USC Dornsife
Center for Economic and Social Research
USC Schaeffer
Leonard D. Schaeffer Center for Health Policy & Economics

A Theory of Socioeconomic Disparities in Health Over the Life Cycle

Titus J. Galama, Hans van Kippersluis

Paper No: 2015-016

CESR-SCHAEFFER WORKING PAPER SERIES

A Theory of Socioeconomic Disparities in Health Over the Life Cycle

Titus J. Galama and Hans van Kippersluis*

First version: July 22, 2010
This version: July 8, 2015

Abstract

Understanding of the substantial disparity in health between socioeconomic status (SES) groups is hampered by the lack of a sufficiently comprehensive theoretical framework to interpret empirical facts and to predict yet untested relations. Motivated by the observation that medical care explains only a relatively small part of the SES-health gradient, we present a life-cycle model that incorporates several additional behaviors that potentially explain (jointly) a large part of the observed disparities. In our model, choices regarding lifestyle, working conditions, labor-force participation, living conditions, and health investment, provide mechanisms through which SES, health, and longevity are related. As a result, the model provides not only a conceptual framework for the SES-health gradient but also more generally an improved framework for the production of health.

Keywords: socioeconomic status, health, human capital, health behavior
JEL Codes : D91, I10, I12, I14, J24

*Galama: University of Southern California, Dornsife Center for Economic and Social Research (CESR), USA, galama@usc.edu. Van Kippersluis: Erasmus School of Economics, The Netherlands, PO Box 1738, 3000 DR Rotterdam; and Tinbergen Institute, The Netherlands, hvankippersluis@ese.eur.nl

1 Introduction

Disparities in health across socioeconomic status (SES) groups – often called the SES-health gradient – are substantial. For example, Case and Deaton (2005) show how in the United States, a 20 year old low-income (bottom quartile of family income) male, on average, reports to be in similar health as a 60 year old high-income (top quartile) male. In cross sectional data the disparity in health between low and high SES groups appears to increase over the life cycle until ages 50-60, after which it narrows. These patterns exist across a wide range of measures of SES, such as education and wealth, and across all indicators of health, including the onset of chronic diseases, disability and mortality (e.g., Adler et al. 1994; Marmot, 1999). The pattern is also remarkably similar between countries with relatively low levels of protection from loss of work and health risks, such as the U.S., and those with stronger welfare systems, such as the Netherlands (Case and Deaton, 2005; Smith 2007; Van Kippersluis et al. 2010).

Recent significant contributions to the understanding of socioeconomic disparities in health have concentrated on the identification of causal effects, but have stopped short of uncovering the underlying mechanisms that produce the causal relationships. For example, education is found to have a causal protective effect on mortality (Lleras-Muney, 2005) but it is not known exactly how the more educated achieve their health advantage (Cutler and Lleras-Muney, 2010).

Understanding of the relative importance of underlying mechanisms responsible for the observed relationships is hampered by the lack of a sufficiently comprehensive theory. Case and Deaton (2005) argue that it is extremely difficult to understand the relationships between health, education, income, wealth and labor-force status without some guiding theoretical framework. Integrating the roles of proposed mechanisms and their long-term effect into a theoretical framework allows researchers to disentangle the differential patterns of causality and assess the interaction between mechanisms. Such understanding is essential in designing effective policies to reduce disparities (Deaton, 2002). It is no surprise then that several authors (e.g., Case and Deaton, 2005; Cutler, Lleras-Muney and Vogl, 2011) have pointed to the absence of a theory of SES and health over the life cycle and have emphasized the importance of developing one.

This paper develops a conceptual framework for health and SES in which the SES-health gradient is the outcome of rational (constrained) individual choices made over the life cycle. The paper makes two main contributions. The first main contribution is of a fundamental nature and consists of extending the canonical human-capital model for the demand for health (Grossman, 1972a; 1972b) in two ways. First, we employ a relatively straightforward extension by allowing for decreasing returns to scale in health investment of the health-production process (Ehrlich and Chuma, 1990; Galama, 2015) and we use a different interpretation of the "equilibrium" condition for health (Galama, 2015). This addresses a number of issues with the conventional theory that are the result of a mathematical degeneracy. Second, motivated by the observation that differences in medical care usage explain only a small part of the health gradient (e.g., Adler et al.

1993), we include several additional decisions regarding health (besides health investment), such as choices regarding lifestyle (exercise, healthy/unhealthy consumption), working conditions, labor-force participation, and longevity, as mechanisms generating disparities in health. We are the first to develop such a comprehensive theory of the SES-health gradient, by integrating the most important interactions between health, longevity, health behavior, and SES (wealth, education, and earnings) during adulthood.[1]

Our second main contribution consists of deriving detailed predictions from the theory by performing comparative dynamic analyses of the effects of wealth, earnings, education, and health on health behavior. We are the first to successfully perform such analyses for a comprehensive theory with multiple health behaviors (to better model health) and multiple dimensions of SES (to model disparities in these measures).[2] Comparative dynamic analyses enable explorations of the role of SES and other model parameters on health, health behaviors, and longevity. This allows us to generate new predictions and explain stylized facts regarding health behavior and the SES-health gradient. We explain these two main contributions in more detail below.

The Grossman model (Grossman, 1972a; 1972b; 2000) is the canonical (textbook) theory of health, and therefore provides a natural foundation for a framework of the SES-health gradient. However, it has two main limitations. First, the Grossman model and the subsequent literature it spawned assume a health-production process that is linear in investment (See Galama, 2015, and references therein). The Hamiltonian of the associated constrained optimization problem is then also linear in investment and the optimality condition for investment (derived by taking the first derivative of the Hamiltonian with respect to investment) is no longer a function of investment. Thus the optimality condition cannot be employed to determine the optimal level of investment (see Galama, 2015, for detail). This causes the model to essentially break down. As a result of this so-called degeneracy, the Grossman model is not able to explain a number of the most salient features of the SES-health gradient. For example, Case and Deaton (2005) argue that

[1]Several papers contain components of our generalized theory of health. The Grossman model (Grossman, 1972a;b) contains health and health investment and interactions with earnings, and wealth, but suffers from the aforementioned degeneracy and it does not include other health behaviors. Ehrlich and Chuma (1990) were the first to address the degeneracy and to introduce endogenous longevity, but their model does not include other decisions besides health investment. Forster (2001) models the relation between health, longevity, healthy consumption, and unhealthy consumption but does not model health investment, wealth accumulation, or job conditions. Case and Deaton (2005) include unhealthy consumption as well as physical effort on the job, but their model suffers from the degeneracy and they do not model longevity.

[2]Ehrlich and Chuma (1990) generate a set of directional predictions (their Table 3), based on a pioneering comparative dynamic analysis of the Grossman model with endogenous longevity. However, while they are able to generate the sign of the effects (broadly whether an effect is more or less likely to be positive or negative), they do not present dynamic results. Further, they do not consider other health behaviors besides health investment. Ried (1998) presents a comparative dynamic analysis of the Grossman model but his model suffers from the degeneracy. Eisenring (1999) presents a comparative dynamic analysis of a much-simplified Grossman-type model without consumption, without wealth accumulation, and without additional health behaviors.

while the Grossman model can explain differences in the *level* of health between low and high SES groups, it cannot explain differences in the *rate* of health decline. In other words, it cannot account for the widening of the SES-health gradient with age till late middle life, as is observed in empirical studies.[3] In our own work (Galama and Van Kippersluis, 2013; Galama, 2015) we have come to the conclusion that there is a fairly simple, but so far largely misunderstood, solution to the degeneracy. Assuming decreasing-returns-to-scale (DRTS) in the health-production function solves the degeneracy (Ehrlich and Chuma, 1990). Galama (2015), however, has shown that this is not sufficient. A reinterpretation of the "equilibrium" condition for health is also needed to address the conventional model's inability to explain differences in the rate of health decline between SES groups.

The second limitation of the Grossman model is that it only contains health investment, which is broadly identified with medical care and time investments (e.g., doctor visits). We conduct an extensive review of the literature from multiple disciplines to identify the most important mechanisms through which specific socioeconomic characteristics such as wealth, earnings, and education, interact with health. We then incorporate, besides health investment, choices regarding lifestyle (exercise, healthy/unhealthy consumption), working conditions, labor-force participation, and longevity, as mechanisms generating disparities in health. With these two limitations addressed, our formulation can account for a greater number of empirical patterns and suggests that our extended Grossman model provides a suitable life-cycle framework for the SES-health gradient.

The comparative dynamic analyses, which constitute our second main contribution, predict that greater wealth, higher earnings and education induce individuals to invest more in health, shift consumption toward healthy consumption, and enable individuals to afford healthier working environments (associated with lower levels of physical and psychosocial health stresses) and living environments. As a result, they live longer.

The mechanism through which wealth, earnings, and education operate is by increasing the marginal value of health relative to the marginal value of wealth. Intuitively, wealth, earnings, and the higher earnings associated with education, relax the budget constraint and increase the relative importance of health compared to wealth. Additionally, at high levels of wealth, and hence consumption, only limited marginal utility is gained from additional consumption and it is more beneficial to invest in health, thereby extending length of life (Becker, 2007; Hall and Jones, 2007).

A higher marginal value of health relative to wealth, in turn, increases the health benefit of healthy consumption, and the health cost of unhealthy working (and living) environments, and unhealthy consumption. This leads to healthier behavior and gradually to greater health advantage with age. The more rapidly worsening health of low SES individuals may lead to early withdrawal from the labor force and associated lost earnings,

[3]The conventional model has been further critized for its inability to predict the observed negative association between health and medical care (e.g., Zweifel and Breyer, 1997) and for the lack of history in the model's solutions (e.g., Usher, 1976). Other problems with some of the predictions and properties of health-production models have been pointed out in the literature (see Grossman, 2000, for a review and rebuttal of these; Zweifel, 2013; Kaestner, 2013; Dalgaard and Strulik, 2014; Laporte, 2014).

further widening the gradient in early- and mid-age. The model allows for a subsequent narrowing of the SES-health gradient, due to mortality selection and potentially because low SES individuals increase their health investment and improve their health behavior faster as a result of their more rapidly worsening health. Our model is thus able to replicate the life cycle patterns of the SES-health gradient.

The theory can further be employed to generate novel testable predictions. We highlight a few here and discuss these and several others more extensively in section 4. First, we predict a central role for our concept of a "health cost" of unhealthy behaviors. The health cost is the marginal value (in terms of life-time utility) of health lost due to detrimental health behaviors. It takes into account all future consequences of current health behavior. As a result of differences in the health cost, our theory predicts that high SES individuals are more likely to drink moderately but less likely to drink heavily (Van Kippersluis and Galama, 2014); and that individuals are willing to accept unhealthy working conditions in mid-life, given the high monetary benefits during those years, but that their willingness declines later in life due to increasing health cost. Thus, the concept of a health cost has potential for explaining variation in health behaviors over the lifecycle and across SES groups.[4]

Second, we predict that the ability to postpone death (endogenous longevity) is crucial in explaining observed associations between SES and health. Absent the ability to extend life (fixed horizon), associations between SES and health are small. If, however, life can be extended, SES and health are positively associated and the greater the degree of life extension, the greater is their association. The intuition behind this result is that the horizon (longevity) is a crucial determinant of the return to investments in health. This suggests that in settings where it is difficult for wealthier, higher income and higher educated individuals to increase life expectancy (e.g., due to a high disease burden, competing risks, low efficiency of health investment, etc.), health disparities across socioeconomic groups would be smaller.

These are just a few examples of how the theory can be used as a conceptual framework to generate testable predictions for the complex relationships between SES and health. Because of the inclusion of a rich set of health behaviors, the model can also more generally serve as a conceptual framework of health production. Further, the theory can be employed to understand disparities between SES groups in the value of health as well as in the value of life.[5] The theory is rich, and it is impossible to produce an exhaustive list of its possible

[4]While the concept of a health cost (benefit) of unhealthy (healthy) behavior is not new, explicit theoretical modeling is, and so is our formal definition of the concept. The literature on the value of a statistical life (e.g., Viscusi and Aldy, 2003) focuses on the cost of reductions in life (mortality) rather than in health (morbidity) as in our theory. Even the seminal theory of rational addiction (Becker and Murphy, 1988), while arguing conceptually for an effect of unhealthy addictive consumption on health, does not explicitly model this effect. To the best of our knowledge, only Forster (2001) and Case and Deaton (2005) have previously explicitly modeled behavior as a choice variable affecting health. Case and Deaton's (2005) model, however, suffers from the degeneracy. And, while Forster models a health cost / health benefit, by allowing consumption to affect health, he does not formally define or discuss the concept.

[5]Previous papers employing a life cycle model for the value of life include Rosen (1988), Ehrlich (2000),

uses. Researchers can use the detailed comparative dynamic analyses presented here as a template to study their own questions of interest.

The paper is organized as follows. Section 2 reviews the literature on health disparities by SES to determine the essential components required in a theoretical framework. Developing a theory requires simplification and a focus on the essential mechanisms relating SES and health. To keep the model relatively simple we focus on explaining health disparities in adulthood.[6] We highlight potential explanations for the SES-health gradient that a) explain a large part of the gradient and b) are relatively straightforward to include in our theoretical framework. Based on these principles we develop our theoretical formulation in section 3. Section 4 presents the dynamic and comparative dynamic analyses of the model and makes predictions. Section 5 summarizes and concludes.

2 Components of a theory of the gradient

In this section we review the empirical literature to determine the essential components of a theory of health disparities by SES in adulthood. Based on these findings we present our theoretical formulation.

A significant body of research across multiple disciplines (including epidemiology, sociology, demography, psychology, evolutionary biology, and economics) has been devoted to documenting and explaining the substantial disparity in health between low and high SES groups. The pathways linking the various dimensions of SES to health are diverse: some cause health, some are caused by health and some are jointly determined with health (e.g., Cutler, Lleras-Muney and Vogl, 2011). Several key findings can be identified from a review of the literature.

Medical care: Utilization of medical services and access to care explain only a relatively small part of the association between SES and health (e.g., Adler et al. 1993). Therefore, additional mechanisms, besides medical care have to be included in the model.

Work environment and life style: Epidemiological research has used longitudinal studies to examine the role of behavioral, material, psychosocial, and healthcare related pathways in explaining SES-health associations (e.g., House et al. 1994; Lynch, Kaplan and Shema, 1997). These studies highlight the importance of lifestyles (e.g., smoking, drinking, caloric intake, and exercise), psychosocial and environmental risk

and Murphy and Topel (2006). In contrast to these papers, we allow health to be endogenously determined separately from the force of mortality. This enables independent analyses of the value of life and of the value of health.

[6] James Heckman and colleagues have emphasized the role of childhood cognitive and non-cognitive abilities in determining both education and health outcomes in later life (e.g., Heckman, 2007; Cunha and Heckman, 2007; Conti et al. 2010; Cambell et al. 2014; see also Almond and Currie, 2011), and there is strong evidence that parental, especially maternal, SES influences the evolution of child health (Currie, 2009), suggesting that part of the SES-health gradient may be determined very early in life.

factors, neighborhood social environment, acute and chronic psychosocial stress, social relationships and supports, sense of control, fetal and early childhood conditions, and physical, chemical, biological, and psychosocial hazards and stressors at work.

During adulthood, two of those mechanisms appear to be of particular importance: (i) working conditions, and (ii) lifestyles. Using three different datasets from the U.K. and the U.S., House et al. (1994) find that features of the psychosocial working environment, social circumstances outside work, and health behavior jointly account for much of the social gradient in health. Some epidemiological studies suggest that around two thirds of the social gradient in health deterioration could be explained by working environment and life style factors alone (Borg and Kristensen, 2000). Low SES individuals more often perform risky, manual labor than high SES individuals, and their health deteriorates faster as a consequence (Marmot et al. 1997; Ravesteijn et al. 2013). Case and Deaton (2005) find that those who are employed in manual occupations have worse health than those who work in professional occupations and that the health effect of occupation operates at least in part independently of the personal characteristics of the workers. Extensive research further suggests an important role of lifestyle factors, particularly smoking, in explaining SES disparities in health (Mackenbach et al. 2004). Fuchs (1986) argues that in developed countries, it is personal lifestyles that cause the greatest variation in health.

Education: Education appears to be a key dimension of SES and studies suggest education has a causal protective effect on health and mortality (Lleras-Muney, 2005; Conti et al. 2010; Van Kippersluis et al. 2011).[7] Education increases wages (e.g., Mincer, 1974), thereby enabling purchases of health investment goods and services (though higher wages also increase the opportunity cost of time). Education potentially increases the efficiency of medical and preventive care usage and time inputs into the production of health investment (Grossman, 1972a; 1972b). And, the higher educated are better able at managing their diseases (Goldman and Smith, 2002), and benefit more from new knowledge and new technology (Lleras-Muney and Lichtenberg, 2005; Glied and Lleras-Muney, 2008).

Financial measures of SES: Financial measures of SES may have a more limited impact on health than education. Smith (2007) finds no effect of financial measures of SES (income, wealth, and change in wealth) on changes in health. Cutler, Lleras-Muney and Vogl (2011) provide an overview of empirical findings and conclude that the evidence points to no, or a very limited, impact of income or wealth on health (see also Michaud and Van Soest, 2008). Yet, this view is not unequivocally accepted. For example, Lynch, Kaplan and Shema (1997) suggest that accumulated exposure to economic hardship causes bad health, and Herd, Schoeni and House (2008) argue that there might be causal effects of financial resources on health at the bottom of the income or wealth distribution (see also Mani et al. 2013). Income and wealth enable purchases of medical care and thereby

[7] Yet, see, e.g., Albouy and Lequien (2009) and Clark and Royer (2013) who could not establish a causal effect of education on mortality.

potentially allow for better health maintenance. Further, more affluent workers may choose safer working and living environments since safety is a normal good (Viscusi, 1978). But, higher wages are also associated with higher opportunity costs, which would reduce the amount of time devoted to health maintenance.

Health and labor-force withdrawal: In the other direction of causality, studies have shown that perhaps the most dominant causal relation between health and dimensions of SES is the causal impact that poor health has on one's ability to work and hence produce income and wealth (e.g., Case and Deaton, 2005; Smith, 2007). Healthy individuals are also more productive and earn higher wages (Currie and Madrian, 1999).

Joint determination: Fuchs (1986) has argued that the strong correlation between SES and health may be due to differences in the time preferences of individuals, which affects investments in both education and health, and helps to explain variations in cigarette smoking, diet, and exercise. Cutler and Lleras-Muney (2008) argue that differences in individual preferences (risk aversion and discount rates) appear to explain only a small portion of the SES-health gradient, but they also note that preferences are difficult to measure, and that preferences with respect to health may differ from preferences with respect to finance. Other third factors known to contribute to the correlation between SES and health are cognitive and non-cognitive skills, in particular conscientiousness and self-esteem (Auld and Sidhu, 2005; Deary, 2008; Chiteji, 2010; Conti et al. 2010; Savelyev, 2014).

Gradient over the lifecycle: Health inequalities are largest in mid-life and narrow in later life. The literature provides competing explanations for this pattern. The *cumulative advantage* hypothesis states that health inequalities emerge by early adulthood and subsequently widen as economic and health advantages of higher SES individuals accumulate (House et al. 1994; Ross and Wu, 1996; Lynch, 2003). Any apparent narrowing of SES inequalities in late life is largely attributed to mortality selection, i.e., lower SES people are more likely to die which results in an apparently healthier surviving disadvantaged population.[8] The competing *age-as-leveler* hypothesis maintains that later in life deterioration in health becomes more closely associated with age than with any other predictor, i.e. through a greater equalization of health risks and/or of protections (House et al. 1994), with the result that the SES-health gradient narrows.

[8] Beckett (2000) and Baeten, van Ourti and van Doorslaer (2013), however, have demonstrated that the convergence in health inequalities in later life cannot be explained entirely, or even mostly, by mortality selection.

3 Theory

3.1 Theoretical formulation

In this section we formalize the above discussion on the features of a theoretical framework for the SES-health gradient over the life cycle. The aim is to understand the SES-health gradient as the outcome of rational constrained individual behavior.

A natural starting point for a theory of the relation between health and SES is a model of life cycle utility maximization. Our model is based on the Grossman model of the demand for health (Grossman, 1972a; 1972b; 2000) in continuous time (see also Wagstaff, 1986a; Ehrlich and Chuma, 1990; Zweifel and Breyer, 1997; Galama, 2015). The Grossman model provides a framework for the interrelationship between health, financial measures of SES (wealth, wages, and earnings), the demand for consumption, the demand for medical goods and services, and the demand for time investments in health (e.g., visits to the doctor, exercise). Health increases earnings (through reduced sick time) and provides utility. We add six additional features to the model.

First, we assume decreasing-returns-to-scale (DRTS) in investment of the health-production process. This addresses the degeneracy of the solutions for investment and health that characterizes commonly employed linear investment models (Ehrlich and Chuma, 1990; Galama, 2015). It is further attractive in that the health-production process is generally thought of as being subject to diminishing returns (Wagstaff, 1986b).

Second, individuals choose their level of "job-related health stress". The concept of job-related health stress can be interpreted broadly, ranging from physical working conditions (e.g., hard or risky labor) to psychosocial aspects of work (e.g., low social status, lack of control, repetitive work, etc) that are detrimental to health. Individuals may accept risky and/or unhealthy work environments, in exchange for higher pay (Muurinen, 1982; Case and Deaton, 2005), i.e. a compensating wage differential (Smith, 1776; Viscusi, 1978).[9]

Third, we allow consumption patterns to affect the health deterioration rate (Case and Deaton, 2005; see also Forster, 2001). We distinguish healthy consumption (such as the consumption of healthy foods, sports and exercise) from unhealthy consumption (such as smoking, excessive alcohol consumption). Healthy consumption provides utility, and is associated with health benefits in that it lowers the health deterioration rate. We interpret healthy consumption broadly to include decisions regarding housing and neighborhood.[10] Unhealthy consumption provides consumption benefits (utility) but increases the health deterioration rate.

Fourth, the effect of education on income is included in a straightforward manner by assuming a Mincer-type wage relation, in which earnings are increasing in the level of

[9] Evidence is strong that jobs with higher health and mortality risk demand a wage premium (Smith, 1978; Duncan and Holmlund, 1983).

[10] Living in an affluent neighborhood is an expensive, yet health-promoting and utility-generating choice of individuals. The choice of neighborhood (housing) is a constrained choice: low SES individuals cannot afford to live in more affluent areas.

education and in the level of experience of workers (e.g., Mincer, 1974).

Fifth, individuals endogenously optimize length of life (Ehrlich and Chuma, 1990). Longevity is an important health outcome and differential mortality by SES may explain part of the narrowing of the gradient in late life. Moreover, length of life is an essential horizon that determines the duration over which the benefits of health investments and healthy behaviors can be reaped.

Last, we include leisure, which jointly with sick time and time inputs into health investment and health behavior allows for the modeling of an implicit retirement decision. As health declines, increased sick time and increased demand for time inputs into health investment and healthy behavior reduce the amount of time that can be devoted to work, capturing possible reverse causality from health to labor force participation, and thereby financial measures of SES.

Individuals maximize the life-time utility function

$$\int_0^T U(t)e^{-\beta t}dt, \tag{1}$$

where T denotes total lifetime (endogenous), β is a subjective discount factor, and individuals derive utility $U(t) \equiv U[C_h(t), C_u(t), L(t), H(t)]$ from healthy consumption $C_h(t)$, unhealthy consumption $C_u(t)$, leisure $L(t)$, and from health $H(t)$. Time t is measured from the time an individual has completed her education and joined the labor force (e.g., around age 25 or so). Utility is assumed to be strictly concave and increasing in its arguments.

The objective function (1) is maximized subject to the following dynamic equations,

$$\frac{\partial H(t)}{\partial t} = I(t)^{\alpha} - d(t), \tag{2}$$

$$\frac{\partial A(t)}{\partial t} = rA(t) + Y(t) - p_{X_h}(t)X_h(t) - p_{X_u}(t)X_u(t) - p_m(t)m(t), \tag{3}$$

the total time budget Ω,

$$\Omega = \tau_w(t) + L(t) + \tau_I(t) + \tau_{C_h}(t) + \tau_{C_u}(t) + s[H(t)], \tag{4}$$

and we have initial and end conditions: $H(0)$, $H(T)$, $A(0)$ and $A(T)$ are given.[11]

Health $H(t)$ (equation 2) can be improved through investment in health $I(t)$ and deteriorates at the health deterioration rate $d(t) \equiv d[t, C_h(t), C_u(t), z(t), H(t); \xi(t)]$. The health-production function $I(t)^{\alpha}$ is assumed to exhibit DRTS ($0 < \alpha < 1$).[12] The

[11] In Grossman's original formulation (Grossman, 1972a; 1972b) length of life T is determined by a minimum health level $H_{\min}$ below which life cannot be sustained. If health reaches this level $H(t) = H_{\min}$ an individual dies, hence $H(T) \equiv H_{\min}$.

[12] Mathematically, this assumption is equivalent to assuming a linear process ($\alpha = 1$) and DRTS in the relation between the inputs of health investment goods / services $m(t)$ and own time $\tau_I(t)$ (as in Ehrlich and Chuma, 1990).

health deterioration rate depends endogenously on healthy consumption $C_h(t)$, unhealthy consumption $C_u(t)$, job-related health stress $z(t)$, and health $H(t)$.[13] $\xi(t)$ denotes a vector of exogenous environmental conditions. Consumption can be healthy ($\partial d/\partial C_h \leq 0$; e.g., healthy foods, healthy neighborhood) or unhealthy ($\partial d/\partial C_u > 0$; e.g., smoking). Greater job-related health stress $z(t)$ accelerates the "aging" process ($\partial d/\partial z > 0$). Finally, the deterioration rate depends in a flexible way on health, instead of the usual assumption of a linear relationship $d(t) = \delta(t)H(t)$ as in Grossman (1972a;b) and the related literature.[14]

Assets $A(t)$ (equation 3) provide a return r (the return on capital), increase with income $Y(t)$ and decrease with purchases in the market of healthy consumption goods $X_h(t)$, unhealthy consumption goods $X_u(t)$, and medical care $m(t)$, at prices $p_{X_h}(t)$, $p_{X_u}(t)$, and $p_m(t)$, respectively. Income $Y(t) \equiv Y[H(t), z(t); E, x(t)]$ is assumed to be an increasing function of health $H(t)$ ($\partial Y/\partial H > 0$) and of job-related health stress $z(t)$ ($\partial Y/\partial z > 0$; Case and Deaton, 2005). Further, income depends exogenously on the consumer's stock of knowledge (an individual's human capital exclusive of health capital), assumed to be a function of years of schooling E and years of working experience $x(t)$ (e.g., Mincer, 1974). We assume that individuals face no borrowing constraints.[15]

The total time available in any period Ω is the sum of all possible uses $\tau_w(t)$ (work), $L(t)$ (leisure), $\tau_I(t)$ (health investment), $\tau_{C_h}(t)$ (healthy consumption), $\tau_{C_u}(t)$ (unhealthy consumption) and $s[H(t)]$ (sick time). The resulting time budget constraint is shown in equation (4).

Goods and services $m(t)$ (e.g., medical care) as well as own time inputs $\tau_I(t)$ (e.g., exercise, time spent visiting a doctor, etc.) are used in the production of health investment $I(t)$. Similarly, goods $X_h(t)$ and $X_u(t)$ purchased in the market and own time inputs $\tau_{C_h}(t)$ and $\tau_{C_u}(t)$ are used in the production of healthy and unhealthy consumption, $C_h(t)$ and $C_u(t)$, respectively. The efficiency of the production of health investment $\mu_I(t; E)$ is assumed to be a function of the consumer's stock of knowledge E as the more educated are assumed to be more efficient consumers and producers of health investment (Grossman, 1972a; 1972b),

$$I(t) \equiv I[m(t), \tau_I(t), \mu_I(t; E)], \quad (5)$$

$$C_h(t) \equiv C_h[X_h(t), \tau_{C_h}(t), \mu_{C_h}(t)], \quad (6)$$

$$C_u(t) \equiv C_u[X_u(t), \tau_{C_u}(t), \mu_{C_u}(t)]. \quad (7)$$

[13]We follow Grossman (1972a, 1972b) in distinguishing between the production of health $I(t)^\alpha$ and the deterioration of health $d(t)$ and in allowing the deterioration rate $d(t)$ to be a function of health $H(t)$. We follow Case and Deaton (2005) in modeling health behaviors, such as job stress and consumption, as operating through the deterioration rate $d(t)$. These choices are somewhat arbitrary, e.g., behavior could also operate through the production process. Mathematically this would be equivalent, with the exception that with our current choice, investment is not a direct function of health or health behavior since the production process does not explicitly depend on health or health behavior.

[14]We use compact notation whenever possible by omitting the dependence of functions on controls and states (such as for the utility function $U(t)$ and the aging rate $d(t)$) and omitting the time component in derivatives, e.g., $\partial d/\partial C_h$. When confusion may arise, we include the explicit dependence, e.g., $\partial H(t)/\partial H_0$.

[15]Imperfect capital markets itself could be a cause of socioeconomic disparities in health if low income individuals face greater borrowing constraints and therefore cannot optimally invest in health.

Further, we implicitly assume that health investment $I(t)$ and job-related health stress $z(t)$ are non-negative. We do so by assuming DRTS of the health-production function in investment (see equation 2) and diminishing marginal benefits for job-related health stress. The notion here is that one cannot "sell" ones health through negative investment (see Galama and Kapteyn, 2011), nor can one "buy" health through negative job-related health stress.

We follow Grossman (1972a; 1972b; 2000) and assume that income $Y(t)$ is equal to the wage rate $w(t)$ times the amount of time spent working $\tau_w(t)$,

$$Y[H(t)] \equiv w(t)\left\{\Omega - L(t) - \tau_I(t) - \tau_{C_h}(t) - \tau_{C_u}(t) - s[H(t)]\right\}. \tag{8}$$

Individuals receive wages $w(t) \equiv w[t, z(t); E, x(t)]$, which are a function of job-related health stress $z(t)$

$$w(t) = w_*(t)[1 + z(t)]^{\gamma_w}, \tag{9}$$

where $\gamma_w \geq 0$ and $w_*(t) \equiv w_*[E, x(t)]$ represents the "stressless" wage rate, i.e., the wage rate associated with the least job-related health stress $z(t) = 0$.[16] The stressless wage rate $w_*(t)$ is a function of the consumer's education E and experience $x(t)$ (e.g., Mincer, 1974),

$$w_*(t) = w_E e^{\rho_E E + \beta_x x(t) - \beta_{x^2} x(t)^2}, \tag{10}$$

where education E is expressed in years of schooling, $x(t)$ is years of working experience, and ρ_E, β_x and β_{x^2} are coefficients, assumed to be positive.

Thus, we have the following optimal control problem: the objective function (1) is maximized with respect to the control functions $L(t)$, $X_h(t)$, $\tau_{C_h}(t)$, $X_u(t)$, $\tau_{C_u}(t)$, $m(t)$, $\tau_I(t)$, $z(t)$, the parameter T, and subject to the constraints (2, 3 and 4). The Hamiltonian (see, e.g., Seierstad and Sydsaeter, 1987) of this problem is:

$$\Im = U(t)e^{-\beta t} + q_H(t)\frac{\partial H(t)}{\partial t} + q_A(t)\frac{\partial A(t)}{\partial t}, \tag{11}$$

where $q_H(t)$ is the marginal value of health $H(t)$, defined as

$$q_H(t) = \frac{\partial}{\partial H(t)} \int_t^{T^*} U(*)e^{-\beta s} ds, \tag{12}$$

and $q_A(t)$ is the marginal value of wealth, defined as

$$q_A(t) = \frac{\partial}{\partial A(t)} \int_t^{T^*} U(*)e^{-\beta s} ds, \tag{13}$$

[16] Our model concerns individuals who participate in the labor force. Given that our frame of reference is the labor force we associate $z(t) = 0$ with the least amount of job-related health stress possible in employment, and since there is no obvious scale to job-related health stress we employ the simple relationship shown in equation (9).

where T^* denotes optimal length of life and $U(*)$ denotes the maximized utility function (see, e.g., Caputo, 2005). Hence $q_H(t)$ represents the marginal value of remaining life-time utility (from t onward) derived from additional health capital, and $q_A(t)$ represents the marginal value of remaining life-time utility derived from additional financial capital.

The transversality condition for optimal longevity T follows from the dynamic envelope theorem (e.g., Caputo, 2005, p. 293):[17]

$$\frac{\partial}{\partial T}\int_t^{T^*} U(*)e^{-\beta s}ds = \frac{\partial}{\partial T}\int_0^T \Im(t)dt \quad = \quad \Im(T) = 0. \tag{14}$$

3.2 First-order conditions

In this section we discuss the first-order conditions for optimization. We assume that an interior solution to the optimization problem exists. Detailed derivations are provided in Appendix B. The first-order condition for health investment is given by

$$q_{h/a}(t) = \pi_I(t), \tag{15}$$

where $q_{h/a}(t)$ represents the marginal benefit of health investment, defined as the ratio of the marginal value of health, $q_H(t)$, to the marginal value of wealth, $q_A(t)$ (throughout the paper we refer to $q_{h/a}(t)$ as the relative marginal value of health):

$$q_{h/a}(t) \equiv \frac{q_H(t)}{q_A(t)}, \tag{16}$$

and $\pi_I(t)$ represents the marginal cost of health investment,

$$\pi_I(t) \equiv \frac{p_m(t)}{\alpha[\partial I/\partial m]}I(t)^{1-\alpha} = \frac{w(t)}{\alpha[\partial I/\partial \tau_I]}I(t)^{1-\alpha}. \tag{17}$$

The marginal benefit of health investment increases in the marginal value of health $q_H(t)$ and decreases in the marginal value of wealth $q_A(t)$. If the marginal value of health is high individuals invest more in health, and if the marginal value of wealth is high individuals invest less, consume less, and save more. The marginal cost of health investment $\pi_I(t)$ increases with the level of health investment $I(t)$ due to decreasing returns to scale,[18] with the price of medical goods and services purchased in the market $p_m(t)$, and with the

[17]The marginal value of life extension is given by $\Im(T) = U(T)e^{-\beta T} + q_H(T)\,\partial H(t)/\partial t|_{t=T} + q_A(T)\,\partial A(t)/\partial t|_{t=T}$. When dividing by the marginal value of wealth, one obtains a measure for the monetary value of life, $\Im(T)/q_A(T) = U(T)/U_C(T) + q_{h/a}(T)\,\partial H(t)/\partial t|_{t=T} + \partial A(t)/\partial t|_{t=T}$. The monetary value of life is similar to the expressions obtained in Rosen (1988) (his equation 16) and Murphy and Topel (2006) (their equations 7 and 8), for health-neutral consumption. Our measure is richer since it additionally takes into account asset accumulation and health depreciation.

[18]Intuitively, at higher levels of investment, due to concavity of the health-production function $I(t)^\alpha$, an additional increment of investment produces a smaller improvement in health. Thus, the effective 'price' $\pi_I(t)$ increases with the level of investment $I(t)$.

opportunity cost of time $w(t)$, where the latter is a function of job-related health stress, $z(t)$.

The first-order condition for leisure is

$$\frac{\partial U}{\partial L} = q_A(0) w(t) e^{(\beta - r)t}, \tag{18}$$

a standard result equating the marginal utility of leisure $\partial U/\partial L$ to the marginal cost of leisure, which is a function of the marginal value of initial wealth $q_A(0)$, the individual's wage rate $w(t)$, and the difference between the time preference rate β and the return on capital r.

The first-order condition for healthy consumption is

$$\frac{\partial U}{\partial C_h} \quad = \quad q_A(0) \left[\pi_{C_h}(t) - \varphi_{dC_h}(t)\right] e^{(\beta - r)t}, \tag{19}$$

where $\pi_{C_h}(t) \equiv \pi_{C_h}[t, C_h(t), z(t); E, x(t)]$ is the marginal monetary cost of healthy consumption $C_h(t)$

$$\pi_{C_h}(t) \equiv \frac{p_{X_h}(t)}{\partial C_h/\partial X_h} = \frac{w(t)}{\partial C_h/\partial \tau_{C_h}}, \tag{20}$$

and $\varphi_{dC_h}(t) \equiv \varphi_{dC_h}[t, H(t), C_h(t), C_u(t), z(t); E, x(t), \xi(t)]$ is the marginal health benefit of healthy consumption

$$\varphi_{dC_h}(t) \equiv -q_{h/a}(t) \frac{\partial d}{\partial C_h}. \tag{21}$$

The marginal monetary cost of healthy consumption $\pi_{C_h}(t)$ (equation 20) is a function of the price of healthy consumption goods and services $p_{X_h}(t)$ and the opportunity cost of time $w(t)$, and represents the *direct* monetary cost of consumption. The marginal health benefit of healthy consumption $\varphi_{dC_h}(t)$ (equation 21), is equal to the product of the relative marginal value of health $q_{h/a}(t)$ and the "amount" of health saved $\partial d/\partial C_h$, and represents the marginal value of health saved.

Similarly, the first-order condition for unhealthy consumption is

$$\frac{\partial U}{\partial C_u} \quad = \quad q_A(0) \left[\pi_{C_u}(t) + \pi_{dC_u}(t)\right] e^{(\beta - r)t}, \tag{22}$$

where $\pi_{C_u}(t) \equiv \pi_{C_u}[t, C_u(t), z(t); E, x(t)]$ is the marginal monetary cost of unhealthy consumption $C_u(t)$ (*direct* monetary cost)

$$\pi_{C_u}(t) \equiv \frac{p_{X_u}(t)}{\partial C_u/\partial X_u} = \frac{w(t)}{\partial C_u/\partial \tau_{C_u}}, \tag{23}$$

and $\pi_{dC_u}(t) \equiv \pi_{dC_u}[t, H(t), C_h(t), C_u(t), z(t); E, x(t), \xi(t)]$ is the marginal health cost of unhealthy consumption (marginal value of health lost)

$$\pi_{dC_u}(t) \equiv q_{h/a}(t) \frac{\partial d}{\partial C_u}. \tag{24}$$

The first-order condition for unhealthy consumption (22) is similar to the condition for healthy consumption (19). The difference lies in the marginal health cost (rather than health benefit) of unhealthy consumption, which has to be added rather than subtracted from the marginal monetary cost of unhealthy consumption $\pi_{C_u}(t)$.

Last, the first-order condition for job-related health stress is

$$\varphi_z(t) = \pi_{dz}(t), \tag{25}$$

where $\varphi_z(t) \equiv \varphi_z[t, H(t), z(t); E, x(t)]$ is the marginal production benefit of job-related health stress

$$\varphi_z(t) \equiv \frac{\partial Y}{\partial z}, \tag{26}$$

reflecting the notion that job-related health stress is associated with a compensating wage differential (greater earnings), and $\pi_{dz}(t) \equiv \pi_{dz}[t, H(t), C_h(t), C_u(t), z(t); E, x(t), \xi(t)]$ is the marginal health cost of job-related health stress (marginal value of health lost)

$$\pi_{dz}(t) \equiv q_{h/a}(t)\frac{\partial d}{\partial z}. \tag{27}$$

3.3 The health cost and health benefit of health behavior

From the first-order conditions follows that lifestyle decisions regarding consumption and occupation provide utility (directly or indirectly), are associated with a monetary cost and with an opportunity cost. However, in contrast to conventional economic models, in our theory, these lifestyle decisions are additionally associated with a "health benefit" or a "health cost". In our theory, the health cost (benefit) is given by

$$q_{h/a}(t)\frac{\partial d}{\partial x} \tag{28}$$

where the definition of $q_{h/a}(t)$ is given by (12), (13), and (16), and the variable x represents the relevant health behavior (e.g., unhealthy consumption, or hard physical labor). The health cost (benefit) is the amount of health "lost" ("saved") due to the health behavior $\partial d/\partial x$ times the relative marginal value of health $q_{h/a}(t)$ (the "price" or "value" of health, measured in life-time utils). In simple terms, unhealthy consumption worsens health, and the health cost is the value one attaches to the lifetime consequences (see 12, 13) of reduced health.

Assessments of the value of a statistical life (VSL, see Viscusi and Aldy, 2003, for a review) generally involve investigating the risk of death that people are willing to take (usually in a setting of hazardous work) and how much they should be paid for taking these risks. An analogous concept is captured in, e.g., the first-order condition for job-related health stress (25), which weighs the wage premium of engaging in unhealthy / risky jobs today with the costs in terms of the lifetime consequences of reduced health (and associated longevity). In our theory, individuals are also willing to engage in a certain amount of unhealthy consumption for the instantaneous utility it provides, as long as this benefit outweighs the associated health cost: the reduction in life-time utility due to health loss associated with unhealthy consumption. Thus, our theory provides a framework for determining the value of life and the value of health in settings outside of hazardous work, e.g., by exploiting unhealthy behaviors.

As we discuss in the remainder of the paper, the health cost or health benefit of behavior is a promising concept to understanding a wide range of health behaviors, as well as the socioeconomic disparities in these behaviors.

3.4 Assumptions

Apart from the earlier mentioned assumptions of diminishing returns to scale (DRTS) in the health-production function $I(t)^{\alpha}$ $(0 < \alpha < 1)$, and diminishing marginal utilities of healthy $C_h(t)$ and unhealthy consumption $C_u(t)$, of leisure $L(t)$, and of health $H(t)$, in the remainder we assume:

1. Diminishing marginal production benefit of health $\partial^2 Y/\partial H^2 < 0$, diminishing marginal production benefit of job-related health stress $\partial^2 Y/\partial z^2 < 0$, and diminishing marginal health benefit of healthy consumption $\partial^2 d/\partial C_h^2 > 0$.

2. Increasing or constant returns to scale in the marginal health cost of unhealthy consumption $\partial^2 d/\partial C_u^2 \geq 0$ (as in Forster, 2001)[19] and in the marginal health cost of job-related health stress $\partial^2 d/\partial z^2 \geq 0$.

3. Health increases aging at a diminishing rate, i.e. $\partial d/\partial H \geq 0$ and $\partial^2 d/\partial H^2 \leq 0$. This assumption captures the notion that the health of healthy individuals deteriorates faster in absolute terms (since they have more of it) but not in relative terms (as a percentage of total health). It is also consistent with Grossman (1972a;b) and much of the subsequent health-capital literature which assumes $d(t) = \delta(t)H(t)$ (i.e. the standard assumption is a special case of ours).

4. Cobb-Douglas CRTS relations between the inputs (goods/services purchased in the market and own-time) and the outputs health investment $I(t)$, healthy consumption $C_h(t)$, and unhealthy consumption $C_u(t)$: $I(t) = \mu_I(t, E)m(t)^{\kappa_I}\tau_I(t)^{1-\kappa_I}$, $C_h(t) = \mu_{C_h}(t)X_h(t)^{\kappa_{C_h}}\tau_{C_h}(t)^{1-\kappa_{C_h}}$, and $C_u(t) = \mu_{C_u}(t)X_u(t)^{\kappa_{C_u}}\tau_{C_u}(t)^{1-\kappa_{C_u}}$, where κ_I, κ_{C_h}, κ_{C_u} are the elasticities of the outputs with respect to goods and services, and $1-\kappa_I$, $1-\kappa_{C_h}$, $1-\kappa_{C_u}$ are the elasticities of the outputs with respect to time inputs. As

[19] While it seems plausible that the health benefits of health investment and healthy consumption exhibit diminishing returns to scale, the health costs of unhealthy consumption and job-related health stress plausibly exhibit increasing returns to scale. In simple terms: whereas after a certain point more health investment, exercise, or consumption of healthy foods, does not prevent eventual aging, escalating risky behavior (e.g., illicit drug use) or dangerous work can lead to rapid health deterioration.

a result we have (see equations 17, 20, and 23):

$$
\begin{aligned}
\pi_I(t) &= \frac{p_m(t)^{k_I} w(t)^{1-k_I}}{\alpha k_I^{k_I}(1-k_I)^{1-k_I}\mu_I(t,E)} I(t)^{1-\alpha}, \\
&= \pi_I^*(t) I(t)^{1-\alpha}, \qquad (29)
\end{aligned}
$$

where

$$
\pi_I^*(t) \equiv \frac{p_m(t)^{\kappa_I} w(t)^{1-k_I}}{\alpha \kappa_I^{\kappa_I}(1-\kappa_I)^{1-\kappa_I}\mu_I(t,E)}. \qquad (30)
$$

Further,

$$
\pi_{C_h}(t) = \frac{p_{C_h}(t)^{\kappa_{C_h}} w(t)^{1-\kappa_{C_h}}}{\kappa_{C_h}^{\kappa_{C_h}}(1-\kappa_{C_h})^{1-\kappa_{C_h}}\mu_{C_h}(t)}, \qquad (31)
$$

$$
\pi_{C_u}(t) = \frac{p_{C_u}(t)^{\kappa_{C_u}} w(t)^{1-\kappa_{C_u}}}{\kappa_{C_u}^{\kappa_{C_u}}(1-\kappa_{C_u})^{1-\kappa_{C_u}}\mu_{C_u}(t)}. \qquad (32)
$$

5. Diminishing returns to wealth, health, and longevity, i.e. poorer individuals derive greater benefits from an additional increment of wealth than wealthier individuals, unhealthy individuals derive greater benefits from an additional increment in health than healthier individuals, and individuals with shorter longevity benefit more from life extension

$$
\frac{\partial q_A(t)}{\partial A(t)} = \frac{\partial^2}{\partial A(t)^2}\int_t^{T^*} U(*)e^{-\beta s} ds < 0, \qquad (33)
$$

$$
\frac{\partial q_H(t)}{\partial H(t)} = \frac{\partial^2}{\partial H(t)^2}\int_t^{T^*} U(*)e^{-\beta s} ds < 0, \qquad (34)
$$

$$
\frac{\partial \Im(T)}{\partial T} = \frac{\partial^2}{\partial T^2}\int_t^{T^*} U(*)e^{-\beta s} ds < 0, \qquad (35)
$$

where T^* denotes optimal length of life and $U(*)$ denotes the maximized utility function (see, e.g., Caputo, 2005). Further, we assume that poorer individuals derive greater benefits from better health, since the stock of health and the stock of wealth are to some extent substitutable in financing consumption and leisure (e.g., Muurinen, 1982; Case and Deaton, 2005)[20]

$$
\frac{\partial q_A(t)}{\partial H(t)} = \frac{\partial^2}{\partial H(t)\partial A(t)}\int_t^{T^*} U(*)e^{-\beta s} ds < 0. \qquad (36)
$$

[20]Intuitively, health is a resource and having more of it relaxes the dynamic constraint for health. But health also relaxes the dynamic constraint for wealth: being in better health reduces the need for health investment and health provides earnings. Thus health reduces the marginal value of health as well as the marginal value of wealth. Both health and wealth are resources that enable consumption and leisure. We discuss the implications of this assumption in more detail in section 4.2.

6. First-order direct effects dominate higher-order indirect effects for control variables. For example, wealth affects healthy consumption directly, but also indirectly through its effect on unhealthy consumption, since unhealthy consumption affects healthy consumption through its effect on utility and on health deterioration. The assumption, in this particular example, is that the direct effect of wealth on healthy consumption dominates any indirect wealth effect that operates through unhealthy consumption or through any other control variable. We do not make the assumption for state variables since small differences can grow large over time. Thus, sticking to the example, wealth could affect healthy consumption substantially through its cumulative effect on health. Detail on how this assumption is implemented is provided in Appendix C.

4 Dynamics and comparative dynamics

A quick glance at the first-order conditions in section 3.2 shows that an important driver of health behavior is the relative marginal value of health $q_{h/a}(t)$. As the name suggests, if the relative marginal value of health $q_{h/a}(t)$ is high, individuals value health more, invest more in health, and engage in healthier behavior. Hence, to understand health behavior we start with an investigation of the dynamics (section 4.1) and comparative dynamics (section 4.2) of the relative marginal value of health $q_{h/a}(t)$ and of health $H(t)$. We then discuss the implications for health behaviors.[21]

In what follows we use *propositions* in case we can prove a statement (conditional on the assumptions in section 3.4), and use *conjectures* in case a statement is plausible, e.g., because it matches empirical patterns, but we cannot unambiguously prove it. Last, in presenting *predictions* we use language such as "plausible" if the result depends in part on a conjecture.

4.1 Health behavior and health over the life cycle

The evolution of the relative marginal value of health $q_{h/a}(t)$ is given by

$$\frac{\partial q_{h/a}(t)}{\partial t} = q_{h/a}(t)\left[r + \frac{\partial d}{\partial H}\right] - \frac{1}{q_A(0)}\frac{\partial U}{\partial H}e^{-(\beta-r)t} - \frac{\partial Y}{\partial H} \tag{37}$$

(combine equations 41 and 42 of Appendix B). Recall that the relative marginal value of health $q_{h/a}(t)$ is the ratio of the marginal value of health $q_H(t)$ and the marginal value of health $q_A(t)$. The marginal value of health (wealth) represents the effect of a marginal increase in health (wealth) on remaining lifetime utility (see 12 and 13). Naturally, the

[21] Pontryagin's maximum principle (e.g., Caputo 2005) informs us that after solving the optimal control problem, the model's solutions are no longer a function of the controls and can be fully expressed in the state and co-state functions. For this reason it is useful to start with an analysis of the dynamics of the co-state function $q_{h/a}(t)$ and the state function $H(t)$.

marginal value of wealth depreciates with age at the rate of return on capital r (see 41), since the value of assets becomes smaller as the end of life approaches. The marginal value of health $q_H(t)$ however, may increase or decrease with age.[22] As long as $q_H(t)$ appreciates, or depreciates more slowly than $q_A(t)$, the relative marginal value of health $q_{h/a}(t)$ will increase with age.[23]

Conjecture 1: Health eventually declines, and the value of health $q_{h/a}(t)$ plausibly increases over the life-cycle. See Appendix C.1.

Individuals start life generally in good health at $H(0) = H_0$, while the terminal health stock $H(T)$ is constrained to the minimum health level $H_{\min}$, below which life cannot be sustained. This implies that health decreases over the lifecycle. In contrast, the relative marginal value of health $q_{h/a}(t)$ could be either decreasing or increasing over the lifecycle (see Appendix C.1), and it remains to be determined which scenario is more plausible. Empirical evidence suggests that health investments increase with age: medical expenditures peak in the final phase of life (Zweifel, Felder and Meiers, 1999), and other components of health investment either increase or stay relatively flat with age (Podor and Halliday, 2012). This suggests that the relative marginal value of health increases with age (see 15).

Conjecture 2: The health benefit of healthy consumption $-q_{h/a}(t)(\partial d/\partial C_h)$, the health cost of unhealthy consumption $q_{h/a}(t)(\partial d/\partial C_u)$, and the health cost of job-related health stress $q_{h/a}(t)(\partial d/\partial z)$ plausibly increase with age.

Since health decreases with age (conjecture 1), and since $\partial d/\partial H > 0$ (assumption 3), a declining health stock decreases the aging rate $d(t)$ with age.[24] However, the aging

[22]The marginal value of health depreciates over time with the use of the health stock in generating utility $\partial U/\partial H$ and earnings $\partial Y/\partial H$, and appreciates over time with the effect of an increase in health on the deterioration rate $\partial d/\partial H$, since an increase in the aging rate $\partial d/\partial H > 0$ reduces health, thereby increasing the marginal value of health (assumption 5). See Dorfmann (1969) for an economic interpretation of dynamic co-state equations.

[23]An alternative interpretation of the co-state equation (37) follows from reorganizing it as follows

$$\frac{1}{q_A(0)}\frac{\partial U}{\partial H}e^{-(\beta-r)t} + \frac{\partial Y}{\partial H} \quad = \quad q_{h/a}(t)\left[r + \frac{\partial d}{\partial H}\right] - \frac{\partial q_{h/a}(t)}{\partial t}. \tag{38}$$

The left-hand side of (38) represents the marginal benefit of health, consisting of the consumption (first term) and production benefit (second term), and the right-hand side represents the marginal cost of health, consisting of the sum of the effect of health on the rate of aging $\partial d/\partial H$ (a cost if positive, as we assume) and the rate of return on capital r (an opportunity cost as one could invest in the stock market rather than in health) multiplied by the relative marginal value of health $q_{h/a}(t)$. The last term $-\partial q_{h/a}(t)/\partial t$ represents a so-called "adjustment" cost. Note that this relation determines the marginal benefit of health $q_{h/a}(t)$ and not the "equilibrium" health stock (see Galama, 2015).

[24]Depending on the sign of $\partial^2 d/\partial H\partial C_h$, $\partial^2 d/\partial H\partial C_u$, and $\partial^2 d/\partial H\partial z$, this effect may be either reinforced or mitigated.

rate also has an explicit dependence on age t and it is plausible that the direct effect of age on the aging rate is to increase it (e.g., Grossman, 1972b). Further, the marginal value of health $q_{h/a}(t)$ plausibly increases with age (conjecture 1). As a result, the health benefit of healthy consumption, the health cost of unhealthy consumption, and the health cost of job stress could be decreasing or increasing over the lifecycle. Smoking rates are 8.9% among the 65+ compared to 21.6% among the 25-44 (US DHHS, 2014), and intake of fruit and vegetables increases with age (Serdula et al. 2004; Pearson et al. 2005). These patterns suggest that the health benefit of healthy behavior and the health cost of unhealthy behavior increases with age: individuals start caring more about their health when they get older.

Using conjectures 1 and 2, we obtain the following life cycle patterns for health behaviors. Early in life, individuals are generally healthy and therefore value health less (see conjecture 4, section 4.2.3). As a result, they invest less in their health (equation 15), engage more in unhealthy consumption (see 22), and less in healthy consumption (see 19). As individuals age, declining health becomes a burden as poor health reduces utility and increased sick time reduces earnings. As a result, the benefits of health increase and individuals invest more in health, shift toward healthier consumption, and reduce the level of job-related health stress. This general trend of improved health behavior may be partially reversed in mid life, as wages peak, leading to a higher opportunity cost of time. This may result in a reduction in health investment and healthy consumption in mid life, relative to a general trend of improved health behaviors with age.

The pattern for job stress is distinct. Early in life the health cost of job stress $q_{h/a}(t)\,[\partial d/\partial z]$ is low, but so is the marginal benefit of job stress $\partial Y/\partial z$ (see 25). As wages increase, generally plateauing in mid to late age and then potentially declining, the benefit of job-stress increases. After mid age, declining health reduces the marginal benefit of job-stress with age, as sick time reduces the time available for work. This suggests a pattern in which job stress initially increases as the marginal benefit of job-stress increases due to wage growth, followed by a decline in job stress due to plateauing or declining wages, increasing sick time, and an increasing health cost of job stress with age (conjecture 2).

Prediction 1: *Individuals in mid life plausibly accept unhealthy working conditions as they value the associated wage premium, but as they age they seek to engage in healthier work.*

4.2 Variation in health behavior by SES and health

Comparative dynamic analyses allow exploration of the effect of SES and health on the life-cycle trajectories of the control and state variables. We investigate the change in the optimal trajectory in response to variation in initial conditions or other model parameters, by comparing the "perturbed" optimal trajectory with respect to the "unperturbed" (or original) trajectory. Our emphasis is on exploring differences in constraints related to SES and health.[25] Common measures of SES employed in empirical research are wealth, earnings (income), and education. Here we provide an intuitive discussion of the comparative dynamic results, supported by a number of conjectures and propositions, whose formal proof we relegate to the Appendix. It should be kept in mind that we here present first-order (direct) effects, as the results are obtained by assuming that higher order (indirect) effects are small (assumption 6).

The effect of variation in an initial condition or other model parameter δZ, where we are particulary interested in $Z = \{A_0, w_E, E, H_0\}$, on any control or state variable $g(t)$ can be separated into two components[26]

$$\frac{\partial g(t)}{\partial Z} = \left.\frac{\partial g(t)}{\partial Z}\right|_T + \left.\frac{\partial g(t)}{\partial T}\right|_Z \frac{\partial T}{\partial Z}, \tag{39}$$

where the first term on the right-hand side (RHS) represents the response to variation in Z for constant T and the second term on the RHS represents the additional response due to the associated variation in T.

4.2.1 Variation in initial wealth, δA_0

Let us focus first on the hypothetical case where length of life T is fixed. Contrasting the fixed T case with the general case where T is free provides us with useful insights regarding the properties of the model. This scenario may represent a developing nation with a high disease burden (where there may be lack of access to medical or public health technology, and competing risks from many diseases), the developed world, if it were faced with diminishing ability to further extend life, the developed world before the era of the industrial revolution, or individuals with a disease that severely limits longevity, such as Huntington's disease (Oster, Shoulson and Dorsey, 2013).

Proposition 1: Absent ability to extend life, wealthy individuals, ceteris paribus, value health only marginally more than less wealthy individuals. For proof see Appendix C.2.

For fixed length of life T, additional wealth δA_0 increases the relative marginal value of health initially with respect to the unperturbed path, $\partial q_{h/a}(t)/\partial A_0\big|_T > 0$, but eventually the relative marginal value of wealth decreases with respect to the unperturbed path,

[25] Part of the SES-health gradient may be explained by differences in individual's preferences. A lower rate of time preference β operates in a similar manner to wealth, earnings and education. A lower rate of time preference may also lead to greater investment in education (not part of our theory) and hence lead to joint determination of health and education (e.g., Fuchs, 1986).

[26] Note that we can restart the problem at any time t, taking $A(t)$ and $H(t)$ as the new initial conditions. Thus the comparative dynamic results derived for, e.g., variation in initial wealth δA_0 and initial health δH_0 have greater validity, applying to variation in wealth $\delta A(t)$ and in health $\delta H(t)$ at any time $t \in [0, T)$.

$\partial q_{h/a}(t)/\partial A_0\big|_T < 0$. This result is illustrated in Figure 1, where the thick solid line labeled "Unperturbed" represents the unperturbed trajectory of the relative marginal value of health $q_{h/a}(t)$ versus age t, and the dotted line labeled "T fixed" represents the perturbed path for fixed T. Note that both curves end at $t = T$. The intuition is that the relative marginal value of health cannot be higher at all times, as this would be associated with improved health behaviors (see the first-order conditions in section 3.2), and a longer life, violating the transversality condition that end of life occurs at $t = T$ (fixed) at the minimum health level $H(T) = H_{\min}$. The response to additional wealth is thus muted as the individual is forced to invest less later in life in order not to extend life.[27] Hence, the first term on the RHS of equation (39) is small for variation in wealth A_0.

Proposition 2: Wealth raises the consumption benefit (utility) but not the production benefit of health. For proof see Appendix C.3.

The reason that wealthier individuals still value health, despite the lack of ability to extend life in the fixed T case, is that variation in wealth δA_0 increases the consumption benefit of health $q_A(0)^{-1}\partial U/\partial H e^{-(\beta-r)t}$ (see 37 and note that $\partial q_A(0)/\partial A_0|_T < 0$; assumption 5). Indeed, if health does not provide utility, individuals do not adjust their health behavior, and any additional wealth is spent on additional leisure and additional consumption (see Appendix C.3 for proof).[28]

Let us now turn to the more interesting case where individuals can optimally choose T; they not only invest in the quality of life but also in the quantity, or duration, of life. As we will see, ability to extend life changes the picture dramatically as life extension increases the return to health investment by increasing the period over which a multitude of benefits of health can be accrued. The results can be summarized by propositions 3, 4 and 5.

Proposition 3: Wealthy individuals live longer: $\partial T/\partial A_0 > 0$. For proof see Appendix C.4.

Proposition 4: Wealthy individuals value health more. The more life can be extended, the stronger is the increase in the value of health in response to additional wealth. For proof see Appendix C.5.

[27] Note that it can be understood that the opposite pattern, one of disinvestment early in life and increased investment later in life, is inferior as it would be associated with lower health at all ages and therefore a reduced consumption benefit.

[28] The reason wealthier individuals do not value the production benefit of health $\partial Y/\partial H$ (see 37) any differently than less wealthy individuals is that the production benefit (unlike the consumption benefit) is not a function of the marginal value of wealth $q_A(0)$ and thereby not a function of wealth.

Proposition 5: Wealthy individuals are healthier at all ages. For proof see Appendix C.6.

Intuitively, at high values of wealth (and hence consumption), individuals prefer investing in health over consuming, since health extends life, the period over which they can enjoy the benefits of health, leisure and consumption, whereas additional consumption per period would yield only limited marginal utility due to diminishing utility of consumption (see also Becker, 2007; Hall and Jones, 2007). With sufficient wealth one starts caring more about other goods, in particular health. Since a higher relative marginal value of health raises health investment, wealth extends life $\partial T/\partial A_0 > 0$. Wealthier individuals value health more relative to wealth, invest more, and live longer (propositions 3 and 4).

The second part of proposition 4 is best understood by its visual representation in Figure 1. While individuals optimally choose T, the extent to which they are able to extend life using the resources at their disposal, $\partial T/\partial A_0$, depends on the model's parameters r, α, μ_I, etc. These parameters are in turn determined by biology, medical technology, and environmental and other factors. If the environment is unfavorable to life extension (scenario I; small life extension), then individuals value health more early in life for its consumption benefit (proposition 2), but value health less later in life (the perturbed path starts higher, but eventually crosses the unperturbed path). This pattern of initially higher investment, and subsequently lower investment, closely resembles that of the fixed T case (see proposition 1). In contrast, if additional wealth affords considerable life extension (scenario II), e.g., in situations where only the rich have access to the latest medical technology, the relative marginal value of health is higher at all times. Health is not just valued for its consumption benefit but life extension also raises the return to health investment and healthy behaviors. Further, utility from leisure and consumption can be enjoyed with additional years of life. Health also generates additional wealth from work, reinforcing the effect of the initial endowment of wealth δA_0. Together, these various benefits substantially raise the value of health, leading to improved health behaviors, better health throughout life, and greater longevity.

Conjecture 3: The health benefit of healthy consumption $-q_{h/a}(t)(\partial d/\partial C_h)$, the health cost of unhealthy consumption $q_{h/a}(t)(\partial d/\partial C_u)$, and the health cost of job-related health stress $q_{h/a}(t)(\partial d/\partial z)$ are plausibly higher for the wealthy.

Propositions 3 to 5 also allow gauging the predicted response of an increase in wealth on health behaviors. A higher relative marginal value of health directly increases the health benefit of healthy behavior, and the health cost of unhealthy behavior. Plausibly, this represents the dominant effect,[29] consistent with wealthy individuals behaving healthier

[29] An indirect effect operates through the effect that wealth has on health, and health in turn has on the deterioration rate. It is not clear what the signs of these effects are, since signing these terms requires assumptions on the signs of $\partial d^2/\partial H\partial C_u$, $\partial d^2/\partial H\partial C_h$, etc. The effect of wealth on health is gradual and is therefore at least initially unlikely to drive the effect of wealth on health behaviors.

(e.g., Cutler and Lleras-Muney, 2010; Cutler, Lleras-Muney and Vogl, 2011; Cawley and Ruhm, 2012), and consistent with less affluent individuals responding more strongly to an unanticipated wealth shock (Van Kippersluis and Galama, 2014).

Prediction 2: *Wealthy individuals shift consumption toward healthy consumption: they consume more healthy and moderately unhealthy consumption goods and services, but fewer severely unhealthy consumption goods and services.*

The comparative dynamic effect of wealth on healthy consumption can be decomposed into a "direct" and an "indirect" wealth effect. The *direct* wealth effect is positive: an increase in wealth affords more healthy consumption (see equation 19 and assumption 5, $\partial q_A(0)/\partial A_0 < 0$). Yet, wealth also has an *indirect* effect: an increase in wealth leads to a higher relative marginal value of health $q_{h/a}(t)$ (proposition 4), which increases the health benefit of healthy consumption $[-q_{h/a}(t)\partial d/\partial C_h]$ (conjecture 3). Both the *direct* and *indirect* effects operate in the same direction, and wealthy individuals engage more in healthy consumption: $\partial C_h(t)/\partial A_0 > 0$, at least initially.[30]

Similar to healthy consumption, additional wealth enables purchases of more unhealthy consumption goods – the *direct* wealth effect is positive. Yet, additional wealth also increases the marginal health cost of unhealthy consumption $q_{h/a}(t)\partial d/\partial C_u$ (the *indirect* wealth effect), through a higher relative marginal value of health $q_{h/a}(t)$ (conjecture 3). The indirect wealth effect competes with the direct wealth effect.

While we cannot a priori sign the relation between unhealthy consumption and wealth, the two competing effects predict an interesting pattern of behavior. The health cost increases in the severity of its impact on health, $\pi_{dC_u(t)} \propto \partial d/\partial C_u$ (the degree of "unhealthiness" of the consumption good). This suggests that for moderately unhealthy goods the direct wealth effect would dominate, while for severely unhealthy goods the indirect wealth effect would dominate.[31]

[30] If wealth enables limited life extension (scenario I in proposition 4), it is possible that the health benefit decreases near the end of life with respect to the unperturbed path, since in this scenario wealth reduces $q_{h/a}(t)$ late in life compared to the unperturbed path.

[31] A formal definition of severely unhealthy consumption goods and services can be derived from (74). Severely unhealthy goods (defined as goods for which consumption decreases following an increase in wealth) are characterized by

$$\frac{\partial d}{\partial C_u} > \frac{-\left\{ \left[\frac{\partial U}{\partial C_u}\frac{1}{q_A(0)^2}e^{-(\beta-r)t}\right] \times \frac{\partial q_A(0)}{\partial A_0} + \left[q_{h/a}(t)\frac{\partial^2 d}{\partial H \partial C_u}\right] \times \frac{\partial H}{\partial A_0} \right\}}{\frac{\partial q_{h/a}(t)}{\partial A_0}}. \tag{40}$$

Thus what constitutes a severely unhealthy good is not a universal concept. It differs by socioeconomic status (e.g., wealth) and by health.

4.2.2 Variation in wages, δw_E, and education δE

Proposition 6: Permanently higher wages and education operate in a similar manner to an increase in wealth δA_0 (propositions 1 through 5), with some differences: (i) the wealth effect is muted by the increased opportunity cost of time, (ii) permanent wages w_E and education E also raise the production benefit of health, and (iii) education raises the efficiency of health investment. For proof see Appendix C.7.

It is important to distinguish between an *evolutionary wage change* and permanent differences in the wage rate $w(t)$, i.e. *permanent income*. In our model of perfect certainty and perfect capital markets, an evolutionary increase in the wage rate $w(t)$ raises the opportunity cost of time but does not affect the marginal value of wealth $q_A(t)$ (i.e. the life-cycle trajectory is unchanged). In contrast, if wages are permanently higher, i.e. larger w_E in (10), earnings are higher over the entire life cycle,[32] and in addition to the opportunity cost of time effect, there is also a wealth effect (operating by decreasing the marginal value of wealth $q_A(t)$; see assumption 5). Further, the production benefit of health is higher as higher wages increase the value of health in reducing sick time.

There are reasons to believe that the wealth effect and the effect of a higher production benefit of health dominate the opportunity cost of time effect. First, this is consistent with the result by Dustmann and Windmeijer (2000) and Contoyannis, Jones and Rice (2004) that a permanent wage change affects health positively, while a transitory wage increase affects health negatively. Second, it is consistent with the rich literature on SES and health that consistently finds that high-income individuals are generally in better health than low-income individuals.

Permanently higher wages due to education E (see equation 10) are also associated with an increased opportunity cost effect, a wealth effect, and higher production benefits. But, education also increases the efficiency $\mu(t; E)$ of health investment, as the educated are assumed to be more efficient consumers and producers of health. This could explain the stronger evidence for an effect of education on health and the weaker evidence for effects of income and wealth on health (see section 2). Thus, among the socioeconomic indicators, education improves health behaviors and health potentially the most.

From propositions 3, 4, 5, 6, and conjecture 3 we derive the following predictions regarding the effect of SES on health behavior, health, and longevity.

[32]Earnings $Y(t)$ are a function of the wage rate $w(t)$ times the amount of time spent working $\tau_w(t)$ (see equation 8). A higher wage rate w_E implies that the individual has higher earnings $Y(t)$ because the direct effect of higher wages is to increase earnings. There are also two secondary effects. First, individuals may work more because of the higher opportunity cost of not working (substitution effect). Second, individuals may work fewer hours to spend their increased income on leisure or consumption (income effect). Empirical studies suggest that the substitution and income effects are of the same magnitude (e.g., Blundell and MaCurdy, 1999) and hence that the direct effect of a wage increase is to increase earnings, while the secondary effect is small, consisting of two competing effects that roughly cancel out. Thus, a higher wage rate translates into higher earnings.

Prediction 3: *Higher SES individuals invest more in health, behave healthier, are healthier, and live longer.*

Prediction 4: *For a small degree of life extension, investment and health behavior of high SES individuals improves less rapidly with age, while for large life extension, investment and health behavior of high SES individuals improves more rapidly with age, compared to lower SES individuals.*

For a small degree of life extension afforded by the additional resources associated with SES, higher SES individuals invest more in health and behave healthier early in life and invest less and behave unhealthier later in life (flatter profiles with age), while for large life extension afforded by SES, individuals invest more and behave healthier at all ages (steeper profiles with age).

Prediction 5: *Health disparities are larger in environments where the additional resources associated with higher SES can effectively be employed to extend life.*

If life can be extended, SES and health are positively associated and the greater the degree of life extension, the greater is their association (propositions 3 to 6).

4.2.3 Variation in initial health, δH_0

Proposition 7: Absent ability to extend life, healthy individuals, ceteris paribus, value health cumulatively less, $\int_0^T [\partial q_{h/a}(t)/\partial H_0|_T] dt < 0$. For proof see Appendix C.8.

For fixed length of life T, when starting off with a higher level of health, cumulatively the relative marginal value of health has to be lower over the life-cycle, leading to cumulatively unhealthier behavior and lower health investment, in order to arrive at $H_{\min}$ over the same duration of life T. Figure 2 illustrates this: the perturbed fixed T path (dotted line) lies below the unperturbed curve, and both end at $t = T$ since the perturbed path started with higher health.[33]

Proposition 8: Healthy individuals live longer $\partial T/\partial H_0 \geq 0$. For proof see Appendix C.9.

Proposition 9: For small life extension healthy individuals cumulatively value health less $\int_0^T [\partial q_{h/a}(t)/\partial H_0] dt < 0$, for intermediate life extension they value

[33] Cases are also possible where the relative marginal value of health is initially higher but eventually lower. See Appendix section C.8 and Figure 7 for more detail.

health cumulatively more $\int_0^T [\partial q_{h/a}(t)/\partial H_0]dt > 0$, and for large life extension they value health more at all ages, $\partial q_{h/a}(t)/\partial H_0 > 0, \quad \forall t$. For proof see Appendix C.10.

Proposition 10: Individuals with greater endowed health are healthier at all ages, $\partial H(t)/\partial H_0 > 0, \quad \forall t$. For proof see Appendix C.11.

When T can be optimally chosen, healthier individuals live longer (proposition 8). We distinguish between three scenarios: "small", "intermediate", and "large" life extension, as illustrated in Figure 2 (see Appendix section C.10 and Figure 8 for more detail). For small life extension, individuals value health more than in the fixed T case, but cumulatively still less than for the unperturbed path, and life is extended to T_I. For intermediate life extension the relative marginal value of health is cumulatively higher compared with the unperturbed path, but health is still valued less in old age. Life is extended to T_{II}. In the case of large life extension, the relative marginal value of health is higher at all ages, and life is extended to T_{III}. This latter case represents a scenario in which medical technology, institutional, and environmental factors, allow for endowed health to considerably extend life ($\partial T/\partial H_0$ large). Whether this scenario is more likely in developed countries with few competing risks from diseases, universal access to health care, and cutting-edge medical technology, or in developing nations where large gains in longevity can potentially be achieved with relatively low cost interventions such as provision of clean water and improving sanitation, is a-priori not clear and certainly worthy of investigation. In such a scenario, healthy individuals would care more about their health as for them investment pays off in terms of a longer lifespan over which the benefits of health, consumption, and leisure may be enjoyed (proposition 9).

Starting out in better health, under standard economic assumptions regarding the functional forms of the utility and production functions, the relative marginal value of health (and therefore health investment and healthy behavior) will not be reduced to such an extent that health is eventually lower for individuals who started out with a greater endowment of health. Therefore, irrespective of whether the effect of initial health on the relative marginal value of health is positive or negative, individuals with a higher endowed stock of health will be healthier throughout life (proposition 10). These results can be summarized by the following prediction (see also Figure 2).

Prediction 6: *Healthy individuals live longer. For a small degree of life extension afforded by health, healthier individuals invest cumulatively less in health over their life time. For intermediate life extension, individuals invest cumulatively more in health over their lifetime, but less in old age. For large life extension, individuals invest more in health at every age.*

The above discussion concerned the pattern of investment over the lifecycle resulting from endowed health H_0. It is also of interest to explore the contemporaneous relationship between health investment and current health status. Theoretically, it is plausible that the relative marginal value of health decreases with health. This is consistent with the direct effect of health on the marginal value of health $q_H(t)$ being greater than the cross-effect of health on the marginal value of wealth (assumption 6).[34] This scenario is more likely if life extension is small and if the consumption benefit of health is small (see discussion in Appendix C.8).

Casual observation suggests those in poor health consume more medical care, and this is confirmed by empirical evidence. For example, Van de Ven and Van der Gaag (1982), Wagstaff (1986a) and Erbsland, Ried and Ulrich (2002) find statistically significant negative relations between measures of health investment and measures of health.[35] Thus, these empirical and theoretical arguments favor the small degree of life extension and small consumption benefit scenario, in which the relative marginal value of health is lower for those in better health at each instant. This leads to the following conjecture.

Conjecture 4: The relative marginal value of health is plausibly lower for the healthy $\partial q_{h/a}(t)/\partial H(t) < 0$.

The effect of current health status on the health benefit of healthy consumption and the health cost of unhealthy consumption is also theoretically ambiguous. However, conjecture 4, and the empirical observation of relatively strong responses in unhealthy consumption by the most healthy, and muted responses by the least healthy, to positive shocks in wealth (Van Kippersluis and Galama, 2014), suggest that the health cost is higher for those in poor health. This leads to the following conjecture.

Conjecture 5: The health benefit of healthy consumption $-q_{h/a}(t)(\partial d/\partial C_h)$, the health cost of unhealthy consumption $q_{h/a}(t)(\partial d/\partial C_u)$, and the health cost of job-related health stress $q_{h/a}(t)(\partial d/\partial z)$ are plausibly lower for the healthy.

4.3 Variation in work, leisure, and retirement by SES and health

Important variation exists across individuals both in the type of work and in the amount of time spent working, during a day and over the life cycle. We first discuss variation in the type of work, then turn to time spent working.

[34] Intuitively, health is a resource and having more of it relaxes the dynamic constraint for health (2). But health also relaxes the dynamic constraint for wealth (3) since health provides earnings. Hence, both the numerator and the denominator of $q_H(t)/q_A(t)$ decrease with health (assumption 5). It seems plausible that the 'own' effect of $H(t)$ on $q_H(t)$ dominates the 'cross' effect of $H(t)$ on $q_A(t)$, i.e. $-q_H(t)^{-1}(\partial q_H(t)/\partial H(t)) > -q_A(t)^{-1}(\partial q_A(t)/\partial H(t))$. In other words, health affects the marginal value of health more than it affects the marginal value of wealth.

[35] However, after accounting for the endogeneity of health, Galama et al. (2012) find that the effect of health on medical care use becomes statistically insignificant, suggesting the negative association obtained in empirical studies may not be sufficiently robust.

Work and job-related health stress: A higher relative marginal value of health $q_{h/a}(t)$ induced by wealth (proposition 4) increases the health cost of job-related health stress (see 27). Eventually, however, wealth leads to better health (proposition 5) and better health increases the marginal benefit $\partial Y/\partial z$ of job-related health stress through reduced sick time. Permanently higher wages, e.g., through better education, are also associated with the above competing wealth and health effects. In addition, the marginal benefit of job-related health stress $\partial Y/\partial z$ increases directly with the wage rate. Empirical evidence suggests that high SES individuals on average work in less demanding occupations (e.g., Ravesteijn, van Kippersluis and van Doorslaer, 2013). This suggests that higher SES increases the marginal costs of job-related health stress more than it increases the marginal benefits.

The effect of health on job-related health stress is plausibly positive. Better health reduces sick time, which increases the marginal benefit of job-related health stress. Further, if the relative marginal value of health decreases in health (conjecture 4), then healthier individuals will have lower marginal costs of engaging in job-related health stress. With higher benefits and lower costs, healthier individuals will engage more in job-related health stress, in line with empirical evidence (Kemna, 1987). Thus:

Prediction 7: *The healthy and the poor engage in unhealthy jobs.*

Leisure and retirement: To analyze retirement, we informally treat a small amount of time devoted to work $\tau_w(t)$, i.e. below a certain threshold, say τ_R, as a retirement phase. During working life, individuals divide their time between work, leisure, and time inputs into consumption and health investment (see 4). Therefore, we can infer the effect on the time spent working by investigating effects on leisure and time inputs. With declining health, time spent working $\tau_w(t)$ (see 4) gradually decreases, as a result of increasing sick time and the increasing demand for time devoted to health investment (conjecture 1). Hence, the model produces a phase of life in old age that naturally qualifies as retirement.

Wealth increases the demand for leisure, for (time inputs into) healthy and unhealthy consumption, and for (time devoted to) health investment, through a "direct" wealth effect (wealth reduces the marginal value of wealth $q_A(0)$ [assumption 5]) and by increasing the relative marginal value of health (proposition 4), at least initially. This leads wealthier individuals, ceteris paribus, to work less (see 4), and hence retire earlier, in line with empirical evidence (Imbens, Rubin and Sacerdote, 2001; Brown, Coile and Weisbenner, 2010).[36]

Permanently higher wages, e.g., through education, are also associated with the above wealth and value of health effects, but higher wages also increase the cost of time inputs, i.e. of leisure, of healthy and unhealthy consumption, and of health investment (see sections C.12.2 and C.12.3 in the Appendix). The higher opportunity cost of not working

[36] However, wealthy individuals are healthier (proposition 5), which reduces sick time and increases the benefit of work (earnings). The disparity in health grows over time, so that eventually the effect of better health among the wealthy may become important in the retirement decision.

encourages higher educated individuals to retire later. Thus:

Prediction 8: *The wealthy retire earlier, but the higher educated and those with higher permanent wages may retire later.*

Healthier individuals spend more time working, as good health reduces sick time and reduces the demand for (time inputs into) health investment (conjecture 4).[37] This encourages healthier individuals to work more and retire later (see 4). However, health is also associated with a wealth effect, which increases the demand for leisure, and for (time inputs into) healthy and unhealthy consumption, and thereby encourages early retirement.[38] While the net effect is therefore ambiguous, it seems plausible that the direct effect of health on reducing sick time, and reducing time inputs into health investment, outweighs the indirect effect of health on leisure through wealth accumulation. This is consistent with an extensive literature showing quantitatively large effects of health on labor force participaton, with unhealthier individuals retiring earlier (e.g., Currie and Madrian, 1999; Smith, 2007).

Prediction 9: *Under plausible assumptions, healthier individuals retire later. This, combined with an effect of health on earnings, leads to reverse causality as healthier individuals accumulate more wealth by earning more and retiring later.*

5 Discussion and conclusions

We have developed a theory of the relation between health and SES over the lifecycle. Our life-cycle model incorporates health, longevity, wealth, earnings, education, work, job-related physical and psychosocial health stresses, leisure, health investment (e.g., exercise, medical care), and healthy and unhealthy consumption (including housing, neighborhood social environment). Our review of the literature identifies these as essential mechanisms in the formation and evolution of disparities in health.

The theory is capable of reproducing stylized facts characteristic of the SES-health gradient. We find that greater SES, as measured by wealth, earnings, and education, induces a healthy lifestyle: it encourages investment in health, encourages healthy consumption, discourages unhealthy consumption, and protects individuals from the health risks of physically and psychosocially demanding working conditions (e.g., hard labor, limited control), and of unhealthy consumption. The healthier lifestyle of high SES individuals causes the health trajectories of high and low SES individuals to diverge. As a result they are healthier and live longer (prediction 3). In addition, health generates earnings and the worsening health of low SES individuals potentially leads to early

[37] While health may also reduce the demand for healthy consumption, it could increase demand for unhealthy consumption.

[38] The amount of time devoted to work further depends on whether leisure, consumption, and health are complements or substitutes in utility.

withdrawal from the labor force (prediction 9). This reverse causality from health to financial measures of SES potentially reinforces the widening of the SES-health gradient, as documented in empirical studies (e.g., Smith, 2007).

In middle to late life the divergence of health trajectories potentially slows as lower levels of health encourages low SES individuals to invest more in health and engage in healthier behavior in order to slow down their health deterioration (cf. conjectures 4 and 5). Also, mortality selection, i.e. the least healthy among lower SES individuals die sooner, results in an apparently healthier surviving disadvantaged population, potentially narrowing the gradient in late age.[39] Thus, the theory is capable of reproducing the characteristic lifecycle patterns of the SES-health gradient.

Apart from providing a framework to interpret stylized facts, the theory also makes novel testable predictions and provides new intuition. In particular, we emphasize the importance of our concept of a health cost (benefit) of unhealthy (healthy) behaviors, in explaining health behavior. Individuals make decisions regarding health by taking into account not just monetary prices and preferences, but additionally the life-time health consequences of their choices, as embodied by the health cost (benefit). Empirical support for the notion that the health cost is an important determinant of health behavior is provided in Van Kippersluis and Galama (2014).

We predict that individuals in mid life, particularly the healthy and poor, engage in work associated with unhealthy working conditions as they value the associated wage premium (prediction 7). However, as individuals age they engage in healthier work to protect declining health (prediction 1). Further, wealthy individuals can afford to retire early, while higher educated and healthier individuals are likely to retire later (predictions 8 and 9).

Another prediction of the theory (prediction 2) is a pattern in which high SES individuals consume more of moderately unhealthy consumption goods (e.g., moderate alcohol consumption) and less of severely unhealthy consumption goods (e.g., cigarettes, high alcohol consumption, illicit drugs) than do lower SES individuals. Greater wealth permits more consumption but also increases the health cost. This could provide an explanation for the observation that high SES individuals are less likely to smoke cigarettes (bad for health) but are more likely to be moderate drinkers (moderately bad for health) than low SES individuals (e.g., Cutler and Lleras-Muney, 2008).

Finally, we find that (endogenous) longevity is crucial in explaining observed associations between SES and health (cf. propositions 1, 4, 5 and 6, and prediction 4). Absent ability to extend life (fixed horizon), the association between SES and health is small (proposition 1). If, however, life can be extended, SES and health are positively associated and the greater the degree of life extension afforded by SES, the greater is their association (propositions 4, 5, and 6). Thus, health disparities are larger in environments

[39]The narrowing of the gradient due to unhealthier individuals engaging in healthier behavior would represent an economic variant of the *age-as-leveller* hypothesis, while the narrowing of the gradient due to mortality selection would be consistent with a process of *cumulative advantage* (House et al. 1994; Ross and Wu, 1996; Lynch, 2003).

where the additional resources associated with higher SES can effectively be employed to extend life (prediction 5). For example, if the latest medical technology is only available to higher SES individuals (e.g., through better health knowledge), health disparities across SES groups may be larger. The theory also predicts that the increase of health expenditures with age is slower (flatter profile) for higher SES individuals in case the additional resources associated with SES afford small life extension, and faster (steeper profile) in case SES affords large life extension (prediction 4). Analogously, for a small degree of life extension afforded by health, healthier individuals invest cumulatively less in health over their life time (flatter profiles), and for large life extension, individuals invest more in health at every age (steeper profiles; prediction 6).

Future work may extend the model to incorporate the joint determination of SES and health (e.g., Chiteji, 2010; Conti et al. 2010), the evolution of child health (e.g., Case, Lubotsky and Paxson, 2002; Currie and Stabile, 2003; Heckman, 2007), and the impact of fetal and early-childhood conditions on health in adulthood (e.g., Barker et al. 1993; Case, Fertig and Paxson, 2005).[40] Early childhood could be included modeling the production of health by the family, similar to, e.g., Jacobson (2000) and Bolin, Jacobson and Lindgren (2001). We do not explicitly take into account the influence of the wider social context and social relationships of the family or neighborhood on health (e.g., Kawachi and Berkman, 2003), or of social capital on health (e.g., Bolin et al., 2003). Insights from the behavioral-economic and psychological literature regarding myopia and lack of self-control (e.g., Blanchflower, Oswald, and van Landeghem, 2009) might be incorporated following Laibson (1998). Uncertainty (e.g., health shocks) could be included similar to, e.g., Cropper (1977), Dardanoni and Wagstaff (1990), Liljas (1998), and Ehrlich (2000).

Empirical structural- and reduced-form estimation of the model is needed to test the assumptions and the theoretical predictions presented in this work, to assess the relative importance of mechanisms, to study interactions between mechanisms, to disentangle the different patterns of causality, and to simulate the effects of policy interventions.

[40] The potential influence of childhood health on education is not included in our formulation as education is treated as being predetermined by the time individuals join the labor-force. Childhood conditions can be accounted for by treating the health status of an individual joining the labor force and investment in human capital prior to adulthood as initial conditions, i.e., we take initial health $H(0)$ and years of schooling E as given. Our model is therefore limited to explaining the formation of disparities in health from early adulthood till old age but not during childhood or the fetal period. See Galama and Van Kippersluis (2015) for a theory of endogenous education and health.

Acknowledgements

Research reported in this publication was supported by the National Institute on Aging of the National Institutes of Health under Award Numbers K02AG042452, R01AG037398, R01AG030824, P30AG012815 and P01AG022481. The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health. Van Kippersluis thanks Netspar (Health and Income, Work and Care across the Lifecycle II), and the Netherlands Organization of Scientific Research (NWO Veni grant 016.145.082) for support. Galama is grateful to the School of Economics of Erasmus University Rotterdam for a Visiting Professorship in the Economics of Human Capital. We thank Mauricio Avendano, Megan Beckett, Jan Brinkhuis, Michael Grossman, Bas Jacobs, Arie Kapteyn, Erik Meijer, Owen O'Donnell, Rosalie Pacula, Bastian Ravesteijn, Eddy van Doorslaer, Tom Van Ourti, Arthur van Soest, and participants in the Health Inequality Network (HINet) meeting at the University of Chicago, the 8^{th} iHEA World Congress on Health Economics in Toronto, the 12^{th} Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics, the 9^{th} Annual SHESG Conference at the University of Memphis and the 2011 RAND Conference on Health, Aging and Human Capital, as well as seminar participants at Erasmus University, the Harvard Center for Population and Development Studies, the London School of Economics, the RAND Corporation, the University of Chicago, the University of Lausanne, the University of Southern California and the University of Tilburg, for useful discussions and comments.

References

Adler, N.E., Boyce, T., Chesney, M., Folkman, S., & Syme, L. (1993), "Socioeconomic inequalities in health: No easy solution", Journal of the American Medical Association 269: pp. 3140-3145.

Adler, N.E., Boyce, T., Chesney, M.A., Cohen, S., Folkman, S., Kahn, R.L., Syme, S.L. (1994), "Socioeconomic status and health: The challenge of the gradient", American Psychologist 49(1): pp. 15-24.

Albouy, V., and L. Lequien (2009), "Does compulsory education lower mortality?", Journal of Health Economics 28(1): pp. 155-168.

Almond, D., and J. Currie (2011), "Killing Me Softly: The Fetal Origins Hypothesis", Journal of Economic Perspectives 25(3): pp. 153-172.

Auld, M.C. and N. Sidhu (2005), "Schooling, cognitive ability, and health", Health Economics, 14(10): pp. 1019-1034.

Baeten, S., T. Van Ourti, and E. van Doorslaer (2013), "The socioeconomic health gradient across the life cycle: What role for selective mortality and institutionalization?", Social Science and Medicine 97: pp. 66-74.

Barker, D., Gluckman, P.D., Godgrey, K.M., Harding, J.E., Owens, J.A., Robinson, J.S. (1993), "Fetal nutrition and cardiovascular disease in adult life", Lancet 341(8850): pp. 938-941.

Becker, G. (2007), "Health as human capital: synthesis and extensions", Oxford Economic Papers 59(3), pp. 379-410.

Becker, G. and Murphy, K. (1988), "A Theory of Rational Addiction", Journal of Political Economy 96(4): 675-700.

Beckett, M. (2000), "Converging Health Inequalities in Later Life: An Artifact of Mortality Selection?", Journal of Health and Social Behavior 41: pp. 106-119.

Blanchflower, D.G., Oswald, A.J., van Landeghem, B. (2009), "Imitative Obesity and Relative Utility", Journal of the European Economic Association 7: pp. 528-538.

Blundell, R., and T. MaCurdy (1999), "Labor Supply" in Orley Ashenfelter and David Card, eds., Handbook of Labor Economics, Amsterdam: North-Holland.

Bolin, K., Jacobson, L. and Lindgren, B. (2001), "The family as the producer of health - when spouses are Nash bargainers", Journal of Health Economics 20: pp. 349-362.

Bolin, K., Lindgren, B., Lindstrom, M., and Nystedt, P. (2003), "Investments in social capital – implications of social interactions for the production of health", Social Science & Medicine 56(12): pp. 2379-2390.

Borg, V. and Kristensen, T.S. (2000), "Social class and self-rated health: can the gradient be explained by differences in life style or work environment?", Social Science & Medicine 51: pp. 1019-1030.

Brown, J.R., C.C. Coile, and S.J. Weisbenner (2010), "The Effect of Inheritance Receipt on Retirement", Review of Economics and Statistics 92(2): 425-434.

Caputo, M.R. (2005), *Foundations of Dynamic Economic Analysis*, Cambridge University Press.

Campbell, F., Conti, G., Heckman, J. J., Moon, S. H., Pinto, R., Pungello, E., and Pan, Y. (2014), "Early childhood investments substantially boost adult health", Science, 343(6178): pp. 1478-1485.

Case, A., Lubotsky, D. and Paxson, C. (2002), "Economic status and health in childhood: The origins of the gradient", American Economic Review 92: pp. 1308-1334.

Case A., A. Fertig, and C. Paxson (2005), "The lasting impact of childhood health and circumstance", Journal of Health Economics 24(2): pp. 365-389.

Case, A., Deaton, A. (2005), "Broken Down by Work and Sex: How our Health Declines", in: David A. Wise (ed.) (2005), *A*nalyses in the Economics of Aging, The University of Chicago Press, Chicago, pp. 185-212.

Chiteji, N. (2010), "Time Preference, Noncognitive skills and Well Being across the Life Course: Do Noncognitive Skills Encourage Healthy Behavior?", American Economic Review: Papers & Proceedings 100: pp. 200-204.

Clark, D., and H. Royer (2013), "The Effect of Education on Adult Mortality and Health: Evidence from Britain." American Economic Review, 103(6): pp. 2087-2120.

Contoyannis, P., Jones, A.M. and Rice, N. (2004), "The dynamics of health in the British Household Panel Survey", Journal of Applied Econometrics, 19(4): pp. 473-503.

Cropper, M.L. (1977), "Health, investment in health, and occupational choice", Journal of Political Economy 85: pp. 1273-1294.

Cunha, F. and J. J. Heckman (2007), "The technology of skill formation", American Economic Review 97(2): pp. 31-47.

Currie, J. and Madrian, B.C. (1999), "Health, Health Insurance and the labour market", In: Ashenfelter O, Card, D. (eds.) *Handbook of labour economics* 3: pp. 3309-3415, Amsterdam: Elsevier Science B.V.

Currie, J. and Stabile, M. (2003), "Socioeconomic status and child health: why is the relationship stronger for older children?", American Economic Review 93: pp. 1813-1823.

Currie, J. (2009). "Healthy, wealthy, and wise: Socioeconomic status, poor health in childhood, and human capital development", Journal of Economic Literature 47(1): pp. 87-122.

Cutler, D.M. and A. Lleras-Muney (2008), "Education and Health: Evaluating Theories and Evidence", in Robert F. Schoeni et al., eds., Making Americans Healthier: Social and Economic Policy As Health Policy (National Poverty Center Series on Poverty and Public Policy).

Cutler, D.M., and A. Lleras-Muney (2010), "Understanding differences in health behaviors by education", Journal of Health Economics 29(1): pp. 1-28.

Cutler, D.M., Lleras-Muney, A. and Vogl, T. (2011), "Socioeconomic Status and Health: Dimensions and Mechanisms", Oxford Handbook of Health Economics.

Dalgaard, C-J., and Strulik, H. (2014), "Optimal aging and death: understanding the Preston curve", Journal of the European Economic Association 12(3), pp. 672-701.

Dardanoni, V., Wagstaff, A. (1990), "Uncertainty and the demand for medical care", Journal of Health Economics 9: pp. 23-38.

Deary, I. (2008), "Why do intelligent people live longer?" Nature 456: pp. 175-176.

Deaton, A. (2002), "Policy implications of the gradient of health and wealth", Health Affairs 21(2): pp. 13-30.

DHHS (2014), "Current cigarette smoking among adults - United States, 2005-2012", US Department of Health and Human Services, MMWR 63(2): pp. 29-46.

Dorfman, R. (1969), "An Economic Interpretation of Optimal Control Theory", The

American Economic Review 59(5): pp. 817-831.

Duncan, G.J. and Holmlund, B. (1983), "Was Adam Smith Right After All? Another Test of the Theory of Compensating Wage Differentials", Journal of Labor Economics 1(4): pp. 366-379.

Dustmann, C., and Windmeijer, F. (2000), "Wages and the Demand for Health – A Life Cycle Analysis", IZA Discussion Paper 171, Germany.

Ehrlich, I. and Chuma, H. (1990), "A Model of the Demand for Longevity and the Value of Life Extension", Journal of Political Economy 98(4): pp. 761-782.

Ehrlich, I. (2000), "Uncertain lifetime, life protection, and the value of life saving", Journal of Health Economics 19(3): pp. 341-367.

Erbsland, M., Ried, W. and Ulrich, V. (2002), "Health, health care, and the environment: econometric evidence from German micro data", In: *Econometric Analysis of Health Data*, John Wiley & Sons, Ltd, pp. 25-36.

Forster, M. (2001), "The meaning of death: some simulations of a model of healthy and unhealthy consumption, Journal of Health Economics, 20: pp: 613-638.

Fuchs, V. R. (1986), *The Health Economy*, Harvard University Press, Cambridge, MA.

Galama, T.J. and Kapteyn, A. (2011), "Grossman's missing health threshold", Journal of Health Economics 30(5): pp. 1044-1056.

Galama, T.J. (2015), "A Contribution to Health Capital Theory", CESR Schaeffer Working Paper 2015-004.

Galama, T.J. and H. van Kippersluis (2013), "Health Inequalities through the Lens of Health Capital Theory: Issues, Solutions, and Future Directions", In: Research on Economic Inequality, Ed: John Bishop. Volume 21: Health and Inequality, Eds: Pedro Rosa Dias, Owen O'Donnell.

Galama, T.J. and H. van Kippersluis (2015), "A Theory of Education and Health", CESR-Schaeffer Working Paper Series 2015-001.

Galama, T.J., Hullegie, P., Meijer, E., and Outcault, S. (2012), "Is there empirical evidence for decreasing returns to scale in a health capital model?", Health Economics, 21(9): pp. 1080-1100.

Glied, S., and Lleras-Muney, A. (2008), "Technological Innovation and Inequality in Health", Demography, 45(3): pp. 741-761.

Goldman, D.P. and Smith, J.P. (2002), "Can Patient Self-Management Help Explain the SES Health Gradient?", Proceedings of the National Academy of Science 99: pp. 10929-10934.

Grossman, M. (1972a), "The Demand for Health - A theoretical and Empirical Investigation", New York: National Bureau of Economic Research.

Grossman, M. (1972b), "On the Concept of Health Capital and the Demand for Health", Journal of Political Economy 80(2): pp. 223-255.

Grossman, M. (2000), "The Human Capital Model", in *Handbook of Health Economics*, Volume 1, pp. 347-408, A.J. Culyer and J.P. Newhouse (Eds.), Elsevier Science.

Hall, R.E., and Jones, C.I. (2007), "The Value of Life and the Rise in Health Spending", The Quarterly Journal of Economics 122(1): pp. 39-72.

Heckman, J.J. (2007), "The economics, technology, and neuroscience of human capability formation", Proceedings of the national Academy of Sciences, 104(33): pp. 13250-13255.

Herd, P., Schoeni, R.F. and House, J.S. (2008), "Upstream solutions: does the supplemental security income program reduce disability in the elderly?", Milbank Quarterly 86(1): pp. 5-45.

House, J.S., Lepkowski, J.M., Kinney, A.M., Mero, R.P., Kessler, R.C., Regula Herzog, A. (1994), "The Social Stratification of Aging and Health", Journal of Health and Social Behavior 35(3): pp. 213-234.

Imbens, G., D. Rubin, and B. Sacerdote (2001), "Estimating the Effect of Unearned Income on Labor Supply, Earnings, Savings and Consumption: Evidence from a Survey of Lottery Players", American Economic Review 91(4): pp. 778-794.

Jacobson, L. (2000), "The family as producer of health - an extension of the Grossman model", Journal of Health Economics 19: pp. 611-637.

Kaestner, R. (2013), "The Grossman model after 40 years: A reply to Peter Zweifel", European Journal of Health Economics, 14: 357-360.

Kawachi, I., and Berkman, L. (Eds.) (2003), *Neighborhoods and health*, Oxford, U.K.: Oxford University Press.

Kemna, H. (1987), “Working conditions and the relationship between schooling and health”, Journal of Health Economics 6: pp. 189-210.

Laibson, D. (1998), “Life-cycle consumption and hyperbolic discount functions”, European Economic Review 42: pp. 861-871.

Laporte, A. (2014), “Should the Grossman model retain its iconic status in Health Economics?”, Canadian Centre for Health Economics Working Paper 2014-04.

Liljas, B. (1998), “The demand for health with uncertainty and insurance”, Journal of Health Economics 17(2): pp. 153-170.

Lleras-Muney, A. and Lichtenberg, F. (2005), “The Effect of Education on Medical Technology Adoption: Are the More Educated More Likely to Use New Drugs”, Annales d’Economie et Statistique, special issue in memory of Zvi Griliches, No 79/80.

Lleras-Muney, A. (2005), “The relationship between education and adult mortality in the United States”, Review of Economic Studies 72(1): pp. 189-221.

Lynch, J.W., Kaplan, G.A., Shema, S.J. (1997), “Cumulative Impact of Sustained Economic Hardship on Physical, Cognitive, Psychological, and Social Functioning”, New England Journal of Medicine 337(26): pp. 1889-1895.

Lynch, S.M. (2003), “Cohort and Life-Course Patterns in the Relationship between Education and Health: A Hierarchical Approach”, Demography 40: pp. 309-331.

Mackenbach, J.P., Huisman, M., Andersen, O., Bopp, M., Borgan, J.K., Borrell, C., et al. (2004), “Inequalities in lung cancer mortality by the educational level in 10 European populations”, European Journal of Cancer 40(1): pp. 126-135.

Mani, A., S. Mullainathan, E. Shafir, and J. Zhao (2013), “Poverty Impedes Cognitive Function”, *Science* 341(6149): pp. 976-980.

Marmot, M., Ryff, C.D., Bumpass, L.L., Shipley, M., Marks, N.F. (1997), “Social inequalities in health: next questions and converging evidence”, Social Science & Medicine 44(6): pp. 901-910.

Marmot, M. (1999), “Multi-level approaches to understanding social determinants”, in Lisa Berkman and Ichiro Icawachi (eds.), *Social Epidemiology*, Oxford: Oxford University Press.

Michaud, P.-C. and A. van Soest (2008), "Health and wealth of elderly couples: Causality tests using dynamic panel data models", Journal of Health Economics 27: pp. 1312-1325.

Mincer, J.A. (1974), "Schooling, Experience, and Earnings", Columbia University Press.

Muurinen, J-M. (1982), "Demand for health: a generalized Grossman model", Journal of Health Economics 1: pp. 5-28.

Murphy, K.M., and Topel, R.H. (2006), "The value of health and longevity", Journal of Political Economy, 114(5): pp. 871-904.

Oster, E., Shoulson, I. and Dorsey, E.R. (2013), "Limited Life Expectancy, Human Capital and Health Investments", American Economic Review, 103(5): 1977-2002.

Pearson, T., Russell, J., Campbell, M. J., and Barker, M. E. (2005), "Do 'food deserts' influence fruit and vegetable consumption? - A cross-sectional study" Appetite, 45(2): pp. 195-197.

Podor, M., and T.J. Halliday (2012), "Health Status and the Allocation of Time", Health Economics 21, pp. 514-527.

Ravesteijn, B., Van Kippersluis, H., and Van Doorslaer, E. (2013), "The contribution of occupation to health inequality." In: Research on Economic Inequality, Volume 21: Health and Inequality, Pedro Rosa Dias and Owen ODonnell (eds), Emerald Group Publishing.

Ried, W. (1998), "Comparative dynamic analysis of the full Grossman model", Journal of Health Economics 17: pp. 383-425.

Rosen, S. (1988), "The value of changes in life expectancy", Journal of Risk and Uncertainty, 1: pp. 285304.

Ross C.E., and Wu C.L. (1996), "Education, Age, and the Cumulative Advantage in Health", Journal of Health and Social Behavior 37: pp. 104-20.

Savelyev, P. (2014), "Socioemotional Skills, Education, and Longevity of High-Ability Individuals", Working Paper, Vanderbilt University.

Seierstad, A. and K. Sydsaeter (1987), "Optimal Control Theory With Economic Applications", in *Advanced Texbooks in Economics*, Volume 24, Series Editors: C.J. Bliss and M.D. Intriligator, Elsevier, North Holland.

Serdula, M. K., Gillespie, C., Kettel-Khan, L., Farris, R., Seymour, J., and Denny, C. (2004), “Trends in fruit and vegetable consumption among adults in the United States: behavioral risk factor surveillance system, 1994-2000”, American Journal of Public Health 94(6): pp. 1014-1018.

Smith, A. (1776), “An inquiry into the nature and causes of the wealth of nations”, Reprinted Roy H. Campbell and Andrew S. Skinner, eds. Oxford: Clarendon Press, 1976.

Smith, R.S. (1978), “Compensating Wage Differentials and Public Policy: A Review”, Industrial and Labor Relations Review 32(3): pp. 339-352.

Smith, J.P. (2007), “The impact of socioeconomic status on health over the life course”, Journal of Human Resources 42(4): pp. 739-764.

Usher, D. (1976), “Comments on the correlation between health and schooling”, In: N.E. Terleckyj, ed., Household Production and Consumption (Columbia University Press for the National Bureau of Economic Research, New York) pp. 212-220.

Van de Ven, W. and van der Gaag, J. (1982), “Health as an unobservable: A MIMIC-model of the demand for health care”, Journal of Health Economics 1: pp. 157-183.

Van Kippersluis, H., O’Donnell, O., van Doorslaer, E., Van Ourti, T., (2010), “Socioeconomic Differences in Health over the Life Cycle in an Egalitarian Country”, Social Science and Medicine 70(3): pp. 428-438.

Van Kippersluis, H., and Galama, T.J. (2014), “Wealth and health behavior: Testing the concept of a health cost”, European Economic Review 72: pp. 197-220.

Viscusi, K.W. (1978), “Wealth Effects and Earnings Premiums for Job Hazards”, Review of Economics and Statistics 60: pp. 408-416.

Wagstaff, A. (1986a), “The demand for health: Some new empirical evidence”, Journal of Health Economics 5: pp. 195-233.

Wagstaff, A. (1986b), “The demand for health: Theory and applications”, Journal of Epidemiology and Community Health 40: pp. 1-11.

, P. and Breyer, F., (1997), *Health Economics*, Oxford University Press, New York.

Zweifel, P., Felder, S., and Meiers, M. (1999), “Ageing of population and health care expenditures: a red herring?”, Health Economics 8(6): pp. 485-496.

Zweifel, P. (2013), “The Grossman model after 40 years”, European Journal of Health Economics, 13: pp. 677-682.

A Figures

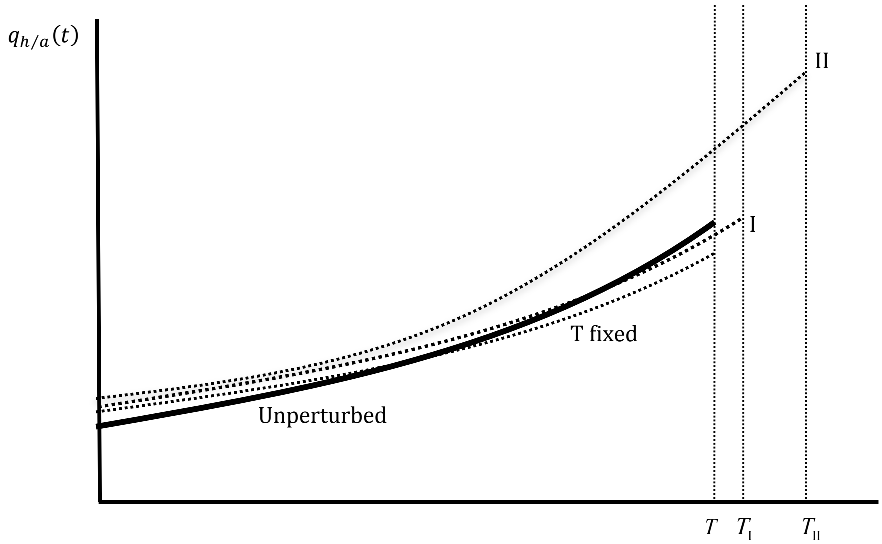

Figure 1: *Evolution of the relative marginal value of health $q_{h/a}(t)$ with age due to variation in A_0. The solid thick line, labeled "Unperturbed", represents the unperturbed path. The perturbed paths are shown for the T fixed case (small dotted line, labeled "T fixed"), scenario I, associated with small life extension T_{I} (dashed line, labeled "I"), and scenario II, associated with large life extension T_{II} (dash dotted line labeled "II").*

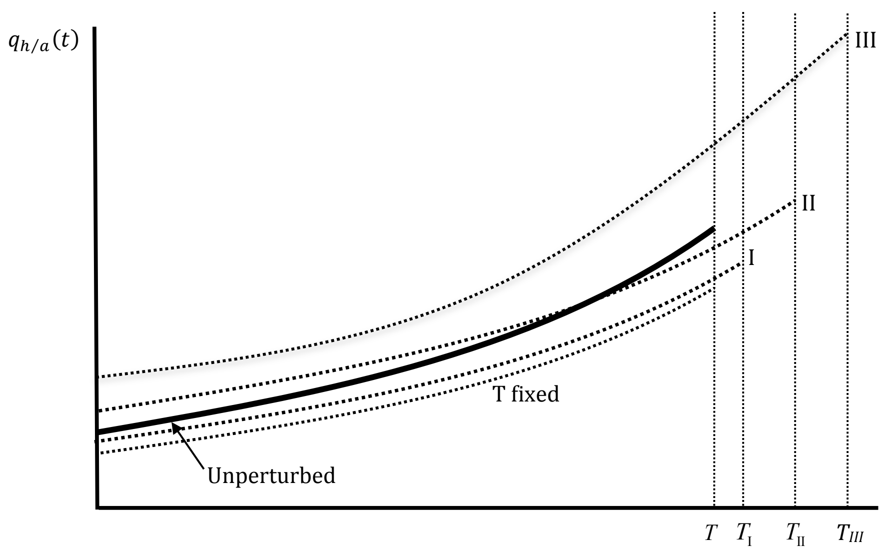

Figure 2: *Evolution of the relative marginal value of health $q_{h/a}(t)$ with age due to variation in H_0. The solid thick line, labeled "Unperturbed", represents the unperturbed path. The perturbed paths are shown for the T fixed case (small dotted line, labeled "T fixed"), scenario I, associated with small life extension T_{I} (dashed line, labeled "I"), scenario II, associated with intermediate life extension T_{II} (long dash dotted line labeled "II"), and scenario III, associated with large life extension (dash dotted line labeled "III").*

B First-order and transversality conditions

Associated with the Hamiltonian (equation 11) we have the following conditions:

$$\begin{aligned}
\frac{\partial q_A(t)}{\partial t} &= -\frac{\partial \Im}{\partial A} \Rightarrow \\
\frac{\partial q_A(t)}{\partial t} &= -r q_A(t) \Leftrightarrow \\
q_A(t) &= q_A(0) e^{-rt}, \qquad (41)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial q_H(t)}{\partial t} &= -\frac{\partial \Im}{\partial H} \Rightarrow \\
\frac{\partial q_H(t)}{\partial t} &= q_H(t)\frac{\partial d}{\partial H} - \frac{\partial U}{\partial H} e^{-\beta t} - q_A(0)\frac{\partial Y}{\partial H} e^{-rt}, \qquad (42)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial L} &= 0 \Rightarrow \\
\frac{\partial U}{\partial L} &= q_A(0) w(t) e^{(\beta - r)t}, \qquad (43)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial X_h} &= 0 \Rightarrow \\
\frac{\partial U}{\partial C_h} &= q_A(0)\frac{p_{X_h}}{\partial C_h / \partial X_h} e^{(\beta - r)t} + q_H(t)\frac{\partial d}{\partial C_h} e^{\beta t}, \qquad (44)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial \tau_{C_h}} &= 0 \Rightarrow \\
\frac{\partial U}{\partial C_h} &= q_A(0)\frac{w(t)}{\partial C_h / \partial \tau_{C_h}} e^{(\beta - r)t} + q_H(t)\frac{\partial d}{\partial C_h} e^{\beta t}, \qquad (45)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial X_u} &= 0 \Rightarrow \\
\frac{\partial U}{\partial C_u} &= q_A(0)\frac{p_{X_u}(t)}{\partial C_u / \partial X_u} e^{(\beta - r)t} + q_H(t)\frac{\partial d}{\partial C_u} e^{\beta t}, \qquad (46)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial \tau_{C_u}} &= 0 \Rightarrow \\
\frac{\partial U}{\partial C_u} &= q_A(0)\frac{w(t)}{\partial C_u/\partial \tau_{C_u}}e^{(\beta-r)t} + q_H(t)\frac{\partial d}{\partial C_u}e^{\beta t}, \qquad (47)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial m} &= 0 \Rightarrow \\
q_H(t) &= q_A(0)\left\{\frac{p_m(t)I(t)^{1-\alpha}}{\alpha[\partial I/\partial m]}\right\}e^{-rt}, \qquad (48)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial \tau_I} &= 0 \Rightarrow \\
q_H(t) &= q_A(0)\left\{\frac{w(t)I(t)^{1-\alpha}}{\alpha[\partial I/\partial \tau_I]}\right\}e^{-rt}, \qquad (49)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Im}{\partial z} &= 0 \Rightarrow \\
0 &= q_H(t)\frac{\partial d}{\partial z} - q_A(0)\frac{\partial Y}{\partial z}e^{-rt}. \qquad (50)
\end{aligned}$$

Equation (48) and (49) provide the first-order condition for health investment (15). Equation (43) provides the first-order condition for leisure (18). Equations (44) and (45) provide the first-order condition for healthy consumption (19). Equations (46) and (47) provide the first-order condition for unhealthy consumption (22). Last, equation (50) provides the first-order condition for job-related health stress (equation 25).

C Conjectures and proofs of propositions

C.1 Conjecture 1: Health eventually declines, and the value of health $q_{h/a}(t)$ plausibly increases over the life-cycle.

The dynamic equation for the relative marginal value of health is given by (37). The dynamic equation for health can be rewritten as

$$\begin{aligned} \frac{\partial H(t)}{\partial t} &= I(t)^{\alpha} - d(t) \\ &= \left(\frac{q_{h/a}(t)}{\pi_I^*(t)}\right)^{\frac{\alpha}{1-\alpha}} - d(t), \end{aligned} \quad (51)$$

where we have used (15), (29) and (30). Both the condition (37) for the relative marginal value of health $q_{h/a}(t)$ and the condition (51) for the health stock $H(t)$ include time derivatives, illustrating the inherently dynamic nature of the model.

The phase diagram in Figure 3 shows the direction of motion of the optimal solution of the system of first-order differential equations given by (37) and (51), as a function of the relative marginal value of health $q_{h/a}(t)$ (vertical axis) versus the health stock $H(t)$ (horizontal axis).

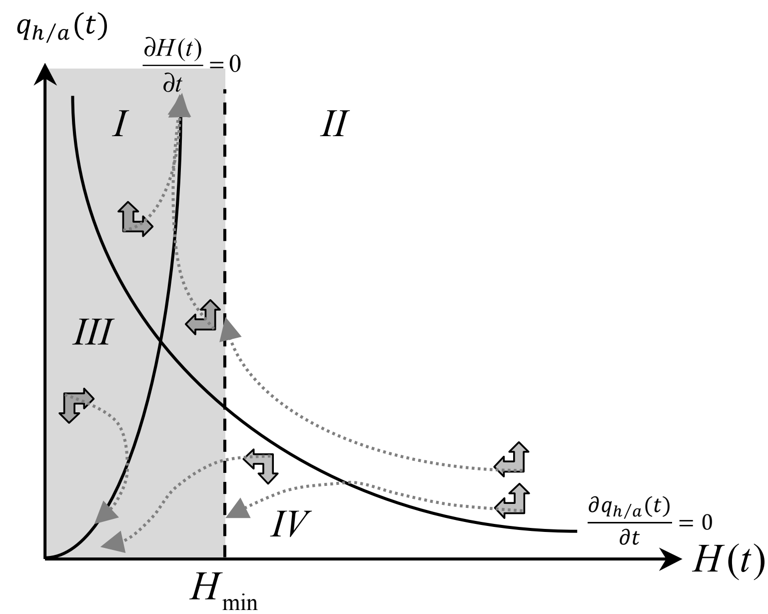

Figure 3: *Phase diagram of the relative marginal value of health $q_{h/a}(t)$ versus health $H(t)$.*

Regime switches occur when $\partial q_{h/a}(t)/\partial t = 0$ and $\partial H(t)/\partial t = 0$. These boundaries between regimes, so called null-clines, are shown by the thick lines in Figure 3 and are obtained by setting the derivatives to zero in (37) and (51), respectively. Because of diminishing marginal consumption $\partial^2 U/\partial H^2 < 0$ and production $\partial^2 Y/\partial H^2 < 0$ benefit (assumption 1), and a diminishing rate of aging with health $\partial^2 d/\partial H^2 \geq 0$ (assumption 3), the null-cline for the relative marginal value of health, $\partial q_{h/a}(t)/\partial t = 0$, is downward sloping. The null-cline for health capital, associated with $\partial H(t)/\partial t = 0$, is upward sloping. It can be either a convex or a concave relation between $q_{h/a}(t)$ and $H(t)$ (see 51). Here, we show a convex relation for illustrative purposes.

The two null-clines define four distinct regions I, II, III and IV. The left-up, right-up, left-down and right-down block arrows indicate the direction of motion in the phase diagram and the grey dotted lines provide example trajectories. For example, every point in region III is associated with an evolution toward lower relative marginal value of health $q_{h/a}(t)$ and higher health $H(t)$.

While the null clines are functions of age (directly and also indirectly through dependence on other control functions such as consumption and job stress), and therefore shift over time, the nature of the diagram is essentially unchanged as the system evolves, i.e., there are always four dynamic regions, the $\partial q_{h/a}(t)/\partial t$ null-cline is always downward sloping, and the $\partial H(t)/\partial t$ null cline is always upward sloping and intersects the origin.

The intersection of the two null-clines defines the steady state, at which both $q_{h/a}(t)$ and $H(t)$ would be temporarily at a stand-still. The steady state, is however of little interest as a potential solution for the system. First, it is saddle-point unstable. This is clear from visual inspection of the phase diagram: a small deviation (perturbation) from the steady state will evolve away from the steady state, except if the deviation landed on an infinitesimally narrow trajectory (the unique trajectory that eventually leads to the steady state).[41] Second, if the trajectory starts at a point that is not a steady state, it cannot reach a steady state in a finite amount of time (Theorem 13.4, p. 350, Caputo 2005).[42] Thus, the steady state requires infinite length of life, and the absence of any perturbations (no matter how small).

The starting point of the optimal trajectory for health is given by the initial condition $H(0) = H_0$. End of life occurs when health deteriorates to a minimum health level $H(T) = H_{\min}$. A priori we do not know the location of $H_{\min}$; here it is shown to the right of the steady state for illustration. Life is not sustainable below $H_{\min}$ as illustrated by the shaded area.

By definition health eventually decreases over the lifecycle since end of life is determined by the minimum health level $H(T) = H_{\min}$, below which life is not sustainable.

[41] A formal proof of the instability of the steady state can be straightforwardly obtained by calculating the Jacobian $J(q^*_{h/a}, H^*)$ of the linearized system at the steady state $(q^*_{h/a}, H^*)$ and showing that the two eigenvalues are real, non-zero and unequal $(\{\mathrm{tr}[J(q^*_{h/a}, H^*)]\}^2 > 4\det[J(q^*_{h/a}, H^*)])$. See Theorem 13.6, p. 354 of Caputo (2005).

[42] Caputo's theorem 13.4 holds for an autonomous system, a system for which the steady state is fixed (does not move). For such a system the steady state is reached after an infinitely long time. If, however, the steady state itself evolves, as is true for our system, the trajectory might never reach the steady state, not even after an infinitely long time. It is thus even less likely in such cases that the steady state is meaningful as a potential solution of the system.

Thus eventually the trajectory has to enter regions II or IV of the phase diagram so that health declines.[43] However, the relative marginal value of health $q_{h/a}(t)$ could be either decreasing or increasing over the lifecycle. Health investments increase with age (e.g., Grossman 1972a,b). This suggests that optimal solutions are best described by region II: declining health capital and increasing relative marginal value of health (and hence increasing levels of health investment, see equation 15) with age.[44]

C.2 Proof of proposition 1: Absent ability to extend life, wealthy individuals, ceteris paribus, value health only marginally more than less wealthy individuals.

The comparative dynamic effect of initial wealth on the relative marginal value of health, keeping length of life T fixed, is obtained by taking the derivate of (37) with respect to intial wealth A_0

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{\partial q_{h/a}(t)}{\partial A_0}\bigg|_T &= \left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial H}e^{-(\beta-r)t}\right]\times\frac{\partial q_A(0)}{\partial A_0}\bigg|_T \\
&+ \left[\frac{\partial d}{\partial H}+r\right]\times\frac{\partial q_{h/a}(t)}{\partial A_0}\bigg|_T \\
&+ \left[q_{h/a}(t)\frac{\partial^2 d}{\partial C_h\partial H}-\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial C_h\partial H}e^{-(\beta-r)t}\right]\times\frac{\partial C_h(t)}{\partial A_0}\bigg|_T \\
&+ \left[q_{h/a}(t)\frac{\partial^2 d}{\partial C_u\partial H}-\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial C_u\partial H}e^{-(\beta-r)t}\right]\times\frac{\partial C_u(t)}{\partial A_0}\bigg|_T \\
&- \left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial L\partial H}e^{-(\beta-r)t}\right]\times\frac{\partial L(t)}{\partial A_0}\bigg|_T \\
&+ \left[q_{h/a}(t)\frac{\partial^2 d}{\partial z\partial H}-\frac{\partial^2 Y}{\partial z\partial H}\right]\times\frac{\partial z(t)}{\partial A_0}\bigg|_T \\
&- \left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial H^2}e^{-(\beta-r)t}+\frac{\partial^2 Y}{\partial H^2}-q_{h/a}(t)\frac{\partial^2 d}{\partial H^2}\right]\times\frac{\partial H(t)}{\partial A_0}\bigg|_T .
\end{aligned} \tag{52}$$

[43] Note that since the null clines shift over time, the trajectory may enter regions II or IV by being overtaken by the null clines. Thus the trajectory does not allways have to be in regions II or IV but may reside for some time in regions I or III.

[44] This contrasts markedly with human capital (skill, knowledge) which is best characterized by region III: decreasing levels of investment with age, and initially increasing then potentially declining levels of human capital.

Likewise, for the health stock, the comparative dynamic effect of A_0, keeping length of life T fixed, is obtained by taking the derivative of equation (51) with respect to A_0

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{\partial H(t)}{\partial A_0}\bigg|_T &= \left[\frac{\alpha}{1-\alpha}\frac{I(t)^\alpha}{q_{h/a}(t)}\right] \times \frac{\partial q_{h/a}(t)}{\partial A_0}\bigg|_T \\
&- \left[\frac{\partial d}{\partial C_h}\right] \times \frac{\partial C_h(t)}{\partial A_0}\bigg|_T \\
&- \left[\frac{\partial d}{\partial C_u}\right] \times \frac{\partial C_u(t)}{\partial A_0}\bigg|_T \\
&- \left[\frac{\gamma_w(1-\kappa_I)\alpha}{1-\alpha}\frac{I(t)^\alpha}{1+z(t)} + \frac{\partial d}{\partial z}\right] \times \frac{\partial z(t)}{\partial A_0}\bigg|_T \\
&- \left[\frac{\partial d}{\partial H}\right] \times \frac{\partial H(t)}{\partial A_0}\bigg|_T. \qquad (53)
\end{aligned}$$

Note that (52) and (53) are expressed in terms of the effect of initial wealth on the control variables, $\partial L(t)/\partial A_0|_T$, $\partial C_h(t)/\partial A_0|_T$, $\partial C_u(t)/\partial A_0|_T$, and $\partial z(t)/\partial A_0|_T$, on the state variable health $\partial H(t)/\partial A_0|_T$, on the marginal value of (initial) wealth (a co-state variable) $\partial q_A/\partial A_0|_T$, and on the relative marginal value of health (a co-state variable) $\partial q_{h/a}/\partial A_0\big|_T$. One can develop similar relations for the comparative dynamic effect of initial wealth on the controls: $L(t)$, $C_h(t)$, $C_u(t)$, and $z(t)$ (see section C.12). These too are expressed in terms of variation in controls, in health, in the marginal value of initial wealth and in the relative marginal value of health.

Pontryagin's maximum principle (e.g., Caputo 2005) informs us that the solution of the optimal control problem is no longer a function of the controls and can be fully expressed in the state and co-state functions. Indeed, one can substitute the expressions for variation in the control variables, $\partial L(t)/\partial A_0|_T$, $\partial C_h(t)/\partial A_0|_T$, $\partial C_u(t)/\partial A_0|_T$, and $\partial z(t)/\partial A_0|_T$, into one another such that the final result (not shown) are comparative dynamic expressions in terms of the effects of initial wealth on health $\partial H(t)/\partial A_0|_T$, on the marginal value of initial wealth $\partial q_A(0)/\partial A_0|_T$, and on the relative marginal value of health $\partial q_{h/a}(t)/\partial A_0\big|_T$. Thus the expressions contain variation of state and co-state functions but no longer contain variation in control functions. These expressions however are unwieldy with a cumbersome mix of opposite-sign coefficients from which it is hard to draw firm conclusions. Therefore, we assume that first-order (direct) effects dominate higher-order (indirect) effects (assumption 6 in section 3.4).

As an example, wealth (operating through the marginal value of initial wealth $q_A(0)$) affects the rate of change of variation in the relative marginal value of health $(\partial/\partial t)(\partial q_{h/a}(t)/\partial A_0\big|_T)$ directly (first term on the right-hand side [RHS] of 52), but also indirectly through, for example, the effect that wealth has on healthy consumption, and healthy consumption in turn has on $(\partial/\partial t)(\partial q_{h/a}(t)/\partial A_0\big|_T)$ (third term on the RHS of 52). The comparative dynamic effect of wealth on healthy consumption is given by (73).

Combining (73) with (52), we obtain the combined wealth effect

$$\left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial H}e^{-(\beta-r)t}\right]\left[1+\frac{\frac{1}{q_A(0)^2}\frac{\partial U}{\partial C_h}}{\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial C_h^2}-q_{h/a}(t)\frac{\partial^2 d}{\partial C_h^2}e^{(\beta-r)t}}\right]\times\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T,$$

where the first term, 1, in the large term in brackets represents the first-order (direct) effect of wealth, and the second term represents the second-order (indirect) effect operating through healthy consumption. The assumption is that the direct effect of wealth dominates any indirect wealth effect that operates through healthy consumption or through any other control variable. This simplifies the expressions considerably. We have:

$$\begin{aligned}\frac{\partial}{\partial t}\left.\frac{\partial q_{h/a}(t)}{\partial A_0}\right|_T \approx\;& \left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial H}e^{-(\beta-r)t}\right]\times\left.\frac{\partial q_A(0)}{\partial A_0}\right|_T\\ &+\left[\frac{\partial d}{\partial H}+r\right]\times\left.\frac{\partial q_{h/a}(t)}{\partial A_0}\right|_T\\ &-\left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial H^2}e^{-(\beta-r)t}+\frac{\partial^2 Y}{\partial H^2}-q_{h/a}(t)\frac{\partial^2 d}{\partial H^2}\right]\times\left.\frac{\partial H(t)}{\partial A_0}\right|_T,\end{aligned}\tag{54}$$

and

$$\begin{aligned}\frac{\partial}{\partial t}\left.\frac{\partial H(t)}{\partial A_0}\right|_T \approx\;& \left[\frac{\alpha}{1-\alpha}\frac{I(t)^\alpha}{q_{h/a}(t)}\right]\times\left.\frac{\partial q_{h/a}(t)}{\partial A_0}\right|_T\\ &-\frac{\partial d(t)}{\partial H(t)}\times\left.\frac{\partial H(t)}{\partial A_0}\right|_T.\end{aligned}\tag{55}$$

Figure 4 shows the phase diagram for the motion paths of the variation of the relative marginal value of health with respect to initial wealth $\partial q_{h/a}(t)/\partial A_0\big|_T$ (y-axis) versus the variation of health with respect to initial wealth $\partial H(t)/\partial A_0|_T$ (x-axis). The boundaries between regimes, the so called null-clines, are shown (for illustration only as they move over time) by the thick lines in the figure and are obtained by setting the derivatives $(\partial/\partial t)\left(\partial q_{h/a}(t)/\partial A_0\right)\big|_T$ and $(\partial/\partial t)\left(\partial H(t)/\partial A_0\right)|_T$ to zero. Since we know the signs of all coefficients in (54) and (55), in particular $\partial q_A(0)/\partial A_0|_T < 0$ (assumption 5), $\partial^2 U/\partial H^2 < 0$, $\partial^2 Y/\partial H^2 < 0$ (assumption 1), $\partial d/\partial H \geq 0$ and $\partial^2 d/\partial H^2 \leq 0$ (assumption 3), we can predict the sign of $(\partial/\partial t)\left(\partial q_{h/a}(t)/\partial A_0\right)\big|_T$ and $(\partial/\partial t)\left(\partial H(t)/\partial A_0\right)|_T$ in the four dynamic regions of the phase diagram, defined by the null-clines. The block arrows indicate the direction of motion in each of the four dynamic regions and the dotted lines provide example trajectories.[45] Since both initial health $H(0) = H_0$ and end-of-life health

[45] While the null clines are functions of age and shift over time the nature of the diagram is essentially unchanged, for the assumed signs of the coefficients in (54) and (55). That is, there are always four dynamic regions, the $(\partial/\partial t)\left(\partial q_{h/a}(t)/\partial A_0\right)\big|_T$ null-cline is always downward sloping and intersects the x-axis to the right of the origin, and the $(\partial/\partial t)\left(\partial H(t)/\partial A_0\right)|_T$ null cline is always upward sloping and intersects the origin.

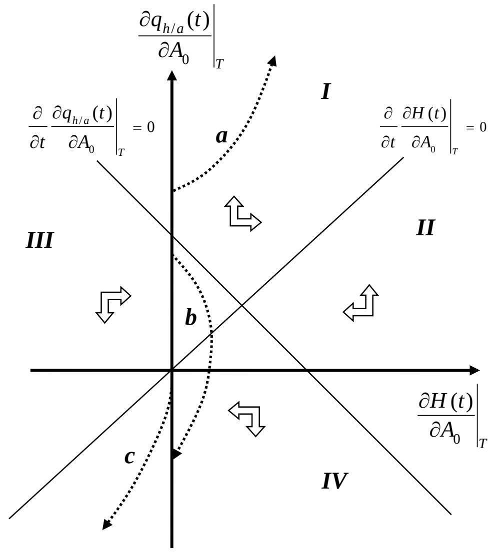

Figure 4: *Phase diagram of the deviation from the unperturbed path, resulting from variation in initial wealth* δA_0*, of the relative marginal value of health* $\partial q_{h/a}(t)/\partial A_0\big|_T$ *and of the health stock* $\partial H(t)/\partial A_0|_T$*, for fixed* T*.*

$H(T) = H_{\min}$ are fixed, it follows that $\partial H(0)/\partial A_0|_T = \partial H(T)/\partial A_0|_T = 0$. Thus, in the phase diagram all admissible paths start and end at the vertical axis.

Consider a path that starts at the vertical axis, but below the horizontal axis (corresponding to $\partial q_{h/a}(0)/\partial A_0 < 0\big|_T$). Such a path will move toward the South-West, and stay there indefinitely, as illustrated by the dotted line (c). Similarly, a path starting at the vertical axis, but above the $(\partial/\partial t)\left(\partial q_{h/a}(t)/\partial A_0\right)\big|_T$ null-cline, will move toward the North-East and stay there indefinitely, never returning to the vertical axis in finite time, as illustrated by trajectory (a).

Now consider a path starting at the vertical axis, between the $(\partial/\partial t)\left(\partial q_{h/a}(t)/\partial A_0\right)\big|_T = 0$ null-cline and the origin. This path is associated with $\partial q_{h/a}(0)/\partial A_0\big|_T > 0$, and returns to the vertical axis in finite time if it crosses the horizontal axis and enters dynamic region IV at some point over the lifecycle. This path satisfies all conditions, and an example trajectory (b) is shown for illustrative purposes.[46]

We conclude from this (see trajectory b) that wealth increases the relative marginal

[46]More complicated paths are possible (given that the null clines shift with time) that may temporarily enter regions I and/or II, but only those paths that start on the vertical axis above the horizontal axis and that eventually end on the vertical axis below the horizontal axis are admissible, producing broadly similar patterns, and leading to the same conclusions.

value of health $\partial q_{h/a}(t)/\partial A_0\big|_T > 0$ initially, but decreases it $\partial q_{h/a}(t)/\partial A_0\big|_T < 0$ eventually. In a model with a fixed life span T, health is higher at all ages, $\partial H(t)/\partial A_0|_T > 0 \quad \forall t$ (trajectory a lies to the right of the vertical axis), except for $t = 0$ and $t = T$. Because length of life T is fixed, any improved health behavior and any additional health investment associated with the higher relative marginal value of health (see the first-order conditions in section 3.2) has to be balanced by reductions in the relative marginal value of health and associated reductions in health investment to ensure that health reaches the minimum health level $H_{\min}$ over the unchanged horizon T. Fixed length of life thus mutes the response to additional wealth. Q.E.D.

C.3 Proof of proposition 2: Wealth raises the consumption benefit (utility) but not the production benefit of health.

If health provides no utility, the first term in (54) disappears ($\partial U/\partial H = 0$), while (55) is unchanged. In the phase diagram, the null cline for the variation of the relative marginal value of health with respect to wealth, $(\partial/\partial t)(\partial q_{h/a}(t)/\partial A_0\big|_T) = 0$, is still downward sloping but now crosses the origin. The only trajectory consistent with the condition that it starts and ends on the vertical axis is now the origin itself, i.e. the perturbed path is identical to the unperturbed path. There is no variation in the relative marginal value of health, $\partial q_{h/a}(t)/\partial A_0\big|_T = 0 \quad \forall t$, or in health $\partial H(t)/\partial A_0|_T = 0 \quad \forall t$. Absent a consumption benefit, the benefit of health consists solely of the production benefit. In this case, and absent ability to extend life, wealthy individuals do not value the production benefit differently than do less wealthy individuals. Q.E.D.

C.4 Proof of proposition 3: Wealthy individuals live longer: $\partial T/\partial A_0 > 0$.

Taking into account that in the optimum the condition $\Im(T) = 0$ has to be satisfied (see 14), we have

$$\left.\frac{\partial \Im(T)}{\partial A_0}\right|_T + \left.\frac{\partial \Im(T)}{\partial T}\right|_{A_0} \frac{\partial T}{\partial A_0} = 0, \tag{56}$$

where $\partial f(t)/\partial A_0|_T$ denotes variation in an endogenous function $f(t)$ with respect to initial wealth A_0, keeping length of life T fixed, and $\partial f(t)/\partial T|_{A_0}$ denotes variation in an endogenous function $f(t)$ with respect to T, keeping A_0 fixed (in this case the notation is used for $\Im(T)$).

The change in life expectancy due to an increase in initial wealth $\partial T/\partial A_0$ can then be identified from the identity

$$\frac{\partial T}{\partial A_0} = -\frac{\left.\frac{\partial \Im(T)}{\partial A_0}\right|_T}{\left.\frac{\partial \Im(T)}{\partial T}\right|_{A_0}}. \tag{57}$$

Using the expression for the Hamiltonian (11) we obtain

$$\begin{aligned}
\left.\frac{\partial \Im(T)}{\partial A_0}\right|_T &= \left.\frac{\partial \Im(T)}{\partial \mathbb{C}(T)}\frac{\partial \mathbb{C}(T)}{\partial A_0}\right|_T + \left.\frac{\partial \Im(T)}{\partial A(T)}\frac{\partial A(T)}{\partial A_0}\right|_T + \left.\frac{\partial \Im(T)}{\partial H(T)}\frac{\partial H(T)}{\partial A_0}\right|_T \\
&\quad + \left.\frac{\partial \Im(T)}{\partial q_A(T)}\frac{\partial q_A(T)}{\partial A_0}\right|_T + \left.\frac{\partial \Im(T)}{\partial q_H(T)}\frac{\partial q_H(T)}{\partial A_0}\right|_T \\
&= \left.\frac{\partial q_A(T)}{\partial A_0}\right|_T \left.\frac{\partial A(t)}{\partial t}\right|_{t=T} + \left.\frac{\partial q_H(T)}{\partial A_0}\right|_T \left.\frac{\partial H(t)}{\partial t}\right|_{t=T} \\
&= \left.\frac{\partial q_A(0)}{\partial A_0}\right|_T e^{-rT} \left.\frac{\partial A(t)}{\partial t}\right|_{t=T} \\
&\quad + \left\{ \left.\frac{\partial q_A(0)}{\partial A_0}\right|_T e^{-rT} q_{h/a}(T) + q_A(0)e^{-rT} \left.\frac{\partial q_{h/a}(T)}{\partial A_0}\right|_T \right\} \left.\frac{\partial H(t)}{\partial t}\right|_{t=T}. \qquad (58)
\end{aligned}$$

where $\mathbb{C}(t)$ denotes the vector of control variables, $\mathbb{C}(t) \equiv [C_h(t), C_u(t), L(t), I(t), z(t)]$. In the derivations we have used $\partial \Im(T)/\partial \mathbb{C}(T) = 0$, and $\partial \Im(T)/\partial H(T) = - \left.\partial q_H(t)/\partial t\right|_{t=T}$, which follow from the first-order conditions, and $\left.\partial A(T)/\partial A_0\right|_T = \left.\partial H(T)/\partial A_0\right|_T = 0$, since $A(T)$ and $H(T)$ are fixed.

Note that we distinguish in notation between $\left.\partial f(t)/\partial t\right|_{t=T}$, which represents the derivative with respect to time t at time $t = T$, and $\left.\partial f(t)/\partial T\right|_{t=T}$, which represents variation with respect to the parameter T at time $t = T$.

Using (57) and (58), and assuming diminishing returns to life extension $\left.\partial \Im(T)/\partial T\right|_{A_0} < 0$ (cf. assumption 5 in section 3.4), wealth increases longevity $\partial T/\partial A_0 > 0$ if $\left.\partial \Im(T)/\partial A_0\right|_T > 0$. In (58), both $\left.\partial A(t)/\partial t\right|_{t=T}$ and $\left.\partial H(t)/\partial t\right|_{t=T}$ are negative since health declines near the end of life as it approaches $H_{\min}$ from above, and assets decline near the end of life in absence of a strong bequest motive. For diminishing returns to wealth $\left.\partial q_A(0)/\partial A_0\right|_T < 0$ (cf. assumption 5 in section 3.4),[47] a sufficient requirement for length of life to increase in response to positive variation in wealth is $\left.\partial q_{h/a}(T)/\partial A_0\right|_T \leq 0$. As we established in section C.3, indeed $\left.\partial q_{h/a}(T)/\partial A_0\right|_T \leq 0$, under plausible assumptions, and thus $\partial T/\partial A_0 > 0$. Q.E.D.

C.5 Proof of Proposition 4: Wealthy individuals value health more. The more life can be extended, the stronger is the increase in the value of health in response to additional wealth.

In the previous section C.4, we showed that wealthy individuals live longer. This result could be obtained by considering dynamic responses to variation in wealth for fixed length of life T. We are now interested in understanding the full comparative dynamic response

[47] Technically, assumption 5 is made for free T, not fixed T as used here. The assumption of diminishing returns to wealth is equally, if not more, plausible for fixed T, a case in which individuals are more constrained, lacking ability to choose T optimally, in using any additional resources to improve their well being.

of the relative marginal value of health $q_{h/a}(t)$ and the health stock $H(t)$ to an increase in initial wealth, employing the full model with optimally chosen T. The total differentials

$$\frac{\partial q_{h/a}(t)}{\partial A_0} = \left.\frac{\partial q_{h/a}(t)}{\partial A_0}\right|_T + \left.\frac{\partial q_{h/a}(t)}{\partial T}\right|_{A_0}\frac{\partial T}{\partial A_0}, \tag{59}$$

$$\frac{\partial H(t)}{\partial A_0} = \left.\frac{\partial H(t)}{\partial A_0}\right|_T + \left.\frac{\partial H(t)}{\partial T}\right|_{A_0}\frac{\partial T}{\partial A_0}, \tag{60}$$

consist of the effect of variation in wealth keeping T fixed (first term on the RHS in 59 and in 60), which we explored in section C.3, and the effect that operates through responses in the optimal length of life T, keeping initial wealth A_0 fixed (second term on the RHS in 59 and in 60).

The coefficients in the comparative dynamic expressions for the response in the relative marginal value of health and the health stock to variation in length of life T, holding initial wealth A_0 constant, are identical to the coefficients derived for the response to variation in initial wealth A_0, holding length of life T fixed, shown in (54) and (55). That is, we simply have to replace the partial differentials with their total differentials, i.e. $\partial q_A(0)/\partial A_0|_T$ with $\partial q_A(0)/\partial T$, $\partial q_{h/a}(t)/\partial A_0|_T$ with $\partial q_{h/a}(t)/\partial T$, and $\partial H(t)/\partial A_0|_T$ with $\partial H(t)/\partial T$, in (54) and (55) to obtain the total comparative dynamic effect.

There is, however, one important difference compared to the previous case in which we explored variation with respect to A_0 for fixed T: the terminal value of the variation in health with respect to T, for fixed A_0, is positive, $\partial H(t)/\partial T|_{A_0,t=T} > 0$. Thus the total differential with respect to wealth A_0 is too, $\partial H(T)/\partial A_0 > 0$, implying that all admissible paths end to the right of the vertical axis in the phase diagram (and not on the vertical axis as is the case for fixed T in section C.3). This can be seen as follows. First, solve the state equation for health (2):

$$H(t) = H(0) + \int_0^t [I(s)^\alpha - d(s)]ds. \tag{61}$$

Then take the derivative of (61) with respect to T to obtain

$$\left.\frac{\partial H(t)}{\partial T}\right|_{A_0} = \int_0^t \left\{\alpha I(s)^{\alpha-1}\left.\frac{\partial I(s)}{\partial T}\right|_{A_0} - \left.\frac{\partial d(s)}{\partial T}\right|_{A_0}\right\} ds. \tag{62}$$

Now take the derivative of (61) with respect to T for $t = T$

$$\begin{aligned}\left.\frac{\partial H(T)}{\partial T}\right|_{A_0} &= \left.\frac{\partial H_{\min}}{\partial T}\right|_{A_0} = 0 \\ &= I(T)^\alpha - d(T) + \left.\frac{\partial H(t)}{\partial T}\right|_{A_0,t=T} \\ &= \left.\frac{\partial H(t)}{\partial t}\right|_{A_0,t=T} + \left.\frac{\partial H(t)}{\partial T}\right|_{A_0,t=T},\end{aligned} \tag{63}$$

where we distinguish in notation between $\partial H(t)/\partial t|_{A_0,t=T}$, which represents the derivative with respect to time t at $t = T$, and $\partial H(t)/\partial T|_{A_0,t=T}$, which represents variation with respect to parameter T at $t = T$.

The derivative of health with respect to time at $t = T$ is negative since we approach $H_{\min}$ from above. Thus we have

$$\left.\frac{\partial H(t)}{\partial T}\right|_{A_0,t=T} = -\left.\frac{\partial H(t)}{\partial t}\right|_{A_0,t=T} > 0.$$

Intuitively, if length of life is extended to $T + \delta T$ the health stock has to be higher at the previous point of death T, and it is higher by exactly the change in health over a small period of time. Thus, $\partial H(T)/\partial A_0 = \partial H(T)/\partial A_0|_T + (\partial H(t)/\partial T|_{A_0,t=T})(\partial T/\partial A_0) = (\partial H(t)/\partial T|_{A_0,t=T})(\partial T/\partial A_0) > 0$.

Figure 5 presents the comparative dynamic results. The initial condition $\partial H(0)/\partial A_0 = 0$, implies that all admissible paths start on the vertical axis, and the end-condition $\partial H(T)/\partial A_0 > 0$, implies that all admissible paths end to the right of the vertical axis, at $\partial H(T)/\partial A_0$ (indicated by the four dotted vertical lines in the figure). As the phase diagram shows, trajectories a, b, c and d, corresponding to four different levels of $\partial H(T)/\partial A_0$, are feasible. This implies that $\partial H(t)/\partial A_0 > 0 \quad \forall t$ (except for $t = 0$), $\partial q_{h/a}(t)/\partial A_0 > 0$ initially, and while potentially the relative marginal value is lower $\partial q_{h/a}(t)/\partial A_0 < 0$ after some $t = t^\dagger$ (as for trajectory d, but not for a, b or c), cumulatively it is higher $\int_0^T \left[\partial q_{h/a}(t)/\partial A_0\right] dt > 0$, which proves the first part op proposition 4.

Consider equation (60) for $t = T$. In proposition 2, section C.3, we established that for fixed T, $\partial H(T)/\partial A_0|_T = 0$. Thus the value of the total differential $\partial H(T)/\partial A_0$ is determined by the second term on the RHS of (60). Since $\partial H(t)/\partial T|_{A_0,t=T} = -\,\partial H(t)/\partial t|_{A_0,t=T} = -[I(T)^\alpha - d(T)H(T)]$ (compare with 63), it is the same for all four scenarios as it represents the negative of the derivate with respect to time t at $t = T$ of the unperturbed (unchanged) path. Thus the end point $\partial H(T)/\partial A_0$ is proportional to the degree of life extension afforded by additional wealth $\partial T/\partial A_0$: it thus lies further to the right in the phase diagram (vertical dotted lines) for greater $\partial T/\partial A_0$.

In scenario d (see Figure 5), if life extension due to additional wealth is small, wealthier individuals will value health more cumulatively, but not necessarily at all times, the relative marginal value of health increases less rapidly over the life cycle, compared to the unperturbed path (less steep increase), and the trajectory eventually crosses the unperturbed path. Associated with the limited increase in the relative marginal value of wealth are relatively low levels of health investment, moderately healthier behavior, marginal improvement in health, and small life extension. This scenario of initial investment and subsequent reduced investment most closely resembles that of the fixed T case (compare with the only admissible path b in Figure 4). In the limit where the degree of life extension approaches zero, scenario d becomes the fixed T scenario. Moving from scenario d to c to b and finally to a, the marginal value of health increases progressively and so does life extension. This proves the second part of proposition 4. In all four scenarios

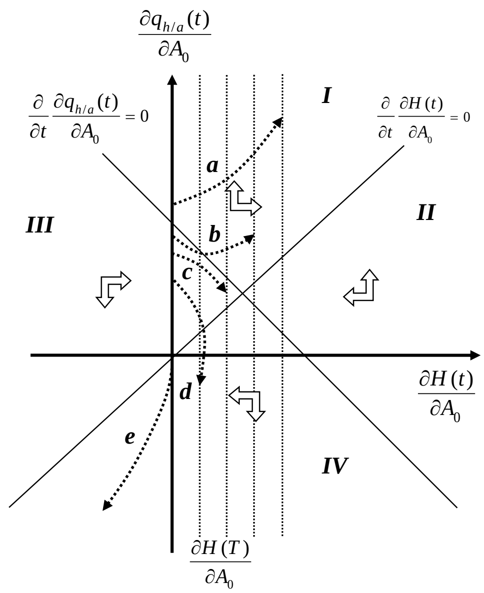

Figure 5: *Phase diagram of the deviation from the unperturbed path, resulting from variation in initial wealth δA_0, of the relative marginal value of health $\partial q_{h/a}(t)/\partial A_0$ and of the health stock $\partial H(t)/\partial A_0$, allowing length of life T to be optimally chosen. The four vertical dotted lines represent different potential values for the end point $\partial H(T)/\partial A_0$.*

individuals value health more cumulatively $\int_0^T \left[\partial q_{h/a}(t)/\partial A_0\right] dt > 0$ but they may value health less at certain ages (for example in scenario d individuals value health less late in life, compared to the unperturbed path). Q.E.D.

C.6 Proof of Proposition 5: Wealthy individuals are healthier at all ages.

The discussion for the proof of Proposition 4 in the previous section C.5 also provides the proof for Proposition 5. Note in particular, that all feasible trajectories in Figure 5 lie to the right of the vertical axis, i.e. $\partial H(t)/\partial A_0 \geq 0$. Q.E.D.

C.7 Proof of Proposition 6: Permanently higher wages and education operate in a similar manner to an increase in wealth δA_0 (propositions 1 through 5), with some differences: (i) the wealth effect is muted by the increased opportunity cost of time, (ii) permanent wages w_E and education E also raise the production benefit of health, and (iii) education raises the efficiency of health investment.

Permanent wages w_E The comparative dynamic effect of a permanent increase in the wage rate $w(t)$, through, e.g., an increase in the parameter w_E in (10), on the relative marginal value of health $q_{h/a}(t)$ can be obtained by taking the derivate of (37) with respect to w_E and keeping first-order terms (total differentials, free T):

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\partial q_{h/a(t)}}{\partial w_E} \quad &\approx \quad \frac{w_*(t)}{w_E}\left[1+z(t)\right]^{\gamma_w}\frac{\partial s}{\partial H}\\ &+\left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial H}e^{-(\beta-r)t}\right]\times\frac{\partial q_A(0)}{\partial w_E}\\ &+\left[\frac{\partial d}{\partial H}+r\right]\times\frac{\partial q_{h/a}(t)}{\partial w_E}\\ &-\left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial H^2}e^{-(\beta-r)t}+\frac{\partial^2 Y}{\partial H^2}-q_{h/a}(t)\frac{\partial^2 d}{\partial H^2}\right]\times\frac{\partial H(t)}{\partial w_E}. \end{aligned} \tag{64}$$

The first term on the RHS of (64) represents a wealth effect. Permanently higher wages raise the production benefit of health, as health is more valuable in reducing sick time (freeing time for work) when wages are higher. In addition there is the usual wealth effect (second term on the RHS). Both wealth terms are negative since sick time decreases with health $\partial s/\partial H < 0$, and $\partial q_A(0)/\partial w_E|_T < 0$ (in line with assumption 4) because w_E raises lifetime earnings (permanent income) and relaxes the budget constraint (3). Variation in permanent wages δw_E is thus distinct from variation in wealth δA_0 in that it not only raises the consumption benefit of health (as is the case for variation in δA_0) but also the production benefit of health (which is not the case for variation in δA_0).

Likewise, the comparative dynamic effect of a permanent increase in the wage rate on health $H(t)$ is obtained by taking the derivative of (51) with respect to w_E and keeping first-order terms:

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\partial H(t)}{\partial w_E} \quad &\approx \quad -\frac{\alpha}{1-\alpha}\frac{(1-\kappa_I)}{w_E(t)}I(t)^\alpha\\ &+\left[\frac{\alpha}{1-\alpha}\frac{I(t)^\alpha}{q_{h/a}(t)}\right]\times\frac{\partial q_{h/a}(t)}{\partial w_E}\\ &-\frac{\partial d}{\partial H}\times\frac{\partial H(t)}{\partial w_E}, \end{aligned} \tag{65}$$

where the first term on the RHS of equation (65) represents the negative effect of the opportunity cost of time on health investment, and in turn on health.

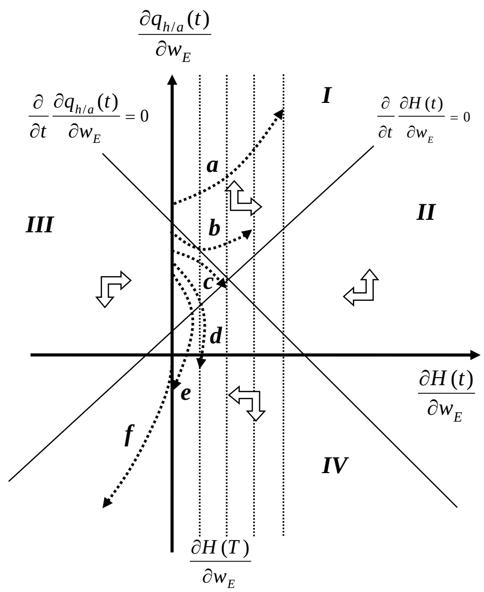

Figure 6: *Phase diagram of the deviation from the unperturbed path, resulting from variation in permanent wages δw_E, of the relative marginal value of health $\partial q_{h/a}(t)/\partial w_E$ and of the health stock $\partial H(t)/\partial w_E$, allowing length of life T to be optimally chosen. The four vertical dotted lines represent different potential values for the end point $\partial H(T)/\partial w_E$.*

The corresponding phase diagram is shown in Figure 6. It is nearly identical to the phase diagram for variation in initial wealth δA_0 (Figure 5), except that the $(\partial/\partial t)(\partial H(t)/\partial w_E)$ null cline crosses the vertical $\partial q_{h/a}(t)/\partial w_E$ axis at $q_{h/a}(t)(1 - \kappa_I)/w_E$ and not at the origin. This term represents the effect of a permanent increase in wages w_E on the opportunity cost of investing time in health. The $(\partial/\partial t)(\partial q_{h/a}(t)/\partial w_E)$ null cline crosses the vertical $\partial q_{h/a}(t)/\partial w_E$ axis at $(-\partial s/\partial H)\,(w_*(t)/w_E)\,[1+z(t)]^{\gamma_w}\,/\,[\partial d/\partial H + r] - \left[q_A(0)^{-2}\partial U/\partial H e^{-(\beta-r)t}\right]\partial q_A(0)/\partial w_E$. This expression represents a wealth, or permanent income, effect: permanently higher wages increase the production benefit of health (first term) and increases wealth, thereby raising the consumption benefit of health (second term; operating through $\partial q_A(0)/\partial w_E < 0$). In the scenario depicted in Figure 6, it is assumed that the opportunity cost of time effect is small compared to the wealth / permanent income effect.

Following similar steps as in sections C.3 to C.5 for variation in wealth, we first need to establish whether length of life is extended as a result of a permanent increase in income. This can be accomplished by considering the fixed T case. The comparative dynamic effect of a permanent increase in the wage rate w_E on longevity can be obtained by replacing A_0 with w_E in (56), (57), and (58). Since $\partial q_A(0)/\partial w_E|_T < 0$ (in line with assumption 4), it follows that, similar to the case for variation in initial wealth δA_0 (see section C.4), a sufficient condition for life extension in response to positive variation in permanent wages is $\partial q_{h/a}(T)/\partial w_E\big|_T \leq 0$.

For fixed T all admissible paths in the phase diagram have to start and end at the vertical axis, since $H(0)$ and $H(T)$ are fixed.[48] Trajectory e in the phase diagram of Figure 6 is consistent with these conditions and the trajectory is characterized by $\partial q_{h/a}(T)/\partial w_E\big|_T < 0$. Thus length of life is extended $\partial T/\partial w_E > 0$.

Considering the T free case, the reasoning is identical to the discussions for propositions 3 and 4 in sections C.4 and C.5. Following the logic outlined there, we find that trajectories a, b, c, and d are consistent with life extension. The greater life is extended as a result of greater permanent income, the further to the right is the trajectory's end point $\partial H(T)/\partial w_E$. Example trajectory a is associated with a large increase in the marginal value of health $\partial q_{h/a}(t)/\partial w_E$ and in health $\partial H(t)/\partial w_E$, compared to the unperturbed trajectory, and this trajectory is associated with the greatest gain in longevity $\partial T/\partial w_E$. Trajectory b and c represent an intermediary case and trajectory d a case of limited response, the latter most closely resembles the fixed T case, represented by trajectory e. Trajectory f is incompatible with live extension and ruled out. Q.E.D.

These results rely on our assumption that the opportunity cost effect is smaller than the wealth / permanent income effect. If, however, the opportunity cost effect is substantial, the $\partial H(t)/\partial w_E$ null cline is shifted further upward in the phase diagram of Figure 6 than shown. A trajectory similar to e might then end up above the $\partial H(t)/\partial w_E$ axis with a positive end value of $\partial q_{h/a}(T)/\partial w_E\big|_T$, in which case we cannot unambiguously establish that length of life increases.[49]

If the opportunity cost is very high, outweighing the wealth / permanent income effect, the $\partial H(t)/\partial w_E$ null cline could even cross the vertical $\partial q_{h/a}(t)/\partial w_E$ axis above the location where the $\partial q_{h/a}(t)/\partial w_E$ null cline crosses the vertical $\partial q_{h/a}(t)/\partial w_E$ axis. In such a scenario (not shown), for fixed T, any admissible trajectory is characterized by $\partial q_{h/a}(T)/\partial w_E\big|_T > 0$, and we cannot unambiguously establish that length of life increases.

[48] As discussed before, the coefficients of the comparative dynamic equations (64) and (65) are identical for the partial differentials, $\partial q_{h/a}(t)/\partial w_E\big|_T$ and $\partial H(t)/\partial w_E|_T$, for fixed T, and for the total differentials, $\partial q_{h/a}(t)/\partial w_E$ and $\partial H(t)/\partial w_E$, for free T. We can thus use the same phase diagram for the fixed and for the free T case.

[49] It is possible that length of life is still extended $\partial T/\partial w_E > 0$, even if $\partial q_{h/a}(T)/\partial w_E\big|_T > 0$, as long as

$$\left.\frac{\partial q_A(0)}{\partial w_E}\right|_T e^{-rT} q_{h/a}(T) + q_A(0) e^{-rT} \left.\frac{\partial q_{h/a}(T)}{\partial w_E}\right|_T < 0 \tag{66}$$

(see expression 58). It is not clear from the phase diagram that this condition holds, hence we cannot establish whether life is extended.

While theoretically we cannot rule out the scenario where the opportunity cost effect outweighs the wealth effect, empirical evidence suggests that a permanent wage change affects health positively, while a transitory wage increase affects health negatively (e.g., Contoyannis, Jones and Rice 2004), and that high-income individuals are generally in better health than low-income individuals. Thus, in practice it appears the opportunity cost effect is not large.

Education E The comparative dynamic effect of an increase in education E (see 9 and 10), on the relative marginal value of health $q_{h/a}(t)$ is obtained by taking the derivate of (37) with respect to E and keeping first-order terms (total differentials, free T):

$$\begin{aligned} \frac{\partial}{\partial t}\frac{\partial q_{h/a}(t)}{\partial E} \approx\ & \rho_E w_*(t)\,[1+z(t)]^{\gamma_w}\,\frac{\partial s}{\partial H} \\ & + \left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial H}e^{-(\beta-r)t}\right]\times\frac{\partial q_A(0)}{\partial E} \\ & + \left[\frac{\partial d}{\partial H}+r\right]\times\frac{\partial q_{h/a}(t)}{\partial E} \\ & - \left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial H^2}e^{-(\beta-r)t}+\frac{\partial^2 Y}{\partial H^2}-q_{h/a}(t)\frac{\partial^2 d}{\partial H^2}\right]\times\frac{\partial H(t)}{\partial E}. \end{aligned} \tag{67}$$

Likewise, the comparative dynamic effect of an increase in education on health $H(t)$ is obtained by taking the derivative of (51) with respect to E and keeping first-order terms:

$$\begin{aligned} \frac{\partial}{\partial t}\frac{\partial H(t)}{\partial E} \approx\ & \frac{\alpha}{1-\alpha}I(t)^{\alpha}\left[\frac{1}{\mu_I}\frac{\partial \mu_I}{\partial E}-(1-\kappa_I)\rho_E\right] \\ & + \left[\frac{\alpha}{1-\alpha}\frac{I(t)^{\alpha}}{q_{h/a}(t)}\right]\times\frac{\partial q_{h/a}(t)}{\partial E} \\ & - \frac{\partial d}{\partial H}\times\frac{\partial H(t)}{\partial E}, \end{aligned} \tag{68}$$

where we have used (30). Contrasting the results of the comparative dynamics for education E (equations 67 and 68) with those obtained for permanent income w_E (equations 64 and 65) we observe that permanent wages w_E and education E operate in the same way. This should come as no surprise, as they both operate by increasing permanent wages. There is however one important difference: the first term on the RHS of (68) represents both the effect of education on the efficiency of health investment $\partial\mu_I/\partial E$ (the educated are assumed to be more efficient producers and consumers of health) and the effect of education on the opportunity cost of time $\rho_E(1-\kappa_I)$. The efficiency effect of education reduces the opportunity cost of time effect. The phase diagram for the effect of variation in education δE is essentially the same as for variation in permanent income w_E, shown in Figure 6, and replacing w_E by E (for this reason we do not provide a separate phase diagram). Given strong empirical support for a positive association between education and health, it could be that the efficiency effect dominates, in which case the $(\partial/\partial t)(\partial H(t)/\partial E)$ null cline would cross the vertical $\partial q_{h/a}(t)/\partial E$ axis below instead of above the origin. This would make the case for variation in education δE stronger (compared to the case for variation in permanent wages w_E) in ensuring that the condition $\partial q_{h/a}(t)/\partial E\big|_T \leq 0$ is obtained and hence length of life is extended.

C.8 Proof of Proposition 7: Absent ability to extend life, healthy individuals, ceteris paribus, value health cumulatively less, $\int_0^T (\partial q_{h/a}(t)/\partial H_0\big|_T)dt < 0$.

The comparative dynamic effect of variation in initial health δH_0 on the relative marginal value of health, keeping length of life T fixed, is obtained by taking the derivative of (37) with respect to initial health H_0 and keeping first-order terms. Likewise, the comparative dynamic effect of initial health on health, keeping length of life T fixed, is obtained by taking the derivate of (51) with respect to intial health H_0 and keeping first-order terms:

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\partial q_{h/a}(t)}{\partial H_0}\bigg|_T \;\approx\;& \left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial H}e^{-(\beta-r)t}\right]\times\frac{\partial q_A(0)}{\partial H_0}\bigg|_T\\ &+\left[\frac{\partial d}{\partial H}+r\right]\times\frac{\partial q_{h/a}(t)}{\partial H_0}\bigg|_T\\ &-\left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial H^2}e^{-(\beta-r)t}+\frac{\partial^2 Y}{\partial H^2}-q_{h/a}(t)\frac{\partial^2 d}{\partial H^2}\right]\times\frac{\partial H(t)}{\partial H_0}\bigg|_T, \qquad (69)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\partial H(t)}{\partial H_0}\bigg|_T \;\approx\;& \left[\frac{\alpha}{1-\alpha}\frac{I(t)^\alpha}{q_{h/a}(t)}\right]\times\frac{\partial q_{h/a}(t)}{\partial H_0}\bigg|_T\\ &-\frac{\partial d}{\partial H}\times\frac{\partial H(t)}{\partial H_0}\bigg|_T. \qquad (70)\end{aligned}$$

Thus, the coefficients in (69) and (70) are identical to the coefficients in the comparative dynamic relations (54) and (55) for variation with respect to wealth δA_0.

A-priori we don't know the sign of $\partial q_A(0)/\partial H_0|_T$. First, consider the scenario where health reduces the marginal value of wealth, $\partial q_A(0)/\partial H_0|_T < 0$ (assumption 5).[50] In this case the phase diagram for variation in initial health δH_0, shown in Figure 7, is similar to the phase diagram for variation in initial wealth δA_0, shown in Figure 4. Importantly, the $(\partial/\partial t)(\partial q_{h/a}(t)/\partial H_0)\big|_T$ null-cline crosses the vertical $\partial q_{h/a}(t)/\partial H_0\big|_T$ axis above the origin, as is also the case for variation in wealth δA_0. While any admissible path has to end on the vertical $\partial q_{h/a}(t)/\partial H_0\big|_T$ axis, as is also the case for variation in wealth δA_0, an important difference with wealth is that any admissible path has to start at $\partial H(t)/\partial H_0|_{T,t=0}$, which for $t = 0$ is identical to 1. We don't know a-priori where

[50] We discuss the other scenario in section C.9.

$\partial H(t)/\partial H_0|_T = 1$ is located with respect to the steady state (where the two null-clines cross) and we show one case where the initial point lies to the left (vertical dashed line to the left) and another case where it lies to the right (vertical dashed line to the right) of the steady state (Figure 7).

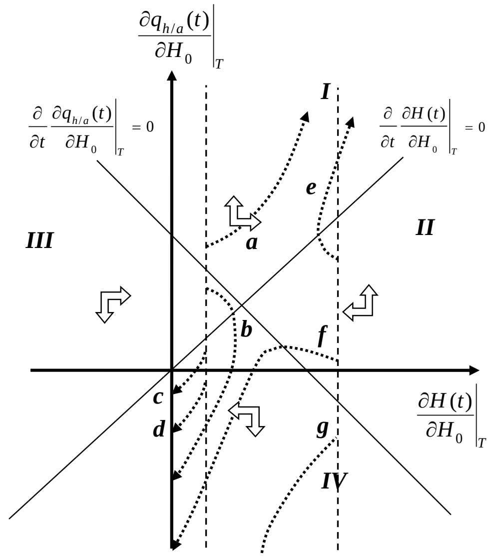

Figure 7: *Phase diagram of the deviation from the unperturbed path, resulting from variation in initial health δH_0, of the relative marginal value of health $\partial q_{h/a}(t)/\partial H_0\big|_T$ and of the health stock $\partial H(t)/\partial H_0|_T$, for fixed T.*

In case $\partial H(t)/\partial H_0|_T = 1$ is located to the left of the steady state, we can rule out trajectory a as it does not end on the vertical axis. For the same reason we can also eliminate trajectory e in case $\partial H(t)/\partial H_0|_T = 1$ is located to the right of the steady state.

Trajectories b, c and f represent an initial increase, followed by a subsequent decrease, in the marginal value of health $q_{h/a}(t)$, with respect to the unperturbed path. Since a higher value of health is associated with greater health investment and better health behavior, and also because individuals start with better initial health $H_0 + \delta H_0$, health is higher at all times (all trajectories stay to the right of the origin), except for $t = T$ (in the fixed T case eventual disinvestment is required to enforce that health reaches the minimum level $H(t) = H_{\min}$ at $t = T$). Thus early in life, greater initial health may increase the demand for health investment and healthy behavior, whereas later in life it decreases it.

Trajectories d and g represent solutions where the relative marginal value of health is lower at all times. Also for these solutions health is higher at all times (except $t = T$) and the individual uses the additional health δH_0 to shift resources from health to other uses.

For these solutions better health reduces the demand for health investment and healthy behavior at all times.

Those trajectories that are feasible (that are consistent with the begin and end conditions), i.e. b, c, d, f and g, all involve a cumulatively lower marginal value of health as initially higher health δH_0 requires lower health investment and less healthy behavior over the life cycle in order for health to reach the minimum health level $H_{\min}$ within the same length of life T. Q.E.D.

Similar to the results for variation in wealth, if health does not provide a consumption benefit (utility), the first term on the RHS of (69) is absent and the steady state is located at the origin (not shown). The admissible trajectory is then characterized by a lower relative marginal value of health at all times (similar to trajectory g). Thus absent ability to extend life, health is valued for its additional consumption benefit, but there is no additional production benefit associated with better health. Also, the greater the consumption benefit of health, the greater the relative marginal value of health (and therefore investment and healthy behavior) and the greater is health (except for $t = T$).

C.9 Proof of Proposition 8: Healthy individuals live longer $\partial T/\partial H_0 \geq 0$.

The comparative dynamic effect of variation in initial health δH_0 on length of life T can be obtained by following the same steps as in section C.4. The result is identical to replacing A_0 with H_0 in conditions (56), (57) and (58). As before, first consider the scenario where health reduces the marginal value of wealth, $\partial q_A(0)/\partial H_0|_T < 0$ (assumption 5). Assuming diminishing returns to life extension $\partial \Im(T)/\partial T|_{H_0} < 0$ (cf. assumption 5 in section 3.4), length of life is extended $\partial T/\partial H_0 > 0$, if:

$$\left.\frac{\partial \Im(T)}{\partial H_0}\right|_T = \left.\frac{\partial q_A(0)}{\partial H_0}\right|_T e^{-rT} \left.\frac{\partial A(t)}{\partial t}\right|_{t=T} + \left\{ \left.\frac{\partial q_A(0)}{\partial H_0}\right|_T e^{-rT} q_{h/a}(T) + q_A(0) e^{-rT} \left.\frac{\partial q_{h/a}(T)}{\partial H_0}\right|_T \right\} \left.\frac{\partial H(t)}{\partial t}\right|_{t=T} > 0. \quad (71)$$

As argued before in section C.4, in (71), both $\partial A(t)/\partial t|_{t=T}$ and $\partial H(t)/\partial t|_{t=T}$ are negative since health declines near the end of life as it approaches $H_{\min}$ from above, and assets decline near the end of life in absence of a very strong bequest motive. Further, in the scenario under consideration, we have $\partial q_A(0)/\partial H_0|_T < 0$. Note that all admissible scenarios b, c, d, f and g, end with negative values for variation in the relative marginal value of health with respect to initial health $\partial q_{h/a}(T)/\partial H_0\big|_T < 0$. Thus, length of life is extended $\partial T/\partial H_0 > 0$ in the scenario where health reduces the marginal value of wealth, $\partial q_A(0)/\partial H_0|_T < 0$ (assumption 5). Q.E.D.

Now briefly consider the scenario where health raises the marginal value of wealth, $\partial q_A(0)/\partial H_0|_T > 0$ (i.e. assumption 5 does not hold). In this scenario the $(\partial/\partial t)(\partial q_{h/a}(t)/\partial H_0)\big|_T$ null-cline shifts downward, crossing the $(\partial/\partial t)(\partial H(t)/\partial H_0)|_T$ null cline to the left of and below the origin. Left as an exercise to the reader, in this case all admissible trajectories start with a lower relative marginal value of health

$\partial q_{h/a}(0)/\partial H_0\big|_T < 0$ and either end with a higher relative marginal value of health $\partial q_{h/a}(T)/\partial H_0\big|_T > 0$, in which case life is not extended but reduced $\partial T/\partial H_0 < 0$ as a result of greater health (see 71), or end with a lower relative marginal value of health $\partial q_{h/a}(T)/\partial H_0\big|_T < 0$, in which case we cannot unambiguously establish that life is extended. We favor the scenario where health reduces the marginal value of wealth (assumption 5), since under this assumption the model unambiguously predicts that healthier individuals live longer, it seems theoretically plausible (see discussion for assumption 5), it is consistent with empirical evidence that worse childhood health is associated with shorter lives (Currie, 2009), and it has a natural intuitive interpretation that health and wealth are to some extent substitutable in financing consumption (see discussion for assumption 5).

C.10 Proof of Proposition 9: For small life extension healthy individuals cumulatively value health less $\int_0^T [\partial q_{h/a}(t)/\partial H_0]dt < 0$, for intermediate life extension they value health cumulatively more $\int_0^T [\partial q_{h/a}(t)/\partial H_0]dt > 0$, and for large life extension they value health more at all ages, $\partial q_{h/a}(t)/\partial H_0 > 0, \quad \forall t$.

Having established that length of life is extended, $\partial T/\partial H_0 > 0$, now consider the more interesting case where T is free. Analogous to the discussion in section C.5 we have

$$\left.\frac{\partial H(t)}{\partial T}\right|_{H_0,t=T} = -\left.\frac{\partial H(t)}{\partial t}\right|_{H_0,t=T} > 0.$$

Thus, $\partial H(T)/\partial H_0 = (\partial H(t)/\partial T|_{H_0,t=T})(\partial T/\partial H_0) > 0$.

Figure 8 presents the comparative dynamic results for free T. The phase diagram on the left shows feasible trajectories a through g for the case where the starting point $\partial H(t)/\partial H_0 = 1$ is located to the left of the steady state, and the phase diagram on the right shows feasible trajectories a through f for the case where the starting point $\partial H(t)/\partial H_0 = 1$ is located to the right of the steady state (the starting values are indicated by the dashed vertical lines in both phase diagrams). The initial condition $\partial H(t)/\partial H_0 = 1$ for $t = 0$, and the end-condition $\partial H(T)/\partial H_0 > 0$, imply that all admissible paths start and end to the right of the vertical axis. Three example end values $\partial H(T)/\partial H_0$ are indicated by the three dotted vertical lines in both figures.

Further, as the result $\partial H(T)/\partial H_0 = (\partial H(t)/\partial T|_{H_0,t=T})(\partial T/\partial H_0) > 0$ shows, also for health the greater life is extended, the further is the end point $\partial H(T)/\partial H_0$ located to the right in the phase diagram. While both phase diagrams are quite complicated, they clearly show that for end points $\partial H(T)/\partial H_0$ (the vertical dotted lines) that lie further to the right (i.e. those associated with a greater degree of life extension), the variation in the value of health $\partial q_{h/a}(t)/\partial H_0$ becomes more and more positive, with some scenarios even allowing for the possibility that healthy individuals value health more at every age. Whereas for end points $\partial H(T)/\partial H_0$ that lie more to the left (i.e. those associated with a smaller degree of life extension), the variation in the value of health $\partial q_{h/a}(t)/\partial H_0$ becomes more and more negative. These latter cases more closely resemble the fixed T case (proposition 7). Q.E.D.

C.11 Proof of Proposition 10: Individuals with greater endowed health are healthier at all ages, $\partial H(t)/\partial H_0 > 0, \quad \forall t$.

The discussion for the proof of Proposition 9 in the previous section C.10 also provides the proof for Proposition 10. In particular, note that all feasible trajectories lie to the right of the vertical axis. Q.E.D.

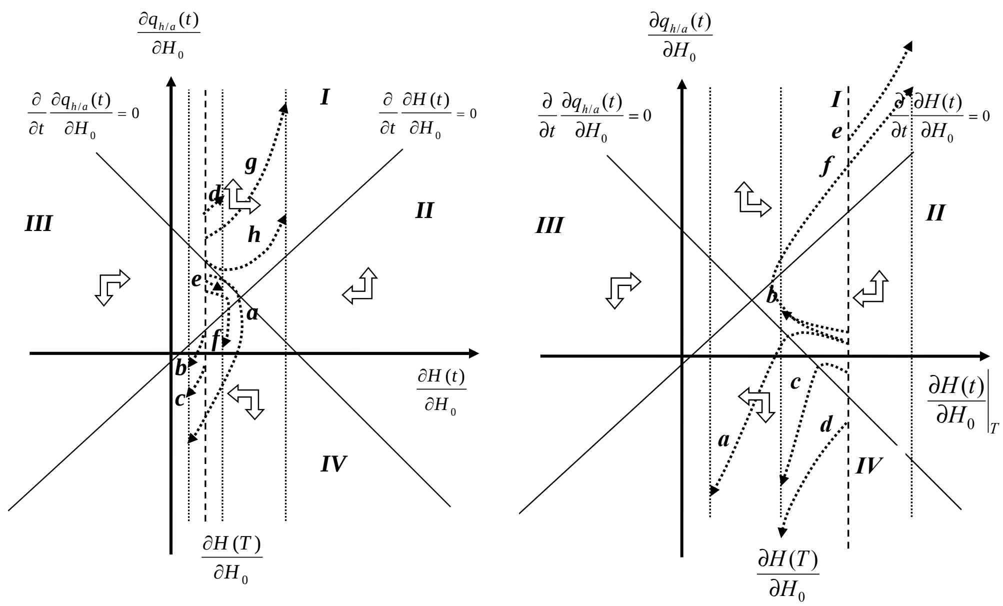

Figure 8: *Phase diagram of the deviation from the unperturbed path, resulting from variation in initial health δH_0, of the relative marginal value of health $\partial q_{h/a}(t)/\partial H_0$ and of the health stock $\partial H(t)/\partial H_0$, for free T. The left shows feasible trajectories a through g for the case where the starting point $\partial H(t)/\partial H_0 = 1$ is located to the left of the steady state, and the phase diagram on the right shows feasible trajectories a through f for the case where the starting point $\partial H(t)/\partial H_0 = 1$ is located to the right of the steady state (the starting values are indicated by the dashed vertical lines in both phase diagrams).*

C.12 Comparative dynamics of the controls

C.12.1 Variation in initial wealth, δA_0

In signing the following comparative dynamic results we rely on assumptions 1 to 6 and propositions 1 to 10.

Health investment For the control variable health investment the comparative dynamic effect of initial wealth A_0 is obtained from (15) and (29)

$$\frac{1-\alpha}{I(t)} \times \frac{\partial I(t)}{\partial A_0} \quad \approx \quad \frac{1}{q_{h/a}(t)} \times \frac{\partial q_{h/a}(t)}{\partial A_0}, \tag{72}$$

where we assume that first-order effects dominate and we focus on the total differential (full model with free T). Since $\partial I(t)/\partial A_0$ is proportional to $\partial q_{h/a}(t)/\partial A_0$, it will mimic the pattern of the variation in the response to wealth of the relative marginal value of health (see propositions 1 and 4).

Healthy and unhealthy consumption For the control variable healthy consumption the comparative dynamic effect of initial wealth A_0 is obtained from (19):

$$\begin{aligned}
& \left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial C_h^2} - q_{h/a}(t)\frac{\partial^2 d}{\partial C_h^2}e^{(\beta-r)t}\right] \times \frac{\partial C_h(t)}{\partial A_0} \\
\approx \quad & \left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial C_h}\right] \times \frac{\partial q_A(0)}{\partial A_0} \\
+ \quad & \left[\frac{\partial d}{\partial C_h}e^{(\beta-r)t}\right] \times \frac{\partial q_{h/a}(t)}{\partial A_0} \\
+ \quad & \left[q_{h/a}(t)\frac{\partial^2 d}{\partial C_h \partial H}e^{(\beta-r)t}\right] \times \frac{\partial H(t)}{\partial A_0}.
\end{aligned} \tag{73}$$

The direct wealth effect (first term on the RHS of 73) as well as the effect of an increase in the relative marginal value of health (second term on the RHS) is positive.[51] The sign of the third term is undetermined, since the sign of $\partial^2 d/\partial H \partial C_h$ is not known. One could imagine that healthier individuals benefit less from healthy consumption, but one could also imagine the opposite scenario. Since the effect of wealth on health $\partial H(t)/\partial A_0$ is gradual and not immediate, the third term is initially small compared to the first two terms. As a result, an increase in endowed wealth increases the demand for healthy consumption, $\partial C_h(t)/\partial A_0 > 0$, initially.

[51] Scenarios are possible in which the effect of wealth on the relative marginal value of health eventually becomes negative after some age. The initial response however is always positive.

Likewise, for unhealthy consumption the comparative dynamic effect of initial wealth is obtained from (22):

$$\begin{aligned}
&\left[\frac{1}{q_A(0)}\frac{\partial^2 U}{\partial C_u^2} - q_{h/a}(t)\frac{\partial^2 d}{\partial C_u^2}e^{(\beta-r)t}\right] \times \frac{\partial C_u(t)}{\partial A_0} \\
\approx\ & \left[\frac{1}{q_A(0)^2}\frac{\partial U}{\partial C_u}\right] \times \frac{\partial q_A(0)}{\partial A_0} \\
+\ & \left[\frac{\partial d}{\partial C_u}e^{(\beta-r)t}\right] \times \frac{\partial q_{h/a}(t)}{\partial A_0} \\
+\ & \left[q_{h/a}(t)\frac{\partial^2 d}{\partial C_u \partial H}e^{(\beta-r)t}\right] \times \frac{\partial H(t)}{\partial A_0}.
\end{aligned} \tag{74}$$

For unhealthy consumption, the effect is ambiguous. While the direct wealth effect is positive (first term on the RHS of 74), the effect of an increase in the marginal health cost (last two terms on the RHS) is negative.

Job-related health stress The comparative dynamic effect of initial wealth A_0 on job-related health stress is obtained from (25):

$$\begin{aligned}
&\left[q_{h/a}(t)\frac{\partial^2 d}{\partial z^2} - \frac{\partial^2 Y}{\partial z^2}\right] \times \frac{\partial z(t)}{\partial A_0} \\
=\ & -\frac{\partial d}{\partial z} \times \frac{\partial q_{h/a}(t)}{\partial A_0} \\
-\ & \left[q_{h/a}(t)\frac{\partial^2 d}{\partial z \partial H} + \frac{\partial w}{\partial z}\frac{\partial s}{\partial H}\right] \times \frac{\partial H(t)}{\partial A_0}.
\end{aligned} \tag{75}$$

where the coefficient of $\partial z(t)/\partial A_0$ is positive under assumptions 1 and 2, but the coefficient of $\partial H(t)/\partial A_0$ cannot be signed. Initially, the term in $\partial H(t)/\partial A_0$ is small as the effect of wealth on health is gradual.

Leisure The comparative dynamic effect of variation in initial wealth A_0 on leisure is obtained from (18):

$$\frac{\partial^2 U}{\partial L^2}\frac{\partial L(t)}{\partial A_0} \approx \frac{1}{q_A(0)}\frac{\partial U}{\partial L} \times \frac{\partial q_A(0)}{\partial A_0} - \frac{\partial^2 U}{\partial L \partial H}\frac{\partial H(t)}{\partial A_0}. \tag{76}$$

For diminishing utility of leisure $\partial^2 U/\partial L^2 < 0$, diminishing returns to wealth $\partial q_A(0)/\partial A(0) < 0$ (assumption 5), the demand for leisure is initially higher as a result of greater wealth. But wealth eventually leads to better health, and if leisure and health are complements in utility $\partial^2 U/\partial H \partial L > 0$, we have $\partial L(t)/\partial A_0 > 0$, $\forall t$. If, however, health and leisure are substitutes in utility, then the demand for leisure is initially higher but could be reduced eventually with improved health.

C.12.2 Variation in the permanent wage rate, δw_E

Health investment For the control variable health investment the comparative dynamic effect of the wage rate w_E is obtained from (72) replacing A_0 by w_E, and adding the term $-(1-\kappa_I)/w_E$ on the RHS.

Healthy and unhealthy consumption For the control variable healthy consumption the comparative dynamic effect of the wage rate w_E is obtained from (73) replacing A_0 by w_E and adding the term $\{(1-\kappa_{C_h})/w_E\}\,\pi_{C_h}(t)e^{(\beta-r)t}$ on the RHS.

Likewise, for unhealthy consumption the comparative dynamic effect of the wage rate w_E is obtained from (74) replacing A_0 by w_E and adding the term $\{(1-\kappa_{C_u})/w_E\}\,\pi_{C_u}(t)e^{(\beta-r)t}$ on the RHS.

Job-related health stress The comparative dynamic effect of w_E on job-related health stress is obtained from (75) replacing A_0 by w_E and adding the term $\{(\partial w/\partial z)\,/w_E\}\,\tau_w(t)$ on the RHS.

Leisure The comparative dynamic effect of the wage rate on leisure is obtained from (76) by replacing A_0 by w_E and adding the term $q_A(0)(w(t)/w_E)e^{(\beta-r)t}$ on the RHS.

C.12.3 Variation in education, δE

Health investment For the control variable health investment the comparative dynamic effect of education E is obtained from (72) replacing A_0 by E, and adding the term $\{-\rho_E(1-\kappa_I)+(1/\mu_I(E))\,(\partial\mu_I/\partial E)\}$ on the RHS.

Healthy and unhealthy consumption For the control variable healthy consumption the comparative dynamic effect of education E is obtained from (73) replacing A_0 by E and adding the term

$$\left(\rho_E(1-\kappa_{C_h})-\frac{1}{\mu_{C_h}(E)}\frac{\partial\mu_{C_h}}{\partial E}\right)\pi_{C_h}(t)e^{(\beta-r)t}$$

on the RHS. Likewise, for unhealthy consumption the comparative dynamic effect of education is obtained from (74) replacing A_0 by E and adding the term

$$\left(\rho_E(1-\kappa_{C_u})-\frac{1}{\mu_{C_u}(E)}\frac{\partial\mu_{C_u}}{\partial E}\right)\pi_{C_u}(t)e^{(\beta-r)t}$$

on the RHS.

Job-related health stress The comparative dynamic effect of education E on job-related health stress is obtained from (75) replacing A_0 by E, and adding the term $\rho_E\,(\partial Y/\partial z)$ on the RHS.

Leisure The comparative dynamic effect of education on leisure is obtained from (76) by replacing A_0 by E and adding the term $\rho_E q_A(0)w(t)e^{(\beta-r)t}$ on the RHS.

C.12.4 Variation in initial health, δH_0

The comparative dynamics of the control variables with respect to H_0 can be directly obtained by using the comparative dynamics for initial wealth in (72) to (76), replacing A_0 by H_0.