The effect of the variation δZ_0 on the optimal path of $g(t)$ can be broken down into variation for fixed longevity T and variation due to the resulting change in the horizon T (see 25). In the below analyses (i) we first analyze the case for fixed T, from which we obtain $\partial g(t)/\partial Z_0|_T$ (see discussion below), (ii) we then determine $\partial T/\partial Z_0$, and (iii) last we obtain $\partial g(t)/\partial T|_{Z_0}$, so that we obtain the full comparative dynamic effect.

Results are summarized in Table 1.

Comparative dynamics of length of life $\partial T/\partial Z_0$: For fixed length of life T we can take derivatives of the first-order conditions and state equations with respect to the initial condition or model parameter and study the optimal adjustment to the lifecycle path in response to variation in an initial endowment or other model parameter.

For free T, however, this is slightly more involved since the additional condition $\Im(T) = 0$ has to be satisfied. Varying the initial condition or model parameter Z_0, and taking into account $\Im(T) = 0$, we have

$$\left.\frac{\partial \Im(T)}{\partial Z_0}\right|_T \delta Z_0 + \left.\frac{\partial \Im(T)}{\partial T}\right|_{Z_0} \delta T = 0. \tag{66}$$

Using the expression for the Hamiltonian (38), taking the first derivative of the transversality condition $\Im(T) = 0$ with respect to the initial conditions or model parameter Z_0, and holding length of life T fixed, we obtain

$$\begin{aligned}\left.\frac{\partial \Im(T)}{\partial Z_0}\right|_T &= \left.\frac{\partial \Im}{\partial \xi}\frac{\partial \xi(T)}{\partial Z_0}\right|_T + \left.\frac{\partial \Im}{\partial \theta}\frac{\partial \theta(t)}{\partial Z_0}\right|_T + \left.\frac{\partial \Im}{\partial A}\frac{\partial A(T)}{\partial Z_0}\right|_T + \left.\frac{\partial \Im}{\partial H}\frac{\partial H(T)}{\partial Z_0}\right|_T \\ &\quad + \left.\frac{\partial \Im}{\partial q_\theta}\frac{\partial q_\theta(t)}{\partial Z_0}\right|_T + \left.\frac{\partial \Im}{\partial q_A}\frac{\partial q_A(T)}{\partial Z_0}\right|_T + \left.\frac{\partial \Im}{\partial q_H}\frac{\partial q_H(T)}{\partial Z_0}\right|_T \\ &= -\left.\frac{\partial q_\theta(t)}{\partial t}\right|_{T,t=T}\left.\frac{\partial \theta(t)}{\partial Z_0}\right|_T + \left.\frac{\partial q_A(T)}{\partial Z_0}\right|_T\left.\frac{\partial A(t)}{\partial t}\right|_{T,t=T} \\ &\quad + \left.\frac{\partial q_H(T)}{\partial Z_0}\right|_T\left.\frac{\partial H(t)}{\partial t}\right|_{T,t=T},\end{aligned} \tag{67}$$

where $\xi(t)$ is the vector of control functions $X_C(t)$, $L(t)$, $X_\theta(t)$, $\tau_\theta(t)$, $X_H(t)$, and $\tau_H(t)$. The first-order conditions imply $\partial\Im(t)/\partial\xi(t) = 0$. Further, $\partial\Im(T)/\partial\theta = -\left.\partial q_\theta(t)/\partial t\right|_{t=T}$, $\partial A(T)/\partial Z_0 = \partial H(T)/\partial Z_0 = 0$ since $A(T)$ and $H(T)$ are fixed, and $\partial q_\theta(t)/\partial Z_0|_T = 0$ since $q_\theta(t) = 0$.

Note that we distinguish in notation between $\partial f(t)/\partial t|_{t=T}$, which represents the derivative with respect to time t at time $t = T$, and $\partial f(t)/\partial T|_{t=T}$, which represents variation with respect to the parameter T at time $t = T$.

From (66) and (67) we have

$$\frac{\partial T}{\partial Z_0} = \frac{q_A(T)\frac{\partial \theta(T)}{\partial Z_0}\Big|_T + \frac{\partial q_A(T)}{\partial Z_0}\Big|_T \frac{\partial A(t)}{\partial t}\Big|_{T,t=T} + \frac{\partial q_H(T)}{\partial Z_0}\Big|_T \frac{\partial H(t)}{\partial t}\Big|_{T,t=T}}{-\frac{\partial \Im(T)}{\partial T}\Big|_{Z_0}}, \tag{68}$$

where we have used $\partial q_\theta(t)/\partial t|_{t=T} = q_\theta(T)d_\theta(T) - q_A(T) = -q_A(T)$ (see 47 and use $q_\theta(T) = 0$).

The denominator of (68) can be obtained from

$$\begin{aligned}\frac{\partial \Im(T)}{\partial T}\Big|_{Z_0} &= -\beta U[.]e^{-\beta T} \\ &\quad + q_A(T)\,\frac{\partial \theta(T)}{\partial T}\Big|_{Z_0} + \frac{\partial q_A(T)}{\partial T}\Big|_{Z_0} \frac{\partial A(t)}{\partial t}\Big|_{t=T} + \frac{\partial q_H(T)}{\partial T}\Big|_{Z_0} \frac{\partial H(t)}{\partial t}\Big|_{t=T},\end{aligned} \tag{69}$$

which follows from differentiating (38) with respect to T and using the first-order conditions (41) to (61), the co-state equations (50) to (53), (49), and the transversality condition $q_{\theta/a}(T) = 0$.

Consistent with diminishing returns to life extension (Ehrlich and Chuma, 1990), we assume

$$\frac{\partial \Im(T)}{\partial T}\Big|_{Z_0} < 0, \tag{70}$$

in which case we can identify the sign of the variation in life expectancy from

$$\text{sign}\left(\frac{\partial T}{\partial Z_0}\right) = \text{sign}\left(\frac{\partial \Im(T)}{\partial Z_0}\Big|_T\right), \tag{71}$$

where,

$$\frac{\partial \Im(T)}{\partial Z_0}\Big|_T = q_A(T)\frac{\partial \theta(T)}{\partial Z_0}\Big|_T + \frac{\partial q_A(T)}{\partial Z_0}\Big|_T \frac{\partial A(t)}{\partial t}\Big|_{T,t=T} + \frac{\partial q_H(T)}{\partial Z_0}\Big|_T \frac{\partial H(t)}{\partial t}\Big|_{T,t=T}. \tag{72}$$

As (71) shows, we can explore variation in initial conditions keeping length of life T initially fixed in order to investigate whether life would be extended as a result of such variation.

Comparative dynamics of variation in length of life $\partial g(t)/\partial T|_{Z_0}$: The derivatives of the control functions, state function and co-state functions with respect to length of life T, holding constant Z_0, are identical for any initial condition or model parameter Z_0. We therefore first obtain their derivatives (using 54 to 65). The symbol $\gtreqless 0$ is used to indicate that the sign cannot unambiguously be determined.

$$
\begin{aligned}
\left.\frac{\partial q_{\theta/a}(t)}{\partial T}\right|_{Z_0} &= e^{-\int_t^T [d_\theta(x)+r]dx} > 0, \\
\left.\frac{\partial q_{h/a}(t)}{\partial T}\right|_{Z_0} &= \frac{-\left.\frac{\partial H(t)}{\partial t}\right|_{t=T} e^{\int_0^t [2d_H(x)+r]dx} e^{\int_t^T d_H(x)dx}}{\frac{\gamma_H}{1-\gamma_H}\int_0^T \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H-1}{1-\gamma_H}} e^{\int_0^s [2d_H(x)+r]dx} ds} > 0, \\
\left.\frac{\partial \theta(t)}{\partial T}\right|_{Z_0} &= \frac{\gamma_\theta}{1-\gamma_\theta}\int_0^t \mu_\theta(s) q_{\theta/a}(s)^{\frac{2\gamma_\theta-1}{1-\gamma_\theta}} \left.\frac{\partial q_{\theta/a}(s)}{\partial T}\right|_{Z_0} e^{-\int_s^t d_\theta(x)dx} ds > 0, \\
\left.\frac{\partial H(t)}{\partial T}\right|_{Z_0} &= \frac{\gamma_H}{1-\gamma_H}\int_0^t \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H-1}{1-\gamma_H}} \left.\frac{\partial q_{h/a}(s)}{\partial T}\right|_{Z_0} e^{-\int_s^t d_H(x)dx} ds > 0, \\
\left.\frac{\partial X_\theta(t)}{\partial T}\right|_{Z_0} &= \frac{1}{1-\gamma_\theta}\frac{\beta_\theta}{p_\theta(t)} \mu_\theta(t) q_{\theta/a}(t)^{\frac{\gamma_\theta}{1-\gamma_\theta}} \left.\frac{\partial q_{\theta/a}(t)}{\partial T}\right|_{Z_0} > 0, \\
\left.\frac{\partial \tau_\theta(t)\left[\theta(t)+H(t)\right]}{\partial T}\right|_{Z_0} &= \frac{1}{1-\gamma_\theta}\alpha_\theta \mu_\theta(t) q_{\theta/a}(t)^{\frac{\gamma_\theta}{1-\gamma_\theta}} \left.\frac{\partial q_{\theta/a}(t)}{\partial T}\right|_{Z_0} > 0, \\
\left.\frac{\partial X_H(t)}{\partial T}\right|_{Z_0} &= \frac{1}{1-\gamma_H}\frac{\beta_H}{p_H(t)} \mu_H(t) q_{h/a}(t)^{\frac{\gamma_H}{1-\gamma_H}} \left.\frac{\partial q_{h/a}(t)}{\partial T}\right|_{Z_0} > 0, \\
\left.\frac{\partial \tau_H(t)\left[\theta(t)+H(t)\right]}{\partial T}\right|_{Z_0} &= \frac{1}{1-\gamma_H}\alpha_H \mu_H(t) q_{h/a}(t)^{\frac{\gamma_H}{1-\gamma_H}} \left.\frac{\partial q_{h/a}(t)}{\partial T}\right|_{Z_0} > 0,
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial q_A(0)}{\partial T}\right|_{Z_0} &= \frac{-\left.\frac{\partial A(t)}{\partial t}\right|_{Z_0, t=T} e^{-rT} - \int_0^T \left.\frac{\partial \phi(s)}{\partial T}\right|_{Z_0} ds}{\frac{\Lambda}{\rho} q_A(0)^{-\left(\frac{1+\rho}{\rho}\right)} \int_0^T p_C(s)^{-\frac{\zeta(1-\rho)}{\rho}} e^{-\frac{(\beta - r(1-\rho))}{\rho}s} ds} \gtreqless 0, \\
\left.\frac{\partial A(t)}{\partial T}\right|_{Z_0} &= e^{rt}\int_0^t \left.\frac{\partial \phi(s)}{\partial T}\right|_{Z_0} \\
&\quad + \left[e^{rt}\frac{\Lambda}{\rho} q_A(0)^{-\frac{(1+\rho)}{\rho}} \int_0^t p_C(s)^{-\frac{\zeta(1-\rho)}{\rho}} e^{-\frac{(\beta - r(1-\rho))}{\rho}s} ds\right] \left.\frac{\partial q_A(0)}{\partial T}\right|_{Z_0} \gtreqless 0,
\end{aligned}
$$

where

$$
\begin{aligned}
\left.\frac{\partial \phi(s)}{\partial T}\right|_{Z_0} \quad \equiv \quad & e^{-rs}\left[\left.\frac{\partial \theta(s)}{\partial T}\right|_{Z_0} + \left.\frac{\partial H(s)}{\partial T}\right|_{Z_0}\right] \\
& - \frac{\gamma_\theta}{1-\gamma_\theta} e^{-rs} \mu_\theta(s) q_{\theta/a}(s)^{\frac{\gamma_\theta}{1-\gamma_\theta}} \left.\frac{\partial q_{\theta/a}(s)}{\partial T}\right|_{Z_0} \\
& - \frac{\gamma_H}{1-\gamma_H} e^{-rs} \mu_H(s) q_{h/a}(s)^{\frac{\gamma_H}{1-\gamma_H}} \left.\frac{\partial q_{h/a}(s)}{\partial T}\right|_{Z_0},
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial X_C(t)}{\partial T}\right|_{Z_0} &= -\frac{\zeta\Lambda}{\rho} q_A(0)^{-\left(\frac{1+\rho}{\rho}\right)} p_C(t)^{-\left(\frac{\rho+\zeta-\zeta\rho}{\rho}\right)} e^{-\left(\frac{\beta-r}{\rho}\right)t} \left.\frac{\partial q_A(0)}{\partial T}\right|_{Z_0} \gtreqless 0, \\
\left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial T}\right|_{Z_0} &= -\frac{(1-\zeta)\Lambda}{\rho} q_A(0)^{-\left(\frac{1+\rho}{\rho}\right)} p_C(t)^{-\left(\frac{\zeta-\zeta\rho}{\rho}\right)} e^{-\left(\frac{\beta-r}{\rho}\right)t} \left.\frac{\partial q_A(0)}{\partial T}\right|_{Z_0} \gtreqless 0. \qquad (73)
\end{aligned}
$$

Comparative dynamics of initial wealth $\partial g(t)/\partial A_0$: First consider the case where T is fixed. Differentiating (64) with respect to A_0, using (56), and differentiating (65) with respect to A_0, one finds $\partial q_{\theta/a}(0)/\partial A_0\big|_T = 0$ and $\partial q_{h/a}(0)/\partial A_0\big|_T = 0$. Using (41) to (58), and (63), we obtain

$$
\begin{aligned}
\left.\frac{\partial q_{\theta/a}(t)}{\partial A_0}\right|_T &= 0,\ \forall t & \left.\frac{\partial q_{h/a}(t)}{\partial A_0}\right|_T &= 0,\ \forall t \\
\left.\frac{\partial \theta(t)}{\partial A_0}\right|_T &= 0,\ \forall t & \left.\frac{\partial H(t)}{\partial A_0}\right|_T &= 0,\ \forall t \\
\left.\frac{\partial X_\theta(t)}{\partial A_0}\right|_T &= 0,\ \forall t & \left.\frac{\partial X_H(t)}{\partial A_0}\right|_T &= 0,\ \forall t \\
\left.\frac{\partial \tau_\theta(t)\left[\theta(t)+H(t)\right]}{\partial A_0}\right|_T &= 0,\ \forall t & \left.\frac{\partial \tau_H(t)\left[\theta(t)+H(t)\right]}{\partial A_0}\right|_T &= 0,\ \forall t
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial A(t)}{\partial A_0}\right|_T &= e^{rt}\left[1-\frac{\int_0^t p_C(s)^{-\zeta(1-\rho)/\rho}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds}{\int_0^T p_C(s)^{-\zeta(1-\rho)/\rho}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds}\right] \geq 0, \\
\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T &= \frac{-1}{\frac{\Lambda q_A(0)^{-\frac{(1+\rho)}{\rho}}}{\rho}\int_0^T p_C(s)^{-\frac{\zeta(1-\rho)}{\rho}}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds} < 0, \\
\left.\frac{\partial X_C(t)}{\partial A_0}\right|_T &= -\frac{1}{\rho}\zeta\Lambda q_A(0)^{-\frac{(1+\rho)}{\rho}}p_C(t)^{-(1-\zeta+\zeta/\rho)}e^{-\frac{\beta-r}{\rho}t}\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T \\
&= \frac{\zeta p_C(t)^{-(1-\zeta+\zeta/\rho)}e^{-\frac{\beta-r}{\rho}t}}{\int_0^T p_C(s)^{-(1-\zeta+\zeta/\rho)}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds} > 0, \\
\left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial A_0}\right|_T &= -\frac{1}{\rho}\left(1-\zeta\right)\Lambda q_A(0)^{-\frac{(1+\rho)}{\rho}}p_C(t)^{-\zeta(1-\rho)/\rho}e^{-\frac{\beta-r}{\rho}t}\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T \\
&= \frac{\left(1-\zeta\right)p_C(t)^{-\zeta(1-\rho)/\rho}e^{-\frac{\beta-r}{\rho}t}}{\int_0^T p_C(s)^{-\zeta(1-\rho)/\rho}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds} > 0.
\end{aligned} \tag{74}
$$

Note that the relation for the variation in wealth has the desired properties $\partial A(0)/\partial A_0|_T = 1$, and $\partial A(T)/\partial A_0|_T = 0$.

Now allow length of life T to be optimally chosen. Using (68) we have

$$
\frac{\partial T}{\partial A_0} = \frac{\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T e^{-rT}\left[\left.\frac{\partial A(t)}{\partial t}\right|_{t=T}+q_{h/a}(T)\left.\frac{\partial H(t)}{\partial t}\right|_{t=T}\right]}{-\partial\Im(T)/\partial T|_{A_0}} > 0, \tag{75}
$$

where we have used $\partial\theta(T)/\partial A_0|_T = 0$ (see 55 and note that $\partial q_{\theta/a}(t)/\partial A_0\big|_T = 0,\ \forall t$), $\partial q_H(T)/\partial A_0|_T = q_{h/a}(T)\,\partial q_A(T)/\partial A_0|_T$ (since $\partial q_{h/a}(T)/\partial A_0\big|_T = 0$), $\partial H(t)/\partial t|_{t=T} < 0$ by definition as health approaches $H_{\min}$ from above, $\partial A(t)/\partial t|_{t=T} < 0$ as individuals draw from their savings in old age, and $-\partial\Im(T)/\partial T|_{A_0} > 0$ (see 70).

Using (25), we obtain the following total responses to variation in wealth

$$\frac{\partial q_{\theta/a}(t)}{\partial A_0} = \left.\frac{\partial q_{\theta/a}(t)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0, \qquad \frac{\partial q_{h/a}(t)}{\partial A_0} = \left.\frac{\partial q_{h/a}(t)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0,$$
$$\frac{\partial \theta(t)}{\partial A_0} = \left.\frac{\partial \theta(t)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0, \qquad \frac{\partial H(t)}{\partial A_0} = \left.\frac{\partial H(t)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0,$$
$$\frac{\partial X_\theta(t)}{\partial A_0} = \left.\frac{\partial X_\theta(t)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0, \qquad \frac{\partial X_H(t)}{\partial A_0} = \left.\frac{\partial X_H(t)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0,$$

$$\begin{aligned}
\frac{\partial \tau_\theta(t)\,[\theta(t)+H(t)]}{\partial A_0} &= \left.\frac{\partial \tau_\theta(t)\,[\theta(t)+H(t)]}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0,\\
\frac{\partial \tau_H(t)\,[\theta(t)+H(t)]}{\partial A_0} &= \left.\frac{\partial \tau_H(t)\,[\theta(t)+H(t)]}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} > 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial A(t)}{\partial A_0} &= \left.\frac{\partial A(t)}{\partial A_0}\right|_T + \left.\frac{\partial A(t)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} \gtreqless 0,\\
\frac{\partial q_A(0)}{\partial A_0} &= \left.\frac{\partial q_A(0)}{\partial A_0}\right|_T + \left.\frac{\partial q_A(0)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} \gtreqless 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial X_C(t)}{\partial A_0} &= -\left(\frac{1}{\rho}\zeta\Lambda q_A(0)^{-\frac{(1+\rho)}{\rho}} p_C(t)^{-(1-\zeta+\zeta/\rho)} e^{-\frac{\beta-r}{\rho}t}\right)\times\\
&\quad \left[\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T + \left.\frac{\partial q_A(0)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0}\right] \gtreqless 0,\\
\frac{\partial L(t)\,[\theta(t)+H(t)]}{\partial A_0} &= -\left(\frac{1}{\rho}(1-\zeta)\,\Lambda q_A(0)^{-\frac{(1+\rho)}{\rho}} p_C(t)^{-\zeta(1-\rho)/\rho} e^{-\frac{\beta-r}{\rho}t}\right)\times\\
&\quad \left[\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T + \left.\frac{\partial q_A(0)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0}\right] \gtreqless 0, \qquad (76)
\end{aligned}$$

where we have used (73). Note that the total response of $q_A(0)$ with respect to initial wealth A_0 is ambiguous, since the additional wealth has to be spread over more time periods ($\partial T/\partial A_0 > 0$). But, a longer horizon also increases the returns to skill investment and to health investment, increasing the stocks, earnings and permanent income (lowering the marginal value of wealth $q_A(0)$). Hence, the effect of initial wealth on $q_A(0)$ and thereby on consumption and leisure is ambiguous for free T. Since wealthy individuals are generally found to consume more and retire earlier (e.g., Imbens et al., 2001; Juster et al., 2006; Brown et al., 2010), it is plausible that the wealth effect dominates $\partial q_A(0)/\partial A_0 < 0$, and consumption goods and services $X_C(t)$ and effective leisure $L(t)\,[\theta(t)+H(t)]$ are higher throughout life.

Comparative dynamics of initial skill $\partial g(t)/\partial\theta_0$: Again, first consider the case where T is fixed. Differentiating (65) with respect to θ_0, one finds $\partial q_{\theta/a}(0)/\partial\theta_0\big|_T = 0$ and differentiating (64) with respect to θ_0, using (56) we find $\partial q_{h/a}(0)/\partial\theta_0\big|_T = 0$. Using (41) to (58), and (63), we find

$$\left.\frac{\partial q_{\theta/a}(t)}{\partial\theta_0}\right|_T = 0,\ \forall t \qquad \left.\frac{\partial q_{h/a}(t)}{\partial\theta_0}\right|_T = 0,\ \forall t$$

$$\left.\frac{\partial\theta(t)}{\partial\theta_0}\right|_T = e^{-\int_0^t d_\theta(x)dx} > 0,\ \forall t \qquad \left.\frac{\partial H(t)}{\partial\theta_0}\right|_T = 0,\ \forall t$$

$$\left.\frac{\partial X_\theta(t)}{\partial\theta_0}\right|_T = 0,\ \forall t \qquad \left.\frac{\partial X_H(t)}{\partial\theta_0}\right|_T = 0,\ \forall t$$

$$\left.\frac{\partial\tau_\theta(t)\left[\theta(t)+H(t)\right]}{\partial\theta_0}\right|_T = 0,\ \forall t \qquad \left.\frac{\partial\tau_H(t)\left[\theta(t)+H(t)\right]}{\partial\theta_0}\right|_T = 0,\ \forall t$$

$$\begin{aligned}
\left.\frac{\partial A(t)}{\partial\theta_0}\right|_T &= e^{rt}\left[\int_0^T e^{-\int_0^s[d_\theta(x)+r]dx}ds\right] \\
&\quad\times\left[\frac{\int_0^t e^{-\int_0^s[d_\theta(x)+r]dx}ds}{\int_0^T e^{-\int_0^s[d_\theta(x)+r]dx}ds} - \frac{\int_0^t p_C(s)^{-\zeta(1-\rho)/\rho}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds}{\int_0^T p_C(s)^{-\zeta(1-\rho)/\rho}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds}\right] \gtreqless 0, \\
\left.\frac{\partial q_A(0)}{\partial\theta_0}\right|_T &= \frac{-\int_0^T e^{-\int_0^s[d_\theta(x)+r]dx}ds}{\frac{\Lambda}{\rho}q_A(0)^{-\left(\frac{1+\rho}{\rho}\right)}\int_0^T p_C(s)^{-\frac{\zeta(1-\rho)}{\rho}}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds} < 0, \\
\left.\frac{\partial X_C(t)}{\partial\theta_0}\right|_T &= -\zeta\Lambda\frac{q_A(0)^{-\frac{(1+\rho)}{\rho}}}{\rho}p_C(t)^{-(1-\zeta+\zeta/\rho)}e^{-\frac{(\beta-r)}{\rho}t}\left.\frac{\partial q_A(0)}{\partial\theta_0}\right|_T \\
&= \frac{\zeta p_C(t)^{-(1-\zeta+\zeta/\rho)}e^{-\frac{(\beta-r)}{\rho}t}\int_0^T e^{-\int_0^s[d_\theta(x)+r]dx}ds}{\int_0^T p_C(s)^{-\frac{\zeta(1-\rho)}{\rho}}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds} > 0, \\
\left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial\theta_0}\right|_T &= -(1-\zeta)\Lambda\frac{q_A(0)^{-\frac{(1+\rho)}{\rho}}}{\rho}p_C(t)^{-\frac{\zeta(1-\rho)}{\rho}}e^{-\frac{(\beta-r)}{\rho}t}\left.\frac{\partial q_A(0)}{\partial\theta_0}\right|_T \\
&= \frac{(1-\zeta)p_C(t)^{-\frac{\zeta(1-\rho)}{\rho}}e^{-\frac{(\beta-r)}{\rho}t}\int_0^T e^{-\int_0^t[d_\theta(x)+r]dx}dt}{\int_0^T p_C(s)^{-\frac{\zeta(1-\rho)}{\rho}}e^{-\frac{(\beta-r(1-\rho))}{\rho}s}ds} > 0.
\end{aligned} \tag{77}$$

Note that the relation for the variation in wealth has the desired properties $\partial A(0)/\partial\theta_0|_T = 0$, and $\partial A(T)/\partial\theta_0|_T = 0$. Further, the wealth effect of additional skill capital $\delta\theta_0$ is proportional to the effect we derived earlier of an additional amount of wealth δA_0,

$$
\begin{aligned}
\left.\frac{\partial q_A(0)}{\partial \theta_0}\right|_T &= \left\{\int_0^T e^{-\int_0^s [d_\theta(x)+r]dx} ds\right\} \left.\frac{\partial q_A(0)}{\partial A_0}\right|_T, \\
\left.\frac{\partial X_C(t)}{\partial \theta_0}\right|_T &= \left\{\int_0^T e^{-\int_0^s [d_\theta(x)+r]dx} ds\right\} \left.\frac{\partial X_C(t)}{\partial A_0}\right|_T, \\
\left.\frac{\partial L(t)\,[\theta(t)+H(t)]}{\partial \theta_0}\right|_T &= \left\{\int_0^T e^{-\int_0^s [d_\theta(x)+r]dx} ds\right\} \left.\frac{\partial L(t)\,[\theta(t)+H(t)]}{\partial A_0}\right|_T.
\end{aligned} \tag{78}
$$

Note further, that

$$
\begin{aligned}
\left.\frac{\partial F_\theta[.]}{\partial \theta_0}\right|_T &= 0, \\
\left.\frac{\partial f_H[.]}{\partial \theta_0}\right|_T &= 0, \\
\left.\frac{\partial Y[.]}{\partial \theta_0}\right|_T &= \left.\frac{\partial \theta(t)}{\partial \theta_0}\right|_T.
\end{aligned} \tag{79}
$$

Now allow length of life T to be optimally chosen. Using (68) we have

$$
\begin{aligned}
\frac{\partial T}{\partial \theta_0} &= \frac{q_A(0)e^{-\int_0^T [d_\theta(x)+r]dx} + \left.\frac{\partial q_A(0)}{\partial \theta_0}\right|_T e^{-rT}\left[\left.\frac{\partial A(t)}{\partial t}\right|_{t=T} + q_{h/a}(T)\left.\frac{\partial H(t)}{\partial t}\right|_{t=T}\right]}{-\left.\frac{\partial \Im(T)}{\partial T}\right|_T} \\
&= \frac{q_A(0)e^{-\int_0^T [d_\theta(x)+r]dx}}{-\left.\frac{\partial \Im(T)}{\partial T}\right|_T} + \left\{\int_0^T e^{-\int_0^s [d_\theta(x)+r]dx} ds\right\} \frac{\partial T}{\partial A_0} > 0.
\end{aligned} \tag{80}
$$

Using (25), we obtain the following total responses to variation in skill capital

$$
\begin{aligned}
\frac{\partial q_{\theta/a}(t)}{\partial \theta_0} &= \left.\frac{\partial q_{\theta/a}(t)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0, &\qquad \frac{\partial q_{h/a}(t)}{\partial \theta_0} &= \left.\frac{\partial q_{h/a}(t)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0, \\
\frac{\partial \theta(t)}{\partial \theta_0} &= \left.\frac{\partial \theta(t)}{\partial \theta_0}\right|_T + \left.\frac{\partial \theta(t)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0, &\qquad \frac{\partial H(t)}{\partial \theta_0} &= \left.\frac{\partial H(t)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0, \\
\frac{\partial X_\theta(t)}{\partial \theta_0} &= \left.\frac{\partial X_\theta(t)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0, &\qquad \frac{\partial X_H(t)}{\partial \theta_0} &= \left.\frac{\partial X_H(t)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \tau_\theta(t)\,[\theta(t)+H(t)]}{\partial \theta_0} &= \left.\frac{\partial \tau_\theta(t)\,[\theta(t)+H(t)]}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0, \\
\frac{\partial \tau_H(t)\,[\theta(t)+H(t)]}{\partial \theta_0} &= \left.\frac{\partial \tau_H(t)\,[\theta(t)+H(t)]}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} > 0,
\end{aligned}
$$

$$\frac{\partial A(t)}{\partial \theta_0} = \left.\frac{\partial A(t)}{\partial \theta_0}\right|_T + \left.\frac{\partial A(t)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} \gtreqless 0,$$
$$\frac{\partial q_A(0)}{\partial \theta_0} = \left.\frac{\partial q_A(0)}{\partial \theta_0}\right|_T + \left.\frac{\partial q_A(0)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} \gtreqless 0,$$

$$\frac{\partial X_C(t)}{\partial \theta_0} = -\frac{\zeta}{\rho} q_A(0)^{-\left(\frac{1+\rho}{\rho}\right)} \Lambda p_C(t)^{-(1-\zeta+\zeta/\rho)} e^{-\left(\frac{\beta-r}{\rho}\right)t} \left[\left.\frac{\partial q_A(0)}{\partial \theta_0}\right|_T + \left.\frac{\partial q_A(0)}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} \right] \gtreqless 0,$$

$$\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial \theta_0} = \left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial \theta_0}\right|_T + \left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial T}\right|_{\theta_0} \frac{\partial T}{\partial \theta_0} \gtreqless 0, \tag{81}$$

where we have used (73).

Comparative dynamics of initial health $\partial g(t)/\partial H_0$: Again, first consider the case where T is fixed. Differentiating (64) with respect to H_0, using (56), and differentiating (65) with respect to H_0, one finds

$$\begin{aligned} \left.\frac{\partial q_{\theta/a}(0)}{\partial H_0}\right|_T &= 0, \\ \left.\frac{\partial q_{h/a}(0)}{\partial H_0}\right|_T &= \frac{-1}{\frac{\gamma_H}{1-\gamma_H}\int_0^T \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H-1}{1-\gamma_H}} e^{\int_0^s [2d_H(x)+r]dx} ds} < 0. \end{aligned} \tag{82}$$

Using (41) to (64), and (82), we obtain

$$\begin{aligned} \left.\frac{\partial q_{\theta/a}(t)}{\partial H_0}\right|_T &= 0, \ \forall t \\ \left.\frac{\partial q_{h/a}(t)}{\partial H_0}\right|_T &= \left.\frac{\partial q_{h/a}(0)}{\partial H_0}\right|_T e^{\int_0^t [d_H(x)+r]dx} < 0, \\ \left.\frac{\partial \theta(t)}{\partial H_0}\right|_T &= 0, \ \forall t \\ \left.\frac{\partial H(t)}{\partial H_0}\right|_T &= e^{-\int_0^t d_H(x)dx} \left[1 - \frac{\int_0^t \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H-1}{1-\gamma_H}} e^{\int_0^s [2d_H(x)+r]dx} ds}{\int_0^T \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H-1}{1-\gamma_H}} e^{\int_0^s [2d_H(x)+r]dx} ds} \right] \geq 0, \\ \left.\frac{\partial X_\theta(t)}{\partial H_0}\right|_T &= 0, \ \forall t \\ \left.\frac{\partial \tau_\theta(t)\left[\theta(t)+H(t)\right]}{\partial H_0}\right|_T &= 0, \ \forall t \\ \left.\frac{\partial X_H(t)}{\partial H_0}\right|_T &= \frac{X_H(t)}{1-\gamma_H} \frac{\left.\frac{\partial q_{h/a}(t)}{\partial H_0}\right|_T}{q_{h/a}(t)} < 0, \end{aligned}$$

$$
\begin{aligned}
\left.\frac{\partial \tau_H(t)\left[\theta(t)+H(t)\right]}{\partial H_0}\right|_T &= \frac{\tau_H(t)\left[\theta(t)+H(t)\right]}{1-\gamma_H} \frac{\left.\frac{\partial q_{h/a}(t)}{\partial H_0}\right|_T}{q_{h/a}(t)} < 0, \\
\left.\frac{\partial A(t)}{\partial H_0}\right|_T &= e^{rt}\left[\int_0^T \epsilon\left[H(s), q_{h/a}(s)\right] ds\right] \\
&\quad \times \left[\frac{\int_0^t \epsilon\left[H(s), q_{h/a}(s)\right] ds}{\int_0^T \epsilon\left[H(s), q_{h/a}(s)\right] ds} - \frac{\int_0^t p_C(s)^{-\zeta(1-\rho)/\zeta} e^{-\frac{(\beta-r(1-\rho))}{\rho}s} ds}{\int_0^T p_C(s)^{-\zeta(1-\rho)/\zeta} e^{-\frac{(\beta-r(1-\rho))}{\rho}s} ds}\right] \gtreqless 0, \\
\left.\frac{\partial q_A(0)}{\partial H_0}\right|_T &= \frac{-\int_0^T \epsilon\left[H(s), q_{h/a}(s)\right] ds}{q_A(0)^{\frac{-(1+\rho)}{\rho}} \frac{\Lambda}{\rho} \int_0^T p_C(s)^{-\zeta(1-\rho)/\rho} e^{-\frac{(\beta-r(1-\rho))}{\rho}s} ds} \\
&= \left\{\int_0^T \epsilon\left[H(s), q_{h/a}(s)\right] ds\right\} \left.\frac{\partial q_A(0)}{\partial A_0}\right|_T < 0, \qquad (83)
\end{aligned}
$$

where

$$
\epsilon\left[H(s), q_{h/a}(s)\right] = \left.\frac{\partial H(s)}{\partial H_0}\right|_T - \frac{\gamma_H}{1-\gamma_H}\mu_H(s) q_{h/a}(s)^{\frac{\gamma_H}{1-\gamma_H}} \left.\frac{\partial q_{h/a}(s)}{\partial H_0}\right|_T e^{-rs} > 0, \;\; \forall s
$$

and we have used $\partial H(s)/\partial H_0|_T > 0$ and $\partial q_{h/a}(s)/\partial H_0|_T < 0$ (see 83).

Further using (60) and (61) it follows that

$$
\begin{aligned}
\left.\frac{\partial X_C(t)}{\partial H_0}\right|_T &= -\frac{1}{\rho}\zeta\Lambda q_A(0)^{-\frac{(1+\rho)}{\rho}} p_C(t)^{-(1-\zeta+\zeta/\rho)} e^{-\frac{\beta-r}{\rho}t} \left.\frac{\partial q_A(0)}{\partial H_0}\right|_T > 0, \\
\left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial H_0}\right|_T &= -\frac{1}{\rho}(1-\zeta)\Lambda q_A(0)^{-\frac{(1+\rho)}{\rho}} p_C(t)^{-\zeta(1-\rho)/\rho} e^{-\frac{\beta-r}{\rho}t} \left.\frac{\partial q_A(0)}{\partial H_0}\right|_T > 0. \qquad (84)
\end{aligned}
$$

Note that the relation for the variation in the health stock has the desired properties $\partial H(0)/\partial H_0|_T = 1$, and $\partial H(T)/\partial H_0|_T = 0$, and the relation for the variation in wealth has the desired properties $\partial A(0)/\partial H_0|_T = 0$, and $\partial A(T)/\partial H_0|_T = 0$. Also note that

$$
\begin{aligned}
\left.\frac{\partial f_\theta[.]}{\partial H_0}\right|_T &= 0, \\
\left.\frac{\partial f_H[.]}{\partial H_0}\right|_T &= \frac{\gamma_H}{1-\gamma_H} \frac{f_H[.]}{q_{h/a}(t)} \left.\frac{\partial q_{h/a}(t)}{\partial H_0}\right|_T < 0, \qquad (85)
\end{aligned}
$$

so that the additional productivity $f_\theta[.]$ from greater health, $\partial H(t)/\partial H_0|_T > 0$, is exactly offset by the reduction in time inputs, $\partial \tau_\theta(t)/\partial H_0|_T < 0$, and, the additional productivity $f_H[.]$ from greater health, $\partial H(t)/\partial H_0|_T > 0$, is more than offset, $\partial f_H[.]/\partial \theta_0|_T < 0$, in order to ensure that length of life remains of the same duration (we assumed fixed T).

Now allow length of life T to be optimally chosen. Using (68) we have

$$\frac{\partial T}{\partial H_0} = \frac{\left.\frac{\partial q_A(0)}{\partial H_0}\right|_T e^{-rT} \left.\frac{\partial A(t)}{\partial t}\right|_{t=T} + \left[q_{h/a}(T) \left.\frac{\partial q_A(0)}{\partial H_0}\right|_T e^{-rT} + q_A(T) \left.\frac{\partial q_{h/a}(T)}{\partial H_0}\right|_T\right] \left.\frac{\partial H(t)}{\partial t}\right|_{t=T}}{-\left.\frac{\partial \Im(T)}{\partial T}\right|_{H_0}}$$

$$= \frac{q_A(T) \left.\frac{\partial q_{h/a}(T)}{\partial H_0}\right|_T \left.\frac{\partial H(t)}{\partial t}\right|_{t=T}}{-\left.\frac{\partial \Im(T)}{\partial T}\right|_{H_0}} + \left\{\int_0^T \epsilon\left[H(s), q_{h/a}(s)\right] ds\right\} \frac{\partial T}{\partial A_0} > 0. \quad (86)$$

Using (25), we obtain the following total responses to variation in skill capital

$$\frac{\partial q_{\theta/a}(t)}{\partial H_0} = \left.\frac{\partial q_{\theta/a}(t)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} > 0, \qquad \frac{\partial q_{h/a}(t)}{\partial H_0} = \left.\frac{\partial q_{h/a}(t)}{\partial H_0}\right|_T + \left.\frac{\partial q_{h/a}(t)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} \gtreqless 0,$$

$$\frac{\partial \theta(t)}{\partial H_0} = \left.\frac{\partial \theta(t)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} > 0, \qquad \frac{\partial H(t)}{\partial H_0} = \left.\frac{\partial H(t)}{\partial H_0}\right|_T + \left.\frac{\partial H(t)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} > 0,$$

$$\frac{\partial X_\theta(t)}{\partial H_0} = \left.\frac{\partial X_\theta(t)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} > 0, \qquad \frac{\partial X_H(t)}{\partial H_0} = \left.\frac{\partial X_H(t)}{\partial H_0}\right|_T + \left.\frac{\partial X_H(t)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} \gtreqless 0,$$

$$\frac{\partial \tau_\theta(t)\left[\theta(t)+H(t)\right]}{\partial H_0} = \left.\frac{\partial \tau_\theta(t)\left[\theta(t)+H(t)\right]}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} > 0,$$

$$\frac{\partial \tau_H(t)\left[\theta(t)+H(t)\right]}{\partial H_0} = \left.\frac{\partial \tau_H(t)\left[\theta(t)+H(t)\right]}{\partial H_0}\right|_T + \left.\frac{\partial \tau_H(t)\left[\theta(t)+H(t)\right]}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} \gtreqless 0,$$

$$\frac{\partial A(t)}{\partial H_0} = \left.\frac{\partial A(t)}{\partial H_0}\right|_T + \left.\frac{\partial A(t)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} \gtreqless 0,$$

$$\frac{\partial q_A(0)}{\partial H_0} = \left.\frac{\partial q_A(0)}{\partial H_0}\right|_T + \left.\frac{\partial q_A(0)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} \gtreqless 0,$$

$$\frac{\partial X_C(t)}{\partial H_0} = -\frac{\zeta}{\rho} q_A(0)^{-\left(\frac{1+\rho}{\rho}\right)} \Lambda p_C(t)^{-(1-\zeta+\zeta/\rho)} e^{-\left(\frac{\beta-r}{\rho}\right)t} \left[\left.\frac{\partial q_A(0)}{\partial H_0}\right|_T + \left.\frac{\partial q_A(0)}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0}\right] \gtreqless 0,$$

$$\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial H_0} = \left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial H_0}\right|_T + \left.\frac{\partial L(t)\left[\theta(t)+H(t)\right]}{\partial T}\right|_{H_0} \frac{\partial T}{\partial H_0} \gtreqless 0, \quad (87)$$

where we have used (73).

A.3 Self-productivity and dynamic complementarity

Here we discuss self-productivity and dynamic complementarity in skill and health.

Self-productivity: Cunha and Heckman (2007a) define self-productivity as "*the skills produced at one stage augment the skills attained at later stages. It embodies the idea that skill acquired in one period persist into future periods. It also embodies the idea that skills are self-reinforcing and cross fertilizing.*"

Self-productivity arises when

$$\frac{\partial g(t)}{\partial g(t')} = \frac{\partial f_g[\cdot]}{\partial g(t')} > 0, \qquad t' < t$$

for $g(t)$ denoting skill $\theta(t)$ or health $H(t)$.

Based on the derivations in A.2, the results for self-productivity for fixed T are as follows:

$$\left.\frac{\partial \theta(t)}{\partial \theta(t')}\right|_T = e^{-\int_{t'}^{t} d_\theta(x)dx} > 0, \tag{88}$$

$$\left.\frac{\partial H(t)}{\partial H(t')}\right|_T = e^{-\int_{t'}^{t} d_H(x)dx}\left[1 - \frac{\int_{t'}^{t} \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H - 1}{1-\gamma_H}} e^{\int_{t'}^{s}[2d_H(x)+r]dx} ds}{\int_{t'}^{T} \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H - 1}{1-\gamma_H}} e^{\int_{t'}^{s}[2d_H(x)+r]dx} ds}\right] \geq 0, \tag{89}$$

$$\left.\frac{\partial \theta(t)}{\partial H(t')}\right|_T = 0, \tag{90}$$

$$\left.\frac{\partial H(t)}{\partial \theta(t')}\right|_T = 0. \tag{91}$$

Hence, for fixed T, self-reinforcing self-productivity exists, but only due to a so-called "carry-over effect" of human capital depreciating (and not by enhancing additional investment). For fixed T, there exists no cross-fertilizing self-productivity.

In sharp contrast, for free T (endogenous longevity), all these derivatives are positive (see 81 and 87). Hence, endogenous longevity gains are a necessary condition for self-productivity, with the caveat that self-reinforcing self-productivity exists for fixed T, but it only derives from the carry-over effect and dies out.

Dynamic complementarity: Cunha and Heckman (2007a,b) define dynamic complementarity as "*Skills produced at one stage rais[ing] the productivity of investment at later stages.*" It arises when

$$\frac{\partial^2 g(t)}{\partial g(t'')\partial I_g(t')} > 0, \qquad t'' \leq t' \leq t, \tag{92}$$

where $g(t)$ equals skill $\theta(t)$ or health $H(t)$, and $I_g(t)$ denotes investment in skill $X_\theta(t)$, $\tau_\theta(t)$ or in health $X_H(t)$, $\tau_H(t)$.

From (41) to (44), we have

$$I_g(t) \quad \propto \quad q_{g/a}(t)^{\frac{1}{1-\gamma_g}}, \tag{93}$$

Investments in skill and in health are increasing functions of the relative marginal value of the stocks $q_{g/a}(t)$. Thus condition (92) implies (taking $t' = t$)

$$\frac{\partial^2 g(t)}{\partial g(t'')\partial q_{g/a}(t)} > 0. \tag{94}$$

Since investment is proportional to the relative marginal value of the stock, $q_{\theta/a}(t)$ or $q_{h/a}(t)$, for analytical convenience we will investigate dynamic complementarity by studying derivatives with respect to the co-state equations.

From (88), (90), and (91) it follows directly that

$$\left.\frac{\partial^2 \theta(t)}{\partial \theta(t'')\partial q_{\theta/a}(t)}\right|_T = 0 \tag{95}$$

$$\left.\frac{\partial^2 \theta(t)}{\partial H(t'')\partial q_{\theta/a}(t)}\right|_T = 0 \tag{96}$$

$$\left.\frac{\partial^2 H(t)}{\partial \theta(t'')\partial q_{h/a}(t)}\right|_T = 0, \tag{97}$$

i.e. no behavioral dynamic complementarity exists for fixed length of life.

The derivation for self-reinforcing dynamic complementarity for health is a bit more cumbersome. Start with (57) and take the derivative of $H(t)$ with respect to $q_{h/a}(t)$. Next use (56) to obtain an expression for $\partial q_{h/a}(s)/\partial q_{h/a}(t)$ and substitute it into the previous result. Next, take the derivative with respect to H_0, use (56) to express $\partial q_{h/a}(s)/\partial H_0$ in terms of $\partial q_{h/a}(0)/\partial H_0$ and use (82) to obtain

$$\left.\frac{\partial^2 H(t)}{\partial H_0 \partial q_{h/a}(t)}\right|_T = \left(\frac{1-2\gamma_H}{1-\gamma_H}\right) \frac{\int_0^t \mu_H(s) q_{h/a}(s)^{\frac{3\gamma_H - 2}{1-\gamma_H}} e^{-\int_s^t [2d_H(x)+r]dx} ds}{\int_0^T \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H - 1}{1-\gamma_H}} e^{\int_0^s [2d_H(x)+r]dx} ds}. \tag{98}$$

One can start the problem at any time t'' so that we have the more general result

$$\left.\frac{\partial^2 H(t)}{\partial H(t'') \partial q_{h/a}(t)}\right|_T = \left(\frac{1-2\gamma_H}{1-\gamma_H}\right) \frac{\int_{t''}^t \mu_H(s) q_{h/a}(s)^{\frac{3\gamma_H - 2}{1-\gamma_H}} e^{-\int_s^t [2d_H(x)+r]dx} ds}{\int_{t''}^T \mu_H(s) q_{h/a}(s)^{\frac{2\gamma_H - 1}{1-\gamma_H}} e^{\int_{t''}^s [2d_H(x)+r]dx} ds}. \tag{99}$$

It follows that the sign of behavioral dynamic complementarity for health depends on the parameter γ_H. Specifically, when $\gamma_H < 0.5$ behavioral dynamic complementarity exists, for $\gamma_H = 0.5$ it does not, and in case $\gamma_H > 0.5$, the opposite of behavioral dynamic complementarity occurs: health produced at one stage *reduces* health investments at later stages.

A.4 First-order conditions calibration model

The Lagrangian of the problem in section 4 is (e.g., Friesz, 2010):

$$
\begin{aligned}
\Im_t &= \sum_{t=0}^{S-1}\left[\frac{U(C_t,H_t)}{(1+\beta)^t}-p_t^S\right]+\sum_{t=S}^{T-1}\frac{U(C_t,H_t)}{(1+\beta)^t}+\\
&\sum_{t=0}^{S-1} q_{t+1}^A\left[-A_{t+1}+(1+r)A_t-p_C C_t-p_m m_t+c_t^S\right]+\\
&\sum_{t=S}^{T-1} q_{t+1}^A\left[-A_{t+1}+(1+r)A_t+\gamma_w w_t\left(S,t-S\right)H_t^{\gamma_H}-p_C C_t-p_m m_t\right]\\
&\sum_{t=0}^{T-1} q_{t+1}^H\left[-H_{t+1}+H_t+\mu_I\left(t,S\right)m_t^{\alpha}-d_t H_t^{\nu}\right],\ t=0,\dots,T-1, \qquad (100)
\end{aligned}
$$

where q_t^H is the adjoint variable associated with the dynamic equation (30) for the state variable health H_t, and q_t^A is the adjoint variable associated with the dynamic equation (32) for the state variable assets A_t. Note that $t=0$ corresponds to age 16, the legal minimum school-leaving age, and S represents the additional years of schooling beyond compulsory.

Associated with the Lagrangian we have the following conditions:

$$
\begin{aligned}
\frac{\partial \Im_t}{\partial A_t} &= 0 \Rightarrow\\
q_{t+1}^A &= q_t^A/(1+r) \Leftrightarrow\\
q_{t+1}^A &= \frac{q_1^A}{(1+r)^t}, \qquad (101)\\
\frac{\partial \Im_t}{\partial H_t} &= 0 \Rightarrow\\
q_{t+1}^H &= \frac{1}{(1-d_t^H\nu H_t^{\nu-1})}\left[q_t^H-\frac{\partial U/\partial H_t}{(1+\beta)^t}-q_{t+1}^A\gamma_w\gamma_H w_t\left(S,t-S\right)H_t^{\gamma_H-1}\right]\Rightarrow\\
q_{t+1}^{h/a} &= \frac{1}{(1-d_t^H\nu H_t^{\nu-1})}\\
&\times\left[q_t^{h/a}(1+r)-\frac{1}{q_1^A}\frac{\partial U}{\partial H_t}\left(\frac{1+r}{1+\beta}\right)^t-\gamma_w\gamma_H w_t\left(S,t-S\right)H_t^{\gamma_H-1}\right], \qquad (102)
\end{aligned}
$$

$$\begin{aligned}
\frac{\partial \Im_t}{\partial C_t} &= 0 \Rightarrow \\
\frac{\partial U}{\partial C_t} &= q_1^A p_C \left(\frac{1+\beta}{1+r}\right)^t && (103) \\
\frac{\partial \Im_t}{\partial m_t} &= 0 \Rightarrow \\
q_{t+1}^H &= q_{t+1}^A \left\{\frac{p_m}{\alpha \mu_I(t,S)}\right\} m_t^{1-\alpha} && (104)
\end{aligned}$$

We start with the initial condition for health, H_0. Initial consumption C_0 then follows from the first-order condition for consumption (103), which, for the assumed utility function (29), can be numerically solved from the equation

$$\mu_U \left[\lambda C_t^\zeta + (1-\lambda) H_t^\zeta\right]^{\frac{1-\rho-\zeta}{\zeta}} \lambda C_t^{\zeta-1} = q_1^A p_C \left(\frac{1+\beta}{1+r}\right)^t. \tag{105}$$

Initial consumption C_0 is a function of initial health H_0, the utility share of consumption λ, the elasticity of substitution between consumption and health $1/(1-\zeta)$, the constant of relative risk aversion ρ, the scale parameter of utility μ_U, the price of goods and services p_C, and the marginal value of initial wealth q_1^A.

Next, the initial level of health investment m_0 follows from the initial relative marginal value of health $q_1^{h/a}$ (see 104). In particular,

$$m_0 = \left[q_1^{h/a} \frac{\alpha \mu_I(0,S)}{p_m}\right]^{\frac{1}{1-\alpha}}. \tag{106}$$

The initial level of health investment m_0 is a function of the price of goods and services p_m, the efficiency of health investment, governed by α and μ_I, and the initial marginal value of wealth q_1^A and of health q_1^H.

Health in the next period H_1 is determined by the dynamic equation (30). Assets in the next period A_1 follow from the initial condition for assets A_0 and the dynamic equations for assets, (31) and (32).

Using the functional form for the utility function (29) and earnings (110), the relative marginal value of health is updated according to

$$\begin{aligned}
q_{t+1}^{h/a} = \frac{1}{1 - d_t^H \nu H_t^{\nu-1}} \Bigg[& q_t^{h/a}(1+r) - \frac{(1-\lambda)\mu_U}{q_1^A} \left[\lambda C_t^\zeta + (1-\lambda) H_t^\zeta\right]^{\frac{1-\rho-\zeta}{\zeta}} H_t^{\zeta-1} \left(\frac{1+r}{1+\beta}\right)^t \\
& -\gamma_w \gamma_H w_t\left(S, t-S\right)\right) H_t^{\gamma_H - 1} \Bigg] \qquad (107)
\end{aligned}$$

The solutions for consumption C_t, health investment m_t, health H_t, and wealth A_t, for every period t are functions of the initial marginal values of wealth q_1^A and health q_1^H.

In the final period, the end conditions for health $H_T = H_{\min}$ and for assets A_T determine q_1^A and q_1^H.

We employ the downhill simplex method (Nelder and Mead, 1965) to iteratively determine the initial marginal value of wealth q_1^A and of health q_1^H that satisfy the end conditions A_T and H_T. We use the usual values $\alpha_{NM} = 1$, $\gamma_{NM} = 2$, $\rho_{NM} = 0.5$ and $\sigma_{NM} = 0.5$ for the reflection, expansion, contraction and shrink coefficients, respectively.

To allow for differential lengths of schooling and longevity one needs to optimize over all possible lengths of schooling S and life T. We achieve this by first solving the problem conditional on length of schooling S and life T, inserting the optimal solutions C_t^*, H_t^* into the "indirect utility function"

$$V_{S,T} \equiv \sum_{t=0}^{S-1} \left(\frac{U(C_t^*, H_t^*)}{(1+\beta)^t} - p_t^S \right) + \sum_{t=S}^{T-1} \frac{U(C_t^*, H_t^*)}{(1+\beta)^t}, \tag{108}$$

and maximizing $V_{S,T}$ with respect to S and T by evaluating $V_{S,T}$ for different values of S and T (Manuelli et al., 2012).

A.5 Calibration

We fix a large number of model parameters by taking values from the literature (see Table 2). In particular, we set $\alpha = 0.75$ in line with Hugonnier et al. (2013, 2020) who estimated values of 0.77 and 0.73, respectively. We follow Scholz and Seshadri (2016) in setting $A_0 = A_T = 0$, and Galama and Van Kippersluis (2019) and Chen et al. (2017) by setting $H_0 = 100$ and $H_T = 15$. The parameters $\rho = 3$, $\zeta = -3.6$, and $\lambda = 0.7$ are taken from Scholz and Seshadri (2016). We follow Blau (2008) by setting the time preference rate $\beta = 0.03$ and the interest rate r equal to the time preference rate β. We set prices of consumption and health investment to 1, and follow Galama and Van Kippersluis (2019) in setting ν equal to 0.3.

We use the Panel Study of Income Dynamics (PSID) between 1999 and 2005 to estimate the earnings function, with prices in 2000 US dollars using the Consumer Price Index.[24] We take men aged 20 to 65, and estimate a Mincer equation for log hourly wages,

$$\ln w_t = \pi_0 + \pi_1 S + \pi_2 (t - S) + \pi_3 (t - S)^2 + \varepsilon_t \qquad S < t < T - 1, \tag{109}$$

and the earnings equation (see 36)

$$\ln Y_t = \ln \gamma_w + \ln w_t + \gamma_H \ln H_t + \varepsilon_t, \tag{110}$$

to estimate the parameters π_0, π_1, π_2, π_3, γ_w, and γ_H.

The remaining parameters to be calibrated are: κ, a, b, p^S and B. The data moments used to calibrate the parameters are

[24] https://fred.stlouisfed.org/series/CPIAUCSL#0

1. five-year averages of health investment for the average high-school graduate over the period 1999-2005
2. five-year averages of a health index for the average high-school graduate over the period 1999-2005
3. life expectancy for the average high-school graduate in 2000
4. years of schooling for the average high-school graduate over the period 1999-2005

We use the U.S. Medical Expenditure Survey (MEPS) in the years 1999-2005 to compute medical expenditures. The MEPS measure of medical expenditures is total expenditures and includes both out-of-pocket expenditures and expenditures covered by the insurance company. Following Fonseca et al. (2020) we scale this variable up to the level of per capita personal health-care spending from the National Health Expenditures (NHE), excluding long-term care (LTC) expenditures.

Following Poterba et al. (2011, 2013), we use PSID data for the years 1999-2005 to construct a health index. We use the first principal component of a rich set of health measures including: (i) a binary indicator of poor self-reported health; (ii) the answer to the questions of whether a doctor ever told the respondent that he was suffering from acute myocardial infarctions (AMI), arthritis, asthma, cancer, diabetes, heart conditions, high blood pressure, learning disorders, lung disease, or stroke; (iii) mental health problems, (iv) activities of daily living (ADLs), and (v) body-mass index (BMI). We arbitrarily scale this health index such that initial health is equal to 100 and health at age 75 is equal to 15. For both health and health investment we only use data moments after ages 25 to ensure that individuals in our data have completed their schooling career. In 2000, male life expectancy at age 25 was 50, hence we adopt $T = 75$ for the average high-school graduate (Sasson, 2016).[25] We compute average years of schooling of male high-school graduates to be 13.2 in the PSID for the period 1999-2005.

In the calibration, we iterate over a grid of parameter values, and compute the weighted sum of the squared deviation between the simulated moments and the data moments, where the deviations are scaled by the inverse of the variance of each observed moment (e.g., DellaVigna, 2018). To ensure a balanced weight of each moment, we reweigh the moments for life expectancy and years of schooling by a factor 10, since the moments for health investment and health each represent ten five-year averages (versus one moment for schooling and one for life expectancy). The parameter values that jointly produce the smallest sum of the squared deviation are presented in Table 2.

Consumption and assets: In our calibration, we use average consumption among 16-74 year olds using years 1999-2001 PSID data and constructed following Andreski et al. (2014) as a moment to check the predictive validity of our model. Figure 12 shows the simulated life-cycle profiles for assets and consumption, respectively.

[25]This life expectancy and its variation over educational groups is similar to those reported by Meara

Table 2: Parameter values for the calibrated simulations

Parameter	Description	Value
Parameters taken from the literature		
α	Degree of decreasing returns in health investment	0.75
r	Interest rate	0.03
β	Time preference rate	0.03
ρ	Coefficient of relative risk aversion	3
ζ	Elasticity of substitution parameter	-3.6
λ	Utility share of consumption vs health	0.7
ν	Dependency of depreciation on health	0.3
H_0	Initial health	100
H_T	Terminal health	15
A_0, A_T	Assets	0
Normalized parameters		
μ_C	Efficiency of consumption	1
p_m	Price of medical care	1
p_c	Price of consumption	1
Estimated parameters		
γ_w	Earnings parameter 1	6.8482
γ_H	Earnings parameter 2	0.0489
π_0	Mincer equation constant	2.0407
π_1	Mincer equation returns to education	0.0932
π_2	Mincer equation experience	0.0068
π_3	Mincer equation experience squared	-0.00018
Calibrated parameters		
κ	Efficiency of health investment	0.015
a	Deterioration rate intercept	0.00069
b	Deterioration rate slope	0.00046
p^S	Psychic cost of schooling	0.0045
B	Constant shifter of utility	0.0022

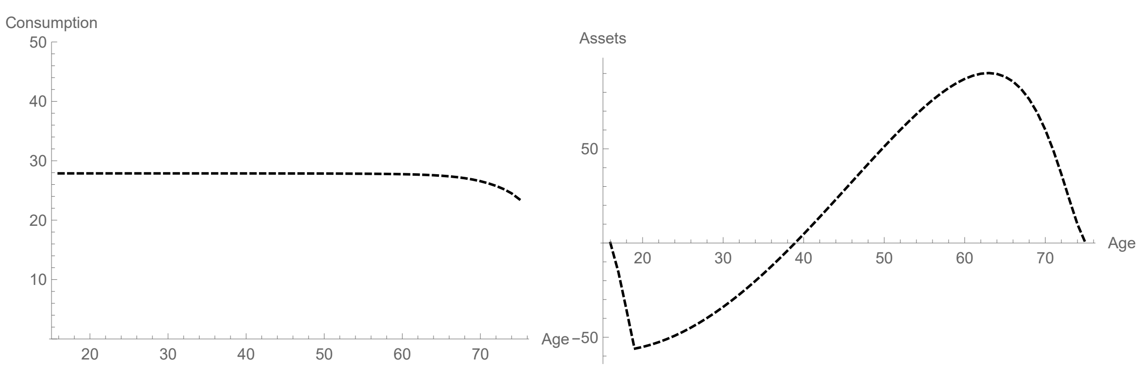

Figure 12: *Consumption and assets fitted profiles.*

A.6 Variation in the price of health investment p_m and life-cycle choices

So far, we have limited the discussion to variations in endowments of the stocks in our model (health, skill and wealth). Here, we explore whether the results also hold when we vary other model parameters. We take the price of health investment as an example, and show the corresponding life-cycle profiles when the price of health investment p_m is reduced by 5%.

As can be seen in Figure 13, our main findings hold also for the model parameter p_m: when the price of health investment is reduced, individuals will invest more in health, optimally choose a longer duration of life and attend school longer. They will be healthier and spend more on consumption. In contrast, when longevity is exogenously fixed, there is hardly any response to a lower price of medical care. Only lifetime consumption is slightly higher, which simply reflects that savings on health investment are being consumed, consistent with the earlier patterns in our simulations and those of the stylized Ben-Porath model (see Table 1).

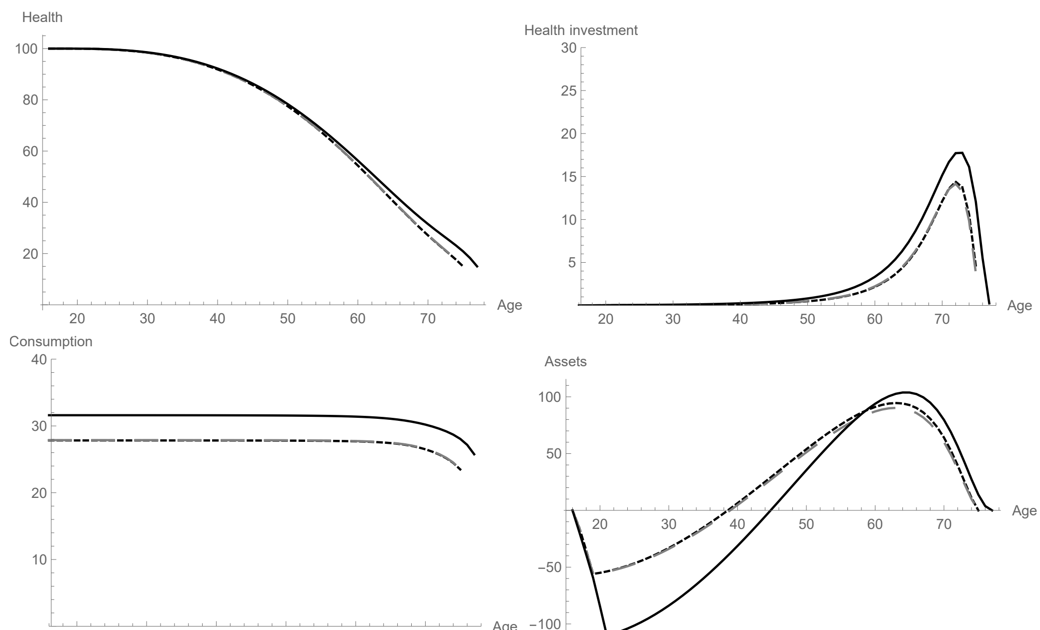

Figure 13: *Top-left: Health (top-left), health investment (top-right), consumption (bottom-left) and assets (bottom-right) over the life-cycle for* $p_m = 1$ *(baseline model; short-dashed line),* $p_m = 0.95$ *(T fixed; long-dashed line) and* $p_m = 0.95$ *(T free; solid line).*

et al. (2008) and Rostron et al. (2010).