Giving for Profit or Giving to Give: 
The Profitability of Corporate Social Responsibility*

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Abstract

Do firms profit from socially responsible behaviors? Measuring the impact of social responsibility on profits is challenging because engaging in social responsible programs may be endogenous to other decisions regarding a firm’s profitability. I overcome these issues by analyzing the demand and supply of social responsibility programs for the case of a fast-growing for-profit company offering charity auctions of celebrities’ belongings. I show both reduced form evidence as well as results from a structural model of auctions with externalities indicating that giving increases bidders’ willingness to pay. However, this increase does not compensate for the amount donated, making non-charity auctions preferable. To understand why the firm donates, I provide both theoretical and empirical evidence showing that the cost of procuring the items decrease in the percentage donated as celebrities favor larger donations as a form of cheap ads. As a result, donating is optimal for the firm. Yet, the firm donates 55% more than what is optimal, leading to less profits by a factor of four. The paper concludes that the firm is maximizing a combination of profit and social impact. This result provides empirical evidence that the objective of firms could extend beyond mere profit maximization.

JEL classifications: L21, L81, D64, C14

Keywords: objectives of the firm, behavioral supply, CSR, charitable giving, auctions with externalities, structural estimation, nonparametric identification

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1 Introduction

Giving is pervasive among most corporate social responsibility (CSR) programs. Donations by the 20 most generous companies of the Fortune 500 list total $3.5 bn in cash in 2015 alone.\(^1\) The benefits of charitable donations include the attraction of otherwise indifferent customers and an increase in their willingness to pay. For example, Apple, who recently became the first company to hit $1 tn of capitalization, pledges a percentage of revenues from its Red branded products to Product Red, a charity fighting HIV and AIDS in Africa. The firm has donated $160 m since 2006. However, such programs come with additional costs, both in terms of lower net revenues, as well as managerial burdens. Thus, while social impact is a commendable pursuit, it is unclear whether it leads to greater profits.

To understand the economic rationale of giving, this paper studies the strategic decisions of Charitystars, an international for-profit firm offering online ascending auctions of celebrities’ belongings for charity.\(^2\) For each item sold on Charitystars a known portion of the final price is donated. This paper relies on variation in the percentage donated to analyze both the demand and supply of charity linked products in this marketplace. On the demand side, the paper builds and estimates a charity auction model showing that the price increase due to charitable giving does not fully compensate for the revenues lost due to the donation. Charity auctions are profitable only after accounting for the cost-savings that giving creates. On the supply side, celebrities view donations as cheap ads and are more willing to provide Charitystars with their goods when Charitystars’ donations are large. Therefore, by looking at the demand and supply of giving, this paper provides causal evidence that donations can be profitable.

Charitystars’ profitability mainly comes from lower costs rather than greater revenues. This suggests that consumers’ willingness to pay is somewhat inelastic to a firm’s donations. Consequently, firms’ prosocial decisions may not be adequately incentivized, as shown by the modest donations observed in most markets. For example donations on Product Red’s website are usually between 5% and 20% of gross revenues. Yet, by comparing marginal net revenues and marginal costs for Charitystars, this paper shows that Charitystars’ median donation exceeds the optimal one by 55%, implying lower profits by a factor of four. This could indicate that Charitystars’ objectives include not only profitability, but may also extend to its social impact. In fact, for some firms giving is a stated goal, as represented by the wave of firms operating according to a one-for-one rule. Toms Shoes is a classic example: for each pair of Toms purchased, the firm donates a pair to children in need.\(^3\) In the last decade, several US states have embraced social

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\(^1\)These firms cover various industries from retail to banking (full story at [https://tinyurl.com/jonuuen](https://tinyurl.com/jonuuen)).

\(^2\)With offices in London, Milan and Los Angeles, Charitystars is a for-profit start-up with almost $4m in equity, gained after several funding rounds from investors like 360 Capital Partners, a major European venture capital player. Since its foundation in 2013, the company generated over $10 million for charities and nonprofits such as Special Olympics, Save the Children, Make-A-Wish and WWF. More information are available on [https://www.charitystars.com](https://www.charitystars.com) and on [https://www.crunchbase.com/organization/charitystars](https://www.crunchbase.com/organization/charitystars).

\(^3\)Education is another area where social impact firms make the difference, as teachers spend up to $1.6 bn in
impact and began to promulgate corporate legislation called “Benefit Corporation” (or B Corp) to support the growth of social firms (Finfrock and Talley, 2014, Battilana and Lee, 2014). The case of Charitystars provides novel evidence that even for-profit firms can forego a portion of their profits to contribute to society.

Evidence that CSR engagement is conducive to higher economic or financial return is sparse. Although Eichholtz et al. (2010) finds a large price premium (16%) for energy friendly buildings, a meta-analysis of over 160 papers found only a small positive correlation between CSR and profits, and suggested that causation may well flow from profits to CSR (Margolis et al., 2007). There is obvious endogeneity, as well as several other issues that plague causal inference in this domain (Bénabou and Tirole, 2010). One of these issues is the large dimensionality of CSR activities, which complicates comparison across firms (Chatterji et al., 2009). Moreover, these dimensions may interact in surprising ways, sometimes reducing the perceived social impact of the firm, which could actually lower consumers willingness to pay. This happens, for example, by disseminating misleading information of environmental friendliness (Lyon and Maxwell, 2011), and by giving to charities to influence connected congressmen (Bertrand et al., 2018).

This paper contributes to the CSR literature in two ways. First, the introduction of a novel dataset using publicly available information from Charitystars provides several advantages to assess the returns to giving. An issue with oligopolistic competition is that the benefits from donating are not limited to a single charity-linked good for multi-product firms (e.g., Apple), but they are spread across the entire offering of the firm. In order to judge the optimality of giving a researcher would require an exogenous shock for each good. This problem does not emerge on Charitystars, as each charity auction can be thought of as a single market. Another critical feature of these data is that the reserve price is set so that the firm breaks even. Thus, the cost of procuring the items is fully observed. Also, this paper focuses on a subset of the range of items on auctions (worn soccer jerseys) and the firm acts as a monopolist in this market. Finally, on Charitystars the percentage donated is clearly shown on each listing and the credibility of donations is supported by certificates issued by Charitystars and the recipient charity. Therefore, these data are particularly suited to address the profitability of giving.

school supply each year. Yoobi, a start-up, tries to fill this gap by selling school supplies online, and by using part of their proceeds to give boxes of supplies to schools free of charge. Since 2014, Yoobi helped over 83,000 classrooms. Source: Yoobi’s social impact report https://tinyurl.com/ychrkdpp.

4B Corp firms have a legally binding fiduciary responsibility to take into account not only the interest of their shareholders but also those of the workers, the community and the environment. Beyond the classic financial reporting duties, as any other limited liability companies do, B Corp must provide independent reports on their social and environmental impact. B Corp’s advantage rests in the greater protection of the social mission before shareholders. Italy and Britain also introduced the B Corp Status. For additional information on Benefit Corporation legislation see https://www.economist.com/business/2012/01/07/firms-with-benefits. Examples of firms with B Corp status include Patagonia, Kickstarter and many subsidiaries of Danone. More information at https://www.economist.com/business/2018/08/09/danone-rethinks-the-idea-of-the-firm. Charitystars does not have B Corp status.

5This paper is also related to a line of work on the existence of a premium for products linked to charities. Several papers are dedicated to the analysis of eBay’s Giving Works data. (e.g., Elfenbein et al., 2012, 2017). The question I
Second, the paper solves the endogeneity between CSR decisions and profitability by studying both the demand and the supply of giving. After establishing that larger donations are not correlated with more bidders in a reduced form manner, the paper builds a charity auction model to investigate the intensive margin of giving. The theoretical model extends the literature on charity auctions to explicitly account for the structure of the donation in the data (Goeree et al., 2005, Engers and McManus, 2007). In charity auctions bidders are compelled by two forces. First, they have a taste for winning the auction and donating (Becker, 1974, Andreoni, 1989). As a result, winning the auction carries additional utility, incentivizing bidders to raise their bids. Second, losing the auction does not leave bidders with zero payoff. Instead, they are gratified by participating in the social contest and helping to drive up the price. The last effect is an externality from the winner to the other bidders, who extract surplus from the former. The model shows that marginal changes in the percentage donated can either increase or decrease the transaction price depending on the relative weights of these two forces.6

Estimation of the model requires the nonparametric identification of the distribution of values and two charitable parameters, indicating each of the two forces discussed above. To gain intuition, imagine two auctions for the same item varying on the percentage of the final price donated, and think of the bids placed in each auction by the same bidder. The difference between the two bids reflects the benefit of winning the auction. Observing multiple bidders, the externality parameter can be inferred by comparing the respective likelihood of winning. Identification relies on the restriction that the distribution of values for the item does not depend on how much is donated. This orthogonality condition means that the consumption value of the item does not change as the auctioneer changes the fraction donated. Orthogonality is formally tested and not rejected by the data. This identification strategy is reminiscent of those for risk-aversion in first-price auctions (Guerre et al., 2009, Lu and Perrigne, 2008).

The model can be taken to the data and shows a good fit, with estimated revenues in a neighborhood of 10% of the realized ones. Prices on Charitystars command a premium as bidders’ willingness to pay increases based on how much the firm donates. However, donations have a large direct cost in terms of foregone revenues. A counterfactual scenario where the firm does not donate evaluates the loss in net revenues to be as much as €260 per listing, or about 70% of the average price observed in the data. Thus, the demand analysis unambiguously calls for the firm to switch to standard non-charity auctions.

Given the large revenue lost due to the donations, how can Charitystars maintain a for-profit status? Why is the company still offering only charity auctions? The simple answer is that revenue diverges from these analyses as it concerns profits rather than revenues. To this extent, the data is collected from Charitystars instead of eBay, as Charitystars’ listings share only one seller (Charitystars itself).

6In particular, bidders will be less aggressive when they enjoy large returns from somebody else’s donation. Thus, charity auctions do not always result in positive premiums, explaining the conflicting results found in the empirical literature. For example, there is both evidence for losses or negligible gains (e.g., Carpenter et al., 2008, Schram and Onderstal, 2009, Isaac et al., 2010), as well as for considerable profits from charity auctions (e.g., Elfenbein and McManus, 2010, Leszczyc and Rothkopf, 2010, Elfenbein et al., 2012).
enues are not profits. Charitystars operates like a platform, procuring items from celebrities and selling them to consumers; the percentage donated is a result of bargaining between each celebrity and Charitystars. Celebrities may be eager to exchange their belongings for cheap and genuine advertising. Also, due to the positive association between prices and donations, this signaling should be stronger when Charitystars is more generous (Glazer and Konrad, 1996, Harbaugh, 1998). A simple bargaining model between the two parties indicates that procurement costs to the firm decrease in the percentage donated and in the bargaining power of the celebrity. Assuming that footballers have greater bargaining powers when jerseys are scarcer, I empirically confirm this result by showing that in the inactive Summer months, when there are no league matches, Charitystars donates more on average. Thus celebrities favor higher donations to charities, resulting in lower costs to Charitystars when it donates more.

Marginal costs are estimated using a price shifter (i.e., the variation in the portion of the reserve price kept by the firm across auctions). The estimated costs are decreasing in the percentage donated, confirming that celebrities prefer more generous auctions. Intersecting the estimated marginal net revenues from the demand side with the marginal cost curve yields an interior solution for the optimal donation. Thus, to understand the full extent of giving on Charitystars’ decisions, both sides of the market must be investigated because donations impact both revenues and costs. CSR programs in other industries function similarly. For instance, socially responsible mutual funds invest in less profitable firms with social impact (Barber et al., 2018, Hong and Kacperczyk, 2009), but also face lower costs of financing as socially responsible investors forego higher returns to invest in these funds (Riedl and Smeets, 2017).\textsuperscript{7}

The analysis concludes that the optimal donation percentage is 30%, yet the median donation in the data is 85% (mean 70%). Charitystars would increase its unitary profit from €25 to €100 on average if it were to adopt the optimal policy.\textsuperscript{8} A possible explanation to this suboptimal outcome is that Charitystars overestimates the elasticity of prices to the amount donated. Such a bias could encourage the firm to donate a larger share of revenues. Although such a behavioral bias cannot be ruled out, given the number of external investors and the large number of auctions hosted by the firm (over a thousand per year), it seems unlikely that the firm holds biased estimates of how prices react to the sharing rule.\textsuperscript{9} Alternatively, considering a dynamic environment the firm could donate more in the current period to increase participation in future auctions. However, this margin does not convincingly explain the data as the number of bidders does not vary with the percentage donated. Another possibility is that high donation standards

\textsuperscript{7}Other examples include green technologies, whose diffusion does not only depend on the willingness to pay of consumers for environmentally friendly products, but also on their cost of production (Kok et al., 2011).

\textsuperscript{8}In this paper the term profits stands for a positive cash flow balance. While many successful firms had negative financial profits in their early years (for example eBay and Amazon), their success critically depended on their ability to boost their cash flow in order to absolve to their daily operations without recurring to external funding. This further shows the importance of the choice of the percentage donated for Charitystars’ shareholders.

\textsuperscript{9}Bloom et al. (2015) show that private equity owned firms (like Charitystars) are well managed and have better management practices than comparable firms – especially with respect to data collection and data analysis.
may discourage other firms from entering the market. Still, Charitystars is a de facto monopolist in its main market (Europe) for the large number and the particularity of the objects it sells. This is because entering such a market requires a huge investment in terms of relations with celebrities and their agents. In fact, Charitystars has dealt with more than 1,000 celebrities and 450 charities in its 5 year history, and is currently gaining traction in US and Asia. Moreover, this paper investigates the specific market of authentic worn soccer jerseys: entering this market would require matching the commitments that Charitystars developed with both European soccer teams and leagues. Finally, evidence of consumer inertia in search markets further increases the barriers to entry (e.g., Hortaçsu et al., 2017b).

Considering all these possible explanations, the fact that Charitystars median donation is 55% more than the profit maximizing level could indicate that the firm cares also for the size of its contributions. Charitystars’ money-making ability and ethical concerns are clearly not separable. Quoting Hart et al. (2017), if consumers and investors value social donations, why “would they not want the companies they invest in to do the same?” Socially responsible shareholders may see this company as an opportunity to sustain their donations over time, while even receiving dividends. Under this logic, Charitystars not only cares for its long term sustainability, but also for its total giving.

Most of the research in economics assumes that firms are profit maximizing agents. An empirical assessment of this assumption is challenging because costs are often not observed but only inferred by inverting the equilibrium mapping resulting from this very assumption. A handful of papers give evidence that firms fail to maximize profits. For example, Ellison et al. (2016) and DellaVigna and Gentzkow (2017) show suboptimal pricing strategies for computer component firms and US retail stores, and attribute them to managerial costs. Others discuss managerial quality as a driver to higher profits (Bloom and Van Reenen, 2007, Goldfarb and Xiao, 2011, Hortaçsu et al., 2017a). However, these conclusions highlight the presence of principal-agent problems rather than identifying the real objective function of firms.

An empirical discussion on profit maximization is presented in papers based on NFL data. Evidence shows that certain plays called by coaches do not maximize the probability of victory (Romer, 2006), and that teams often waste their top picks at the annual draft (Massey and Thaler, 2013). These papers hint that influence from shareholders and fans could sway decisions away from optimality. Yet, empirical research providing similar evidence with business data is lacking. Socially responsible firms provide a fruitful context to analyze profit maximization because they engage in costly activities to persuade concerned consumers. Studying these decisions sheds light on whether the objective function of firms is limited to profits or extends to the greater good.

11 In the health economics literature Kolstad (2013) shows that surgeons respond more to intrinsic incentives (quality report cards) than to monetary incentives. Despite analyzing the decision of people rather than firms, the author suggests that objective functions can extend beyond mere profits.


2 Auctions on Charitystars

Charitystars is a for-profit internet platform helping charities do fundraising. On Charitystars.com famous celebrities place their objects and memorabilia on auction. The higher bidder wins the object, pays his bid, which is then partially donated to a charity. Charitystars guarantees the donations by issuing certificates. The identity of the charity receiving the donation, as well as the fraction of the transaction price being donated are common knowledge before each auction.

In order to bid for an item, potential users sign up on the online platform. Once the account is online, users can bid in any live auction. All auctions employ an open, ascending-bid format analogous to eBay. Auctions involve a single item. Bidders can submit a cutoff price (this tool is called proxy bidding on eBay) instead of a bid. Once a cutoff is set, Charitystars.com will issue a bid equal to the smallest of the highest current price (or the reserve price if there is no bid yet) and the highest submitted cutoff price, plus the minimum increment. Thus, the winner pays the second highest valuation, plus a minimum increment.\footnote{Moreover, Charitystars’ auctions have another feature that make them more appealing than eBay auctions for empirical research. The auction countdown is automatically extended by 4 minutes anytime a bid is placed in the last 4 minutes of the auction. This impedes sniping (i.e. the practice of bidding in the very last seconds of an auction), which is common in eBay and is associated with lower transaction prices and loss of users (Backus et al., 2017).}

A particularity of Charitystars’ auctions is the secrecy of the reserve price: at any point in time bidders only know whether the reserve price is met or not. Although the reserve price is never disclosed, not even after the end of an auction, it can be found in the source code (HTML) behind each listing. Secret reserve prices are not uncommon in online auctions. The empirical auction literature have treated these occurrences by simply adding an additional bidder who keeps the object in case no one bids above the reserve price (e.g., Bajari and Hortacsu, 2003). In addition, on Charitystars after each unsold auction there is an additional step where the highest bidder is given the option to purchase the object by paying the reserve price (7% of the cases). To avoid possible concerns the following analysis is restricted to listings counting at least two bidders which concluded in a direct sale.

The company auctions a very broad spectrum of items, from VIP tickets to the Monaco Gran Prix to famous photographs and arts collectibles. However, soccer is one of the most popular item categories with over 4,000 auctions held in 3 years. Given that the firm’s only operation consists in providing charity auctions and that the firm is a de facto monopoly in the market for worn soccer jerseys, Charitystars represents a good environment in which to investigate the effect of a firm’s generosity on consumers’ decisions. The first sections of the paper are dedicated to the demand side. Section 8 shifts the focus to how the firm procures the items it sells, and it shows that the donations also affect costs, and thus profits. This approach provides first evidence on the relation between social responsibility and the objective of firms.
3 Description of the demand side

This paper uses publicly available data from Charitystars.com that were collected directly from
the website. The dataset contains auctions of authentic soccer jerseys sold between July 1st,
2015 and June 12th, 2017. These dates were chosen as they mark the beginning and the end
(after the Champions League final) of two consecutive football seasons. The dataset includes
1,583 auctions. There is large variation in the percentage that is finally donated though for most
auctions the fundraising corresponds to either 10%, 78% or 85% of the final price. Throughout
the paper $q$ denotes the percentage donated. Figure 1 shows this variation for two subsets of the
data.

Charitystars claims on its website to keep only 15% of each transaction price as revenues,
with the rest shared between the provider of the good (often a football agent, a footballer or a
team) and a charity. However, conversations with some shareholders of the company uncovered
the actual pricing strategy set by the firm. The firm keeps the whole portion that is not donated
to a charity as net revenues. For example, if a percentage $q$ is donated, the firm keeps $1 - q$ of the
price paid. Charitystars’ standard minimum fee is 15%. For this reason, I disregard all auctions
where more than 85% is donated (all auctions to the right of the black solid lines in 1).

All items are posted online on the platform’s website and advertised on the social media
of the firm in a similar fashion. The listing webpage shows pictures of the item on the left of
the screen, while bidders find a short description on the bottom of the website, altogether with
the information on the recipient charity. A picture of a typical webpage at the time of the data
collection is reported in Appendix A (Figure 11). The website layout did not change during the
time period under analysis.

For each auction, all bids placed, the date and time of the bid, the bidder nationality and the
charity receiving the money are observed. Unfortunately, auctions start dates are not available
online, but the average length of Charitystars’ auctions is usually between 1 and 2 weeks. Length
is therefore proxied using the distance in days between the first bid posted and the closing day..
Table 1 gives an overview of the main characteristics of the auctions for listings with transaction
prices larger than €100 and smaller than €1,000, with at least 2 different bidders and whose
minimum increment is within €25. The final price is greater than the reserve price in more than
95% of the listings. On average (median) the winning bid is 2.9 (2) times larger than the reserve
price. This database will be used throughout and consists of 1,108 auctions.

To better address unobservables, data are subset to include only jerseys that were sold at
prices below €400. This upper limit (€400) was chosen because in the Summer of 2017 Char-

\footnote{All other soccer related auctions not involving jerseys (such as shin guards, footballs and shorts) are excluded from the database.}

\footnote{A larger list of auction variables which will be used in most regression tables is available in Table 10 in Appendix B. Figure 12a in Appendix B plots the pdf of the transaction price for the three most frequent auction formats ($q \in \{10\%, 78\%, 85\%\}$).}
Figure 1: Number of auctions by percentage donated

(a) Transaction price ∈ (100, 1000)

(b) Transaction price ∈ (100, 400)

Note: Number of auctions available in the dataset by percentage donated. The plots include only auctions that ended in a transaction and for which the reserve price was greater than 0, the number of bidders was at least 2 and the minimum increment is not greater than €25. Panel (a) shows the number of auctions available for each percentage donated when the dataset is restricted to auctions whose price is in €(100, 1,000). There are 1,188 auctions in total. Panel (b) restricts the dataset to more homogeneous auctions (762 auctions). Charitystars generally withholds a 15% share of the final price and therefore all auctions whose percentage donated is above 85% are excluded from the analysis as these are special one-off charitable events (these are all the auctions to the right of the solid vertical line).

Charitystars decided to set a €50 minimum raise anytime the standing price reaches €400 (i.e. the minimum increment changes during the auction depending on the standing price). This is a high value which may suggest that the company believes that items reaching such a high price may differ under some characteristics. Therefore, the analyses are also replicated with this smaller dataset to ensure that the results are not driven by unobservables.\footnote{Note also that such a high minimum increment undermines the identification of the structural model in Section 5 (e.g., Haile and Tamer, 2003, Chesher and Rosen, 2017). For this reasons auctions with large minimum increments are removed from the dataset. Table 1 shows that the mean of the Minimum increment variable is close to €1, which is also the value at the 90th quantile.}

3.1 Price response to the percentage donated

This section reports results from correlational analyses aimed at highlighting certain features of bidding in Charitystars that should be respected by the theoretical model developed in Section 4. The value of these analyses is twofold because upon estimation of the primitives of the structural model (Section 6), the estimates can be used to simulate bids and replicate some of these results.
Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Q(25%)</th>
<th>Q(50%)</th>
<th>Q(75%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Auction characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage donated (q)</td>
<td>0.70</td>
<td>0.27</td>
<td>0.78</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Transaction price in €</td>
<td>364.25</td>
<td>187.50</td>
<td>222.00</td>
<td>315.00</td>
<td>452.50</td>
</tr>
<tr>
<td>Reserve price in €</td>
<td>179.03</td>
<td>132.02</td>
<td>100.00</td>
<td>145.00</td>
<td>210.00</td>
</tr>
<tr>
<td>Minimum increment in €</td>
<td>1.71</td>
<td>3.15</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Number of bidders (in # days)</td>
<td>7.83</td>
<td>3.27</td>
<td>5.00</td>
<td>7.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Sold at reserve price (d)</td>
<td>0.04</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Length</td>
<td>8.08</td>
<td>3.07</td>
<td>7.00</td>
<td>7.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Extended time (d)</td>
<td>0.43</td>
<td>0.50</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Charity’s activity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helping disables (d)</td>
<td>0.35</td>
<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Infrastructures in developing countries (d)</td>
<td>0.09</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Healthcare (d)</td>
<td>0.23</td>
<td>0.42</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Humanitarian scopes in developing countries (d)</td>
<td>0.14</td>
<td>0.34</td>
<td>0.00</td>
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</tr>
<tr>
<td>Children’s wellbeing (d)</td>
<td>0.84</td>
<td>0.36</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Neurodegenerative disorders (d)</td>
<td>0.06</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Charity belongs to the soccer team (d)</td>
<td>0.10</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Improving access to sport (d)</td>
<td>0.63</td>
<td>0.48</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Overview of the main covariates used in all specifications in the reduced form analysis and in the structural model. (d) stands for dummies. Only auctions with price between €100 and €1000. Prices are in Euro. If the listing was in GBP the final price was converted in Euro using the exchange rate of the last day of auction. There are 1,108 auctions in total.

The hypothesis that bids and donations are positively related implies that higher bids are associated with greater portions of the final price donated (q). To test this hypothesis, I run a series of regressions focusing only on the winning bid (i.e. the transaction price). In fact, all the other bids do not provide useful information in an environment where bidders can update their bids multiple times. Table 2 performs the following OLS regression

$$\log(\text{price}_t) = \gamma_0 + \mathbf{x}_t \gamma + \gamma_q q_t + \epsilon_t$$  \hspace{1cm} (3.1)

where \(t\) indexes the auctions. The matrix of covariates, \(\mathbf{x}_t\), includes all variables other than the percentage donated, \(q\). The columns of the table vary based on the definition of \(\mathbf{x}_t\), which is given in the bottom panel of the table. The variables are defined in detail in Appendix B. The main finding is that \(\gamma_q\) is above 20% and significant (at 1% level) in most columns, suggesting that higher bids are associated with greater donations.

Two additional observations are of interest. First, the transaction price is increasing in the number of bidders. This provides a simple test in favor of a model where bidders draw private values. If instead valuations were common, bidders would optimally shade their bids in order to escape from the winner’s curse. Because the size of the curse grows with the number of bidders, lower bids are expected when an additional bidders joins an auction (Bajari and Hortacu, 2003). A private value model for Charitystars’ auctions is also consistent with bidders having different
Second, the inclusion of the reserve price in Table 2 is responsible for a jump in the Adjusted R-squared from 32% to 46%. According to Roberts (2013), this increase suggests that the reserve price carries unobservable information to the researcher but observable to the bidders and auctioneer. For example, bidders’ willingness to pay may be higher if a player receives an important prize while one of his jerseys is up for auction with a high \( q \). If no regressor in the data reflect such prize, the estimated \( \hat{q} \) may be biased.

Given that the auctioneer anticipates that bidders are affected by the size of \( q \) (after all the word “charity” is part of the name of the firm), one would expect the correlation between the reserve price and the percentage donated to be high if the reserve price were not affiliated with unobservables. In particular, theory shows that the optimal reserve price is increasing in \( q \). However this correlation is close to zero.\(^{16}\) This evidence supports affiliation between reserve prices and unobserved heterogeneity, and it will be exploited in the structural model.\(^{17}\)

\(^{16}\) Appendix C.3 shows that if the bidder who is indifferent between bidding a positive value and not bidding at all bids exactly the reserve price, then the optimal reserve price is increasing in \( q \). However, the data does not find support for this hypothesis given that the Spearman’s rank correlation test is -0.0610 in the large sample and -0.0119 in the small sample.

\(^{17}\) The correlation between the reserve price and the transaction price is 0.5175, implying that the reserve price can explain a large portion of the variance of the price. Table 11 in Appendix E estimates (3.1) on the restricted sample, including only items sold for less than €400. Although the table reports a smaller coefficient for \( q \) (\( \gamma_q \approx 0.13 \)), prices and donations are still positively correlated.
This intuition is also confirmed by conversations with some shareholders, who admitted that the company’s practice is to set the reserve price so that it covers costs. In fact, the reserve price is the lowest price at which the auctioneer is willing to sell the object and therefore embodies information on her willingness to receive money, as well as the cost of procuring the item. To avoid losses, in case that the object is sold at the reserve price, this figure must cover both donations and costs. Therefore, variations in the net reserve price (i.e., \((1 - q) \times R\)) provides information about how procurement costs varies with \(q\). This information will be important in estimating marginal costs in Section 8.

Overall, this suggests that bidders react to giving incentives. Related papers found qualitatively comparable results. For example, Elfenbein and McManus (2010) estimated that prices in eBay charitable listings are on average 6% larger than comparable non-charity ones, while Leszczyc and Rothkopf (2010) using a controlled experiment determined that a 40% donation leads to a 40% price increase. In their investigation on charitable giving as a reputational device, Elfenbein et al. (2012) finds similar returns to those estimated in Table 2 for eBay sellers with poor feedback history (e.g., 25% higher prices when the entire price is donated), but much smaller returns for virtuous sellers. Importantly, since on Charitystars.com, the platform is the only seller, this data is not affected by other confounds like reputations.

The charity auctions literature has also investigated the shape of the relation between \(q\) and prices (e.g., Elfenbein and McManus, 2010). To this end, Table 12 in Appendix E adds \(q^2\) to the covariates used in the OLS regression (3.1). The sign of the coefficients imply a concavity in \(q\). However, \(q\) is only marginally significant in column (III) where a large set of covariates is added, while \(q^2\) is never significant. In contrast, the linear combination of \(q\) and \(q^2\) is positive and significant across all columns (shown in the third panel), failing to reject the linearity of the relation between \(q\) and \(\log(price)\).

Quantile regressions are another commonly used approach to test linearity. Table 3 reports three quantile regressions in the same spirit of regression (3.1).18 All the coefficients of \(q\) are similar across the four columns, and in particular, we cannot reject the null hypothesis that the coefficients computed at the 1st, 2nd and 3rd quartiles are equal (F test p-value 0.82). The same result can be observed graphically in Figure 16 which plots these coefficients, and can also be replicated in the smaller sample (Table 13 and Figure 17). This evidence suggests a linear relation between the fundraising and the logarithm of the price.

Moving from the intensive to the extensive margin of giving, a total of 2,247 bidders compete on average in 5.34 different auctions. This mean increases to 8.51 different auctions when excluding bidders who bid only in one auction only. Most bidders take part in different auction formats (varying over the amount donated): excluding bidders who placed less than 2 bids, more than 80% of the bidders bid at least on two auctions with different amount donated. This is a very large number given that \(q = 85\%\) for half of the auctions in the data (559 out of 1108). Also,

18 Column (I) reports the estimates from the OLS regression in the second column of Table 2.
Table 3: Linearity of the relation between log(Price) and percentage donated

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Q(0.25)</td>
<td>Q(0.50)</td>
<td>Q(0.75)</td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td>0.353***</td>
<td>0.521***</td>
<td>0.370***</td>
<td>0.298***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.037)</td>
<td>(0.030)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>0.298***</td>
<td>0.254***</td>
<td>0.302***</td>
<td>0.354***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.034)</td>
<td>(0.037)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>q</td>
<td>0.234***</td>
<td>0.255***</td>
<td>0.274***</td>
<td>0.308***</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.058)</td>
<td>(0.070)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.348***</td>
<td>1.236***</td>
<td>1.236***</td>
<td>2.726***</td>
</tr>
<tr>
<td></td>
<td>(0.228)</td>
<td>(0.310)</td>
<td>(0.336)</td>
<td>(0.378)</td>
</tr>
</tbody>
</table>

Main Variables Y Y Y Y

Adjusted R-squared 45.90% 31.91% 30.31% 28.80%

N 1,108 1,108 1,108 1,108

Note: OLS Regression and quantile regressions of the logarithm of the transaction price on covariates to test the linearity of donation. Only auctions with price between €400 and €1000. Boostrapped standard errors with 400 repetitions. The null hypothesis that q is the same in column (II), (III) and (IV) is not rejected beyond 70% level. Control variables are defined in Appendix B.

* – p < 0.1; ** – p < 0.05; *** – p < 0.01.

note that the participation decision of Charitystars users is not correlated with the percentage donated.\textsuperscript{19} Therefore bidders seem to participate in all auctions and do not select on the basis of the amount donated. This is confirmed in Table 4 using two sets of Poisson regressions of the form

\[
\log(\mathbb{E}[bidders_t|\mathbf{x}_t, q_t]) = \gamma_0 + \mathbf{x}_t \gamma + \gamma_q q_t + \log(length_t) + \varepsilon_t
\]

where the variable Length is the length of the auction (in days) and is used as the exposure variable (the results are to be interpreted in terms of daily bidders). The first two columns refer to the larger dataset, while the second two are for the smaller dataset. The even columns in the table suggest that the weak relation between q and the number of bidders holds also when conditioning on other covariates.

As a result, contributions to the public good depend disproportionately on the intensive margin rather than on the extensive margin (the Spearman correlation between the number of bidders and q is only 0.084). Therefore, how bidders place their bids is the fundamental question that a revenue optimizing auctioneer should address, disregarding how bidders self-select in different auctions.

A final consideration can be made concerning symmetric bidding. Given the data availability, there are two main sources of asymmetry that can be tested in a reduced form fashion. First,

\textsuperscript{19}The Spearman rank-order correlation test reveals that the correlation between q and the number of bidders is only 0.0855 in the large sample and 0.0650 in the small sample.
since most jerseys belong to Italian teams (63% of the auctions) and most charities are Italian (90% of the auctions), one may ask whether bidders from different nationalities employ different strategies. Table 15 in Appendix E.1 reports the coefficients from three regressions similar to (3.1) where the dummies for each winner’s nationality are added to the covariate matrix, x. Overall, this source of asymmetry is rejected by this analysis.

Asymmetry may also come from recurrent winners as these bidders may be more interested in collecting soccer jerseys than in contributing to a public good. Recurrent winners are not unusual: in fact, the median number of auctions won by each winner in the sample is 3.20 Table 16 in Appendix E.1 investigates whether recurrent winners are willing to pay more on average. This is captured by the dummy variable Recurrent Winner that takes value 1 if the winner won more than 3 auctions. There seems to be no evidence for this type of asymmetry as the correlation in Column (I) between Recurrent Winner and the transaction price seems to be driven by unobservables. In fact, it vanishes when adding another covariate accounting for the level of competition in the auction (the total number of bids placed). Moreover, the same correlation is also not significant in the smaller dataset, where observations are more homogeneous.21

In conclusion, while asymmetries are possible, there is not enough empirical evidence to clearly distinguish bidders across different groups.22 The next section develops and discusses

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20 The first, second and third quartiles of the number of auctions won are the same across the small and large samples and correspond to 1, 3 and 7 items won respectively.

21 Importantly, q hardly varies across specifications and the estimated coefficients are very similar to those reported in Table 2.

22 In related empirical papers with asymmetries the distinction across bidders is immediate. In the procurement literature bidders are often grouped based on different technologies or the size of the firm. For example, in their analysis of timber auction data Athey et al. (2013) distinguish between mills and loggers while Krasnokutskaya

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Table 4: Relation between number of bidders and percentage donated

<table>
<thead>
<tr>
<th></th>
<th>I (100 &lt; p &lt; 1000)</th>
<th>II (100 &lt; p &lt; 400)</th>
<th>III (100 &lt; p &lt; 1000)</th>
<th>IV (100 &lt; p &lt; 400)</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>0.032</td>
<td>-0.002</td>
<td>0.067</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.058)</td>
<td>(0.069)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.223</td>
<td>0.105</td>
<td>0.096</td>
<td>0.394</td>
</tr>
<tr>
<td></td>
<td>(0.235)</td>
<td>(0.241)</td>
<td>(0.334)</td>
<td>(0.360)</td>
</tr>
</tbody>
</table>

| Main Variables | Y | Y | Y | Y |
| Charity Dummies | Y | Y |
| League/Match Dummies | Y | Y |

Pseudo R-squared | 12.49% | 13.92% | 13.05% | 14.32%
N | 1,108 | 1,108 | 713 | 713

Note: Poisson regression of the number of bidders on covariates. The length of the auction is used as exposure variable and is not included among the covariates. Control variables are defined in Appendix B.

* – p < 0.1; ** – p < 0.05; *** – p < 0.01.
an auction model building on the results just mentioned.

4 Charitable motives, auctions and percentage donated

Charitystars profits by leveraging its customers’ motivations to behave prosocially. The underlying assumption is that prosocial motivations impact willingness to pay. Understanding what motivates bidders to pay higher prices is a necessary step towards assessing the profitability of donations. Despite the existence of a large number of reasons for prosocial behavior, as suggested by the many ways one has to give to charities, Bénabou and Tirole (2006) classifies them in three categories: intrinsic, extrinsic and self-image motivation.

Intrinsic motivations are born from the innate care a subject has for the well-being of others, and may arise for example from displays of altruism and reciprocity (e.g., Fehr and Fischbacher, 2003). In Charitystars’ marketplace this is akin to the satisfaction bidders obtain by helping people in need through charitable giving.

The private consumption from winning a charity auction provides bidders with extrinsic motivations (i.e. the set of external incentives for giving). In particular, the higher bidders are willing to pay for the item, the greater the fundraise. Furthermore, extrinsic motives are shared also by losing bidders: because of the equilibrium strategy in ascending auctions, losing bidders can affect the transaction price and the donation by placing the second-highest bid.

Auctions also have a social component: all bidders observe the winner of the auction. Charitystars allows users to navigate through old listings, where the name of the winner is highlighted, and the full bid chronology reporting all bidders and their bids is available. Signaling self-image and social concerns have long been seen as a major factor for prosocial behavior, as demonstrated by people distaste for anonymous contributions (Glazer and Konrad, 1996, Harbaugh, 1998).

Given these premises, the connection between more motivated bidders and greater revenues to the auctioneers seems straightforward. However, this is not generally true. For example, looking at extrinsic motivations, there is empirical evidence showing that it can hamper fundraising (Gneezy and Rustichini, 2000). To the contrary, economic incentives are effective at increasing fundraising.

(2011) compares bids across small and large firms competing for highway contracts in California.

There is broad evidence for the role of self-image on prosociality in many economic situation. In a field experiment Shang and Croson (2009) find that the availability of information about somebody else’s giving makes people donate more. Social pressure operates similarly. In this case not intrinsically motivated subjects decide to donate only because asked by solicitors (e.g., DellaVigna et al., 2012, Huck et al., 2015). Anonymous donations are also closely related to self-image. The New York Times of May 10, 2013 (https://tinyurl.com/y9nsy8dp) reports evidence from a Massachusetts Rabbi who imposed anonymity on religious giving. The Rabbi estimated that this policy accounts for a reduction in fundraising between $10,000 to $20,000 per month. There is also evidence for the converse. In a recent study, Peacey and Sanders (2014) find that anonymous donations for athletes joining the London Marathon exceeded public donations. The authors argue that foregoing the private benefits from public donations is a mean for altruistic agents to signal a charity’s quality. Outside of giving, Funk (2010) documents how voting by mail in Switzerland reduced turnout in small villages as the new law made voting unobservable to other villagers.
the number of blood donors (Lacetera et al., 2012). The seminal paper of Bénabou and Tirole (2006), altogether with the experimental evidence in Ariely et al. (2009), reconciles these contrasting views by drawing from the psychology literature: external forces may impair the attribution of prosocial behavior to intrinsic motives, making prosociality less appealing (called “overjustification effect”). Using a marketplace example, while buying fair trade coffee can benefit a consumer, it can also hurt her if it is believed that its unique flavor is the true reason for purchasing it. In this case, willingness to pay could drop as the consumer knows that she cannot fully signal her prosociality by buying fair trade coffee.

This section builds a parsimonious model of bidding in charity auctions capturing the interplay of the various charitable motives just mentioned, as well as the main features of Charitystars’ auctions discussed in the previous section.

4.1 A bidder’s utility

In an ascending auction, like those on Charitystars.com, the final price is equivalent to the second highest bid, denoted by $b^{II}$, and in equilibrium a bidder will stay in the auction as long as the standing price is smaller than her willingness to pay. Charity auctions differ from standard auctions in that a bidder’s utility depends also on the funds raised, as charitable motives provide bidders with additional satisfaction in proportion of the funds raised.

As discussed at the beginning of this section, winning the auction rewards both intrinsic, extrinsic and self-image motives. This additional utility is modelled by the term $\beta \cdot q \cdot b^{II}$, where $\beta$ transforms the pecuniary contribution, $q \cdot b^{II}$, in utils and indicates the satisfaction from winning and being the donor. While losing bidders do not get to consume the item, they also receive a positive satisfaction from the donation. The rewards to the charitable motives to losing bidders may be different than those to a winning bidder (e.g., less visibility on the website); the term $\alpha \cdot q \cdot b^{II}$ accounts for it. Therefore, the realized utility to a bidder who values the private good $v$ can be summarized by

$$u(v; \alpha, \beta, q) = \begin{cases} v - b^{II} + \beta \cdot q \cdot b^{II} & \text{if } i \text{ wins} \\ \alpha \cdot q \cdot b^{II} & \text{otherwise} \end{cases}$$

This model extends the seminal works on price-proportional auctions and on charity auctions by Engelbrecht-Wiggans (1994) and Engers and McManus (2007) to fit the main characteristics of Charitystars.com (i.e., a fraction $q$ of the final price is donated). As charitable motives cannot explain the full amount of one’s bid, the literature assumes that $\alpha, \beta \in [0, 1)$. As $q \to 0$ a bidders’ utility reverts to selfishness (e.g., $\alpha = \beta = 0$).

Despite the objective impossibility to identify the three charitable motives in Bénabou and Tirole (2006) using only real auction data, the different sizes of the parameters $\alpha$ and $\beta$ are still
informative about their mix. Table 5 summarizes the most common theories of altruism, which are briefly exposed here. The null hypothesis, rejected by the structural estimation in Section 6, is that bidders are not interested in giving. This corresponds to $\alpha = \beta = 0$. In this case, the classic textbook equilibrium applies, and bidding is in no way shaped by altruistic behaviors.

Table 5: Overview of the most common models of giving

<table>
<thead>
<tr>
<th>Model</th>
<th>Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noncharity $(\alpha = \beta = 0)$</td>
<td>Bidders do not pay a premium in charity auctions.</td>
</tr>
<tr>
<td>Pure altruism $(\alpha = \beta &gt; 0)$</td>
<td>Bidders obtain extra utility from donating, and are willing to pay a premium. They do not distinguish across sources of donation.</td>
</tr>
<tr>
<td>Warm glow $(\beta &gt; \alpha &gt; 0)$</td>
<td>Bidders derive greater satisfaction from their own donation (impure altruism).</td>
</tr>
<tr>
<td>See-and-be-seen $(\beta &gt; \alpha = 0)$</td>
<td>Bidders derive utility only from their own donation. Limiting case of warm glow $(\alpha = 0)$.</td>
</tr>
<tr>
<td>Volunteer shill $(\alpha &gt; \beta &gt; 0)$</td>
<td>Bidders obtain greater utility from giving by others.</td>
</tr>
</tbody>
</table>

Source: Leszczyc and Rothkopf (2010).

As described earlier, the three motives interact differently according to the situations, leading to prosocial or antisocial outcomes. Similarly, they may operate differently whether the bidder is expecting to win or lose the auction. However, this does not have to be the case: purely altruistic bidders $(\alpha = \beta > 0)$ receives the same utility in either situation. They are moved by their compassionate concern for others, as a psychological gain that is independent from the identity of the donor (Ottoni-Wilhelm et al., 2017).

Although donating is an altruistic behavior, it may be the result of selfish motives due to the pride inherent in pro-social behaviors (Fisher et al., 2008). This is studied in the impure altruism literature, championed by Andreoni (1989) who proposed a model of warm glow where utility flows from the mere act of being the giver $(\beta > \alpha > 0)$. An extreme case of impure altruism is the see-and-be-seen model, where $\beta > 0$ but $\alpha = 0$ where bidders have no intrinsic motivation. This model captures the role of social status or prestige in donation (e.g., Harbaugh, 1998). Situations where the name of the donors is given to the public belong to this category.

\[24\] A theoretical treatment of these auctions first appeared in Engelbrecht-Wiggans (1994), who analyzed auctions with price-proportional benefits to bidders (all bidders receive an equal share of the final price).

\[25\] Outside of the fundraising literature, a model where $\beta$ is the sole positive parameter is similar to a setting where bidders receive subsidies from the auctioneer for each dollar spent. Set-asides and subsidies are commonly used in procurement auctions for natural resources. For example, in their analyses of Californian timber auctions, Athey et al. (2013) find that policies such as subsidies and entry restrictions are welfare improving over set-asides. In this case, the government would promote a $\beta > 0$ by instituting subsidies.
A model with $\alpha > \beta > 0$ was found to have good fit in a field experiment (Leszczyc and Rothkopf, 2010) where researchers manipulated the auction to understand the extent of over-bidding in charity auctions. This model, called volunteer shill, is characterized by bidders with larger intrinsic valuation, as the large $\alpha$ indicates that bidders welcome more the fundraise (pure altruism) without caring much for the identity of the donor (impure altruism).

Therefore, despite its simplicity and reduced form style, the utility function is flexible enough to accommodate different models of giving, which allows us to infer, as in the work of Bénabou and Tirole (2006), which charitable motives drive decisions. The way charitable motives affect outcomes to the auctioneer will become evident in the next sections which investigates bids and surplus.

4.2 Optimal bid

Turning to the bidders’ optimal strategy in a Charastars’ auction, the following classic assumptions are necessaries.

Assumption 1. Optimality:

1. The values are private and independent.

2. All $n > 1$ bidders draw their values for the private item from a continuous distribution $F(\cdot)$ with probability density $f(\cdot)$ on a compact support $[\underline{v}, \overline{v}]$.

3. The hazard rate of $F(\cdot)$ is increasing.

The first condition requires that bidders hold independent and private valuations for the soccer jerseys. The estimation will relax this assumption by including unobserved heterogeneity, effectively making private values affiliated. This assumption is also supported by the discussion of the results in Table 2. Points two and three of Assumption 1 are simple regularity conditions common to most auction models. In particular, condition three is key in order to establish that the equilibrium bidding function is a global optimum. This condition will also play a central role in proving identification of the primitives in Section 5.

To simplify the theoretical treatment of the English auction I resort to the equivalence with sealed-bid second-price auctions, which was proven by Engers and McManus (2007) for the case $q = 1$. The expected utility to bidder $i$ with valuation $v$ in a charity auction with $n$ bidders is

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26Engers and McManus (2007) demonstrate that the optimal strategy in a second-price charity auction is also optimal in the analogous button auction version. They also note that the observation of bidders’ exiting times is not required for the result to hold. Therefore the equivalence can be extended to more general ascending auctions, like online auctions. In such an auction, the FOC are required to hold only for the second highest bidder at the price at which he or she drops out.
\[
\mathbb{E}_{v,i}[u(v;\alpha,\beta,q)] = \underbrace{\mathbb{E}_{v-i}[v - (1 - q \cdot \beta)b^{II}, i \text{ wins}]}_{i \text{ wins and pays } b^{II}} + q \cdot \alpha \cdot b^{II} \cdot \Pr (i' \text{'s bid is } 2\text{nd}) + \underbrace{q \cdot \alpha \cdot \mathbb{E}_{v-i}[b^{II}, i' \text{'s bid is } 2\text{nd}]}_{i \text{ loses and bids } b_i = b^{II}} \quad \text{(4.1)}
\]

The expected utility is taken over three mutually exclusive events. In the first line of (4.1) \(i\) places the highest bid and wins. This provides him with private consumption \(v\) and altruistic satisfaction by \(\beta \cdot q \cdot b^{II}\). In the second line, \(i\) loses the auction and either set the price by bidding the second highest bid, or bids a price below the second highest bid. In either case \(i\) gains the expected value of \(\alpha \cdot q \cdot b^{II}\). The next proposition provides the bidding function in a symmetric Bayesian Nash equilibrium.

**Lemma 1.** The equilibrium bid in a symmetric second-price charity auction where the auctioneer donates \(q\) is:

\[
b^*(v;\alpha,\beta,q) = \begin{cases} 
\frac{1}{1+q(\alpha-\beta)} \left\{ v + \int_v^\infty \left( \frac{1-F(x)}{1-F(v)} \right)^{\frac{1-q\beta}{\alpha}} + 1 \right\} dx & \text{if } \alpha > 0 \land q > 0 \\
\frac{v}{1-q\beta} & \text{if } \alpha = 0 \lor q = 0
\end{cases}
\quad \text{(4.2)}
\]

**Proof.** See Appendix C.2.

This bid function is also optimal in an ascending auction. In this case, \(b^*(v;\alpha,\beta,q)\) describes the highest value at which winning is worthwhile. In equilibrium a bidder stays in the auction as long as the standing price, \(p\), is below her composite value, \(b(v;\alpha,\beta,q)\), and the winner pays the second-highest bid, \(b^{II}\).

When bidders are not charitable, \(\alpha = \beta = 0\), or when the proceeds of the auction are not used to finance any public good, \(q = 0\), bidders bid their valuation \(b^*(v;0,0,q) = b^*(v;\alpha,\beta,0) = v\). The limit of the function in the top row of (4.2) converges to that in the bottom row as \(\alpha\) goes to 0. Finally, Proposition 8 in *Engers and McManus (2007)* demonstrates that revenues in charity auctions are bounded (if the auctioneer cannot shut down the auction), meaning that under no combination of \(\alpha\) and \(\beta\) bidders make unlimited transfers to the auctioneer. Their proof holds also in this scenario where \(q < 1\).

### 4.3 Comparative statics

How does a marginal change in the environment’s primitives affect bidders? What is the elasticity of revenues to the donations? Addressing these issues requires the assessment of how bids react when either bidders become more altruistic (a change in \(\alpha\) or \(\beta\)), or when the auctioneer
becomes more generous (a change in $q$).

Figure 2: Comparative statics

(a) Derivative of the bid with respect to $\alpha$

(b) Derivative of the bid with respect to $q$

Note: Panel (a). The effect of a marginal increase in $\alpha$. Panel (b). The effect of a marginal increase in $q$. In both graphs $q = 85\%$ and $F(\cdot)$ is a truncated normal in $[0,100]$ with mean 50 and standard deviation $\sigma$ (see the legend).

First, bids are unambiguously increasing in the own donation parameter, $\beta$. The greater the glow from the act of donating, the more bidders are willing to pay for the item. Figure 2a illustrates how bidders change their bids after a marginal increase in $\alpha$. The $x$-axis displays all bidders’ private values, while the derivative of the bid at each value is presented on the $y$-axis. Bidders can be separated in two groups based on whether they revise their bids up or down. Low-value bidders are associated with higher bids, and high-value bidders are associated with smaller bids. The following proposition formalizes the findings from the figure proving conditions for existence and uniqueness of the private value that separates the bidders who increase their bids from the others.

**Lemma 2.** If $\alpha > 0$, there exists a value $\nu^*$ such that all bidders with private values in $(\nu^*, \nu]$ decrease their bids after a marginal change in $\alpha$.

**Proof.** See Appendix C.5.

An interpretation of Lemma 2 views bids as strategic complements or substitutes. Due to the monotonic bidding strategies, bidders with high values are possible winners. A marginal change in $\alpha$ has two effects. It intensifies the degree of substitutability between the bid of a high-

\[\text{If the bid of the lowest value bidder is greater than } \frac{\nu}{1-\beta q}, \text{ the private value separating the two groups is unique. Otherwise either the derivative is always negative, or there are two cutoffs such that bidders at the two extremes (below the first cutoff and above the second cutoff) decrease their bids, and the bidders between the two cutoffs increase their bids. In this case, the lowest value bidders do not increase their bids because they know they cannot affect the transaction price.}\]
value bidder and that of the others: this makes a possible winner willing to trade a larger payoff from losing with some probability of winning. Thus, possible winners decrease their bids. A similar change in $\alpha$ also changes how a low-value bidder perceives her bid vis-à-vis those of her competitors. Given that she will likely lose the auction, she can affect her payoff only by placing the second highest bid. Thus, in order to extract surplus from the possible winners, she revises up her bid: her higher bid is complementary to that of a greater winning bidder. A comparison of the dotted and dashed line in Figure 2a corroborates this idea: more low-value bidders increase their bids when they believe they are pivotal, which happens when the variance of $F(v)$ is larger (dotted line).

An immediate extension of Lemma 2 is that the expected prices are generally decreasing in $\alpha$. In fact, given a large enough number of bidders and keeping $\beta$ and $F(v)$ constant, an increase in $\alpha$ is associated with more bidders shading their bids. A sufficient number of bidders or a sufficiently skewed distribution of values is required so that low-value bidders – those who increase their bids after a marginal change in $\alpha$ – are price takers. To provide an intuition of how bids are affected by $\alpha$ and $\beta$, Figure 3 plots the distribution of bids and private values. In Panel (a), $\beta > \alpha$ and all bidders overbid relative to their private values as they enjoy greater returns from winning and donating. Of course in this case the auctioneer earns gross revenues beyond the non-charity auction. However, charity auctions cannot always guarantee revenues beyond non-charity ones. Panel (b) shows that the auctioneer would improve his records by announcing a non-charity auction instead when $\alpha$ is much larger than $\beta$. The figure shows that high value bidders shade their bids more than the others. When this happens the effect on revenues is uncertain.

This discussion partially extends to the relation between $b$ and $q$. Excluding the trivial case $\alpha = 0$ which is equivalent to an increase in $\beta$, the sign of the derivative of the bid with respect to $q$ depends on the model of giving. Bids rise after a marginal change in $q$ for all bidders under warm glow ($\beta \geq \alpha$). This is depicted by the solid line in Figure 2b. In the same figure, the bid derivatives for the voluntary shill model ($\beta < \alpha$) are strikingly similar to those in Figure 2a, where bidders either post higher or lower bids (dotted and dashed lines). This can be formalized too:

Lemma 3. If $\beta \geq \alpha$, bids are increasing in $q$ for all bidders. If $\alpha > \beta$, there exists a private value $\tilde{v}$ such

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28See Figure 22 in Appendix E.4 for a revenue comparison across different charity and non-charity auctions using the primitives in Figure 3b. See Engers and McManus (2007) for the bidding functions for first-price and all-pay auctions.

29Similarly to Lemma 2, in the case $\alpha > \beta$ there is only one bidder separating those who increase their bids from those who decrease their bids if the lowest value bidder bid more than her private value (e.g., $b(\tilde{v}) \geq \tilde{v}$). Otherwise, there can be either no cutoff or two cutoffs. In the last case, only the bidders with the lowest and the highest valuation will decrease their bids. However, while the high-value bidders decrease their bids because a marginal increase in $q$ is associated with a marginally greater payoff from losing, the low value bidders decrease their bids because they cannot affect the transaction price.
that bidders with private values in $(\bar{v}, \tilde{v})$ decrease their bids after a marginal change in $q$.

**Proof.** See Appendix C.6.

This result can be interpreted in light of the theory in Bénabou and Tirole (2006). When the glow from winning and donating is greater than the externality from somebody else’s contribution, all bidders increase their bids as the auctioneer becomes more generous. In this case, the extrinsic and reputational motives make bids strategic complements, leading to higher prices in expectation. Under voluntary shill instead, bidders already have large intrinsic motives, as represented by the high $\alpha$. In these circumstances, bidders find harder to signal themselves as genuinely prosocial by winning the auction before the society (as everyone shares high intrinsic altruism). This phenomenon drains bids by high-value bidders (Gneezy and Rustichini, 2000); their behavior is related to the classic free-riding in public good games as they exchange a higher probability to win the auction with the consumption of the public good funded by others.

This discussion should be read in light of the lack of agreement in the empirical literature assessing the effect of donations on prices and revenues in charity auctions (e.g., Carpenter et al., 2008, Schram and Onderstal, 2009, Isaac et al., 2010, Elfenbein and McManus, 2010, Leszczyc and Rothkopf, 2010, Elfenbein et al., 2012). For example, Leszczyc and Rothkopf (2010) attributed the widespread overbidding over the non-charity auction that emerged in a series of field exper-

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Note: Distributions of private values (solid line) and bids (dotted line). 50 simulations, 12 bidders. While bidders bid more than their private values in Panel (a), most of the bidders bid below their private values in Panel (b). The primitives of the model are $q = 85\%$, $F(\cdot) \sim N(50, 25)$ on $[0, 100]$. 

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30This setting is different from that in Bénabou and Tirole (2010) for agents hold the same altruistic preferences. Heterogeneity comes entirely from differences in the bidders’ private values and thus from the set of incentives inherent to the auction.
iments as a symptom of large externalities. They concluded that the $\alpha > \beta$ case is conducive of higher revenues. However, this conclusion is only partially true. Higher revenues are only possible if the number of bidders is low – a suspicion confirmed by their data as the average number of bidders is only between 2 and 4 per auction. In contrast, another field experiment found charitable prices to sink below non-charity ones at a school charity auction (Carpenter et al., 2008). Once again this can be explained in terms of high externalities as bidders who are likely to win shed their bid in this case, dumping the price.

4.4 Consumer and producer surplus

How do bidders fare in charity auctions? Despite the possibly higher prices in charity auctions, the consumer surplus is at least as large as that in a similar standard auction.

**Proposition 1.** When $\alpha = 0$, the expected consumer surplus in a charity auction is equal to the consumer surplus in a non-charity auction. It is greater when $\alpha > 0$.

**Proof.** See Appendix C.7.

The proof is constructive and reflects the fact that the boost in utility from losing the auction (through the public good) is greater than the (possible) higher price paid when winning it. In fact, in the $\alpha = 0$ case bidders extract the same utility as in the non-charity case – the donation acts as a discount, and bidders bid more until they fully exhaust their discount. When $\alpha > 0$, the positive externality flowing from the winner to the losers increases bidders’ expected utility further, making bidders in charity auctions better-off.

Finally, does supplying impure public goods boosts a firm’s net revenues? In the absence of externality the higher bid paid does not result in higher net revenues to the auctioneer. The discount passes-through to the bidders, while net revenues to the auctioneer decrease by a factor $q(1 - \beta) / (1 - q\beta)$ compared to standard non-charity auctions. Firms should not hold charity in this case.

Cross-bidders externalities can improve net revenues to a charitable auctioneer. This happens when there is enough uncertainty in the auction so that low value bidders are price-makers, as displayed in Figure 2a. However, the externality cannot be too large in order to prevent high-value bidders from excessively shedding their bids. This highlights the peculiar way charity-linked goods (or impure public goods) affect markets creating complex strategic interactions across agents. In a charity auction prices do not only balance the intensive margin (an agent’s surplus) with the extensive margin (the probability of winning), but also an additional margin represented by free-riding. When the latter is kept at bay firms may find charity auctions prof-
Proposition 2. Define \( \eta \) as the elasticity of the expected price to the donation \( q \). When \( \alpha = 0 \), the auctioneer should not donate. When \( \alpha > 0 \), the optimal donation solves \( \eta = \frac{q}{1-q} \).

Proof. See Appendix C.8.

The proposition defines a general condition under which it is optimal for the firm to donate. This holds not only in the auction setting, but most generally for any revenue dependent CSR activity. The ratio \( q/(1-q) \) is the ratio of what is given versus what is kept, and is convex in \( q \). The firm will increase its donation rate until the marginal benefit from higher bids \( (1-q)\eta \) equates the marginal cost from donating an extra euro \( q \). This is evident from the two panels in Figure 4, where net revenues (right axis) are optimized at the \( q \) that sets \( \eta = q/(1-q) \).

Figure 4: The revenue maximizing donation - a numerical example

(a) Optimal \( q^* \) when \( \alpha = 0.7, \beta = 0.9 \)

(b) Optimal \( q^* \) when \( \alpha = 0.5, \beta = 0.9 \)

Note: Both panels report the optimal percentage donated \( (q^*) \) obtained at the intersection of the elasticity of the expected winning bid with respect to \( q \) and the curve \( q/(1-q) \). The elasticities (solid and dotted curves) are reported on the left vertical axis. The right axis shows the net revenues (dashed curve) as \( (1-q) \int b(v) dF^{(2)}(v) \). The charity parameters \((\alpha \text{ and } \beta)\) are displayed in the title of the two panels. The distribution of values \( F(v) \) is uniform on \([0,100] \), and there are two bidders.

Figure 14 in Appendix C illustrates the results in these propositions. The analysis so far indicates that if two second-price auctions are observed and if they differ only for the proportion donated, simply comparing changes in the final price across auctions could lead the researcher to believe that bids are decreasing in the amount donated. In reality, it could be that high-value bidders are shading their bids in the auction with the greater \( q \) (see Figure 2b and Proposition 3).

\[^{31}\text{The discussion of the producer surplus assumes that costs do not depend on the fraction donated. This assumption is relaxed in Section 8, as decreasing costs in } q \text{ are important to explain why Charitystars donates.}\]
This could mistakenly lead to conclude that bidders do not react to charitable incentives, or that unobservables play a major role in the estimation. Moreover, the producer surplus is damaged by large $\alpha$. In particular, the optimal decision of setting a charity auction (or to engage in CSR) does not immediately follow from the empirical observation of a positive correlation between prices and donations.

In conclusion, charitable incentives may affect bids and profits in surprising ways. These comments highlight the importance of properly understanding bidders’ preferences in markets of impure public goods in order to design mechanisms to simultaneously increase profits while raising funds.

5 Nonparametric identification

This section establishes the nonparametric identification of the primitives, $\alpha$, $\beta$ and $F(v)$, given the observed bids, $b(v; \alpha, \beta, q)$, and the percentage donated, $q$. To set the notation, let $G(b)$ be the observed distribution of bids, and $\lambda(b)$ be its inverse hazard rate. Following the seminal work of Guerre et al. (2000), the bids can be inverted, which implies that the distribution of bids is equal to the distribution of private values. Therefore, $G(b(v)) = F(v)$. Replacing $F(v)$ and $f(v)$ with the bid distribution and density respectively in the FOCs reduces the number of unknowns and gives us an equation that must hold for all bidders in second-price auctions (and second highest bidders in online auctions).

$$v = \xi(b, \alpha, \beta, q) = (1 + q \cdot (\alpha - \beta)) \cdot b - q \cdot \alpha \cdot \lambda(b) \quad (5.1)$$

Obviously, without knowledge of the vector of valuations on the left-hand-side (and a rank condition) it is not possible to identify the charitable parameters.

**Proposition 3.** $\alpha$, $\beta$ and $F(v)$ are not identified without additional restrictions.

**Proof.** See Appendix C.9.

The proof shows that two different bids placed under different models are observationally equivalent. In fact, without additional restrictions, the parameters and the distribution can be combined in different ways yielding the same bids distributions.

A similar nonidentification result holds in the estimation of risk aversion in first-price auctions. In this case the econometrician relies on quantile restrictions and on cross-auction varia-

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32 For simplicity, denote the bid function by $b(v)$ and its realization by $b$. In the remainder of the paper a bid will be denoted by $b(v; \alpha, \beta, q)$ only when it would be confusing otherwise. The inverse hazard rate of the bid distribution is $\lambda(b) = \frac{1 - G(b)}{g(b)}$, while the inverse hazard rate of the distribution of private values is $\lambda(v) = \frac{1 - F(v)}{f(v)}$.

33 Under Assumption 1 the following relations hold: $G(B) = Pr(b(v) < B) \equiv Pr(v < b^{-1}(B)) = F(b^{-1}(B))$. An analogous relation holds for the pdf $g(b(v))b'(v) = f(v)$. 

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tion on the number of bidders to identify the risk parameters (e.g., Guerre et al., 2009, Campo et al., 2011). However, placing restrictions on the number of bidders cannot be a source of identification in the second-price charity auctions because the number of bidders cancels out in the first order conditions.

Let’s turn to (5.1) to figure out the most opportune restriction leading to identification of the primitives. First, \( \zeta(b, \alpha, \beta, q) \) is strictly increasing in \( b \). In fact, the multiplicative term in front of the bid is positive (\( \alpha, \beta \) and \( q \) are percentages), while \( \lambda(b) \) is decreasing in \( b \) because of the increasing hazard rate property (Assumption 1.3). Second, the hazard rate and \( b \) are non-linearly related. Thus, there are no two different combinations of \( \alpha \) and \( \beta \) yielding the same vector of pseudo-private values \( v \).

These two observations are key because they ensure that every distribution \( F(\cdot | \hat{\alpha}, \hat{\beta}, q) \) (one for each \( \hat{\alpha}, \hat{\beta} \) combination, given \( q \)) is unique by Theorem 1 in Guerre et al. (2000).\(^\text{34}\) This theorem relies on the independent private value framework, and on the fact that \( \zeta(\cdot) \) is strictly monotonically increasing and differentiable in \( b \) (Assumption 1). Therefore, using variation in the percentage donated across auctions, the primitives are identified by distribution equality

\[
F(\cdot | \hat{\alpha}, \hat{\beta}, q_A) = F(\cdot | \hat{\alpha}, \hat{\beta}, q_B)
\]

with \( q_A \neq q_B \). This strategy fails if, for example, higher valued bidders self-select in auctions with higher (or lower) amount donated. An additional assumption is required:

**Assumption 2. Identification:** \( F(\cdot) \) and \( q \) are independent.

The identification restriction is an orthogonality condition between the distribution of private values and the percentage donated. It holds if bidders do no select into different auctions (e.g., auctions with different amount donated) based on their valuations. The model exposed in Section 4 automatically satisfies this assumption, because it defines \( F(v) \) as the unconditional distribution of private values. In the model, \( q \) modulates the net utility to a bidder through the origination of a public good: in no way it affects \( v \). Moreover, Section 3.1 finds that Charitystars’ bidders do in fact bid in multiple auctions without regard for \( q \). Also, the same section shows no correlation between the number of bidders and \( q \). These stylized facts support the idea that \( q \) does not affect the values bidders hold for the private good (the auctioned item), but only the composite valuation for the impure good. In addition, the optimization conditions (Assumption 1) which requires that bidders’ valuation come from the same distribution and respect some basic regularity conditions are maintained. Upon estimation of the model, the validity of these

\(^{34}\)Theorem 1 in Guerre et al. (2000) is based on a set of regularity conditions: values \( v \) are private and independent, the bidding function \( b(v; \cdot) \) is increasing and \( \zeta(\cdot) \) is strictly increasing in \([\underline{b}, \overline{b}]\) and differentiable. These properties follow from Assumption 1.
conditions can be tested (see Section 6.3).

**Proposition 4.** In second-price auctions, under Assumptions 1 and 2 the parameters $\alpha$ and $\beta$ and the distribution of values $F(v)$ are identified by variation in $q$ across auctions.

**Proof.** See Appendix C.10.

The intuition behind this proposition is rather simple. Assume that the econometrician observes two auctions, $A$ and $B$, conforming to the assumption of Proposition 4. In a symmetric equilibrium, two bidders with equal valuations (e.g., $v^A = v^B$) taking part in two auctions that differ only with regard to the percentage donated (i.e., $q^A \neq q^B$) will place different bids (i.e., $b(v; \cdot, q^A) \neq b(v; \cdot, q^B)$) according to the bid function (4.2). Because of the monotonicity of bids in $v$, the ranking of their bids on their respective bid distribution will be the same (i.e., $G^A(b(v; \cdot, q^A)) = G^B(b(v; \cdot, q^B))$). This is because for any bidder whose private value is at the $\tau$-quantile of the distribution of private values, also their bids will be at the $\tau$-quantile of the bid distribution. This observation allows us to simplify further the FOCs (5.1) by taking difference across the FOCs for the two auctions along the quantiles of the bid distributions. $\alpha$ and $\beta$ are identified under the full-rank condition of the resulting matrix, while $F(v)$ is identified by plugging the identified parameters in the RHS of (5.1).

This approach is related to that in Guerre et al. (2009) who identify risk-averse utility functions nonparametrically using variation in the number of bidders across auctions (i.e. variation in $q$). Lu and Perrigne (2008) instead use a combination of first-price and English auction to identify risk-aversion. Since risk-aversion does not affect equilibrium bidding in ascending auctions, they first recovered the distribution of values from the open auctions, and then plugged its quantiles in the FOC for the bidders in first-price auctions. This is equivalent to solving (5.1) knowing the private value on the left-hand side. Shifters similar to $q$ have also been used to study correlated private values in English auctions (Aradillas-López et al., 2013), interdependent costs in procurement auctions (Somaini, 2015) and selective entry (Gentry and Li, 2014).

Proposition 4 can also be extended to include a finite number of auction types (e.g., $q \in \{q_1, q_2, ..., q_K\}$), as shown in the following corollary. The proof uses the panel structure of the data to create a projection matrix that cancels out the left-hand side of (5.1) in a way akin to the previous proposition.

**Corollary 1.** In second-price auctions, $\alpha, \beta$ and $F(v)$ are nonparametrically identified also when the dataset includes more than 2 types of auctions.

**Proof.** See Appendix C.11.

Despite these identification results, these procedures do not cover Charitystars’ auctions for two reasons. First, to win an English auction bidders bid the second-highest bid plus an increment. While minimum increments on Charitystars are negligible, and thus constitute no harm...
for identification (Haile and Tamer, 2003, Chesher and Rosen, 2017), this statement means that
the winner’s bid does not solve (5.1). Second, the equivalence between English auctions and but-
ton auctions may not hold for most bidders. Thus, the willingness to pay for all bidders but the
second-highest one is not identified. As a result, most bids do not reflect the private valuations,
and one cannot use the monotonicity of the bidding function to determine the distribution of pri-
vate values from the observed distribution of bids.\footnote{Formally, from the observed bids one cannot guarantee that \( b^{-1}(v) = v \), and thus that \( F(V) = \Pr(v \leq V) = \Pr(b(v) \leq b(V)) = G(b(V)) \).} The next proposition amends Proposition
4 and ensures identification of the primitives under these circumstances.

\textbf{Proposition 5.} \textit{In English auctions, \( \alpha \), \( \beta \) and \( F(v) \) are nonparametrically identified by first deriving the
distribution of bids that would have been observed in parallel second-price auctions, and then by applying
Proposition 4.}

\textbf{Proof.} See Appendix C.12.

The precise meaning of “parallel second-price auction” is a second price auction with the
same primitives as those in the English auction. The distribution of values in a parallel auction
can be derived by the sole observation of the winning bids. In fact, this bid identifies the price
at which the second-highest bidder opts-out of the auction. Therefore, the FOCs (5.1) must hold
at this price for this bidder.\footnote{Denote with a subscript \( w \) the winning bid and its distribution. The distribution of bids from a “parallel second-
price auction”, \( G(B) = \Pr(b(v) \leq B) \), is found by a reparametrization in \( n \) of the distribution of the winning bid \( G_w(b) \) which is obtained from the data. Because of the equilibrium condition, the latter is equal to the distribution
of the second highest bid among \( n \) bidders \( G_{(2)}(b) \). This gives \( G_w(b) = G_{(2)}(b) = n \cdot G(b)^{n-1} + (n-1) \cdot G(b)^n \)
which has a unique solution in \((0, 1)\).} In addition, the existence of a one-to-one mapping between the
distribution of bids and that of the second-highest bids (following the theory of order statistics)
ensures that the former is identified. This would be equal to the distribution of bids observed in a
parallel second-price auction.\footnote{Furthermore, the notion of symmetry adopted in this paper does not imply that bidders cannot bid asym-
metrically across auctions. For example, given a large dataset, symmetry can be relaxed by allowing the giving}

At this point, there are all the ingredients to identify the primitives following Proposition 4 (or Corollary 1).

Symmetric bidding is a crucial assumption. Theorem 6 in Athey and Haile (2002) shows that
the primitives of an asymmetric auction model are identified from the winning bids if the iden-
tity of the bidders is known. Lamy (2012) discusses asymmetry with anonymous bidders (as in
Charitystars’ case), but cannot extend his results to ascending auctions. Still, the assumption of
symmetric bidding in charity auctions may not be so restrictive. First, there does not seem strong
evidence suggesting asymmetric behavior in Charitystars (see Section 3). Second, Elfenbein and
McManus (2010) provide empirical evidence of symmetric bidding for the highest bidders in a
study of eBay charity auctions.\footnote{Note that the minimum increment in Charitystars is \( €1 \) for most auctions.}
6 Estimation method and results

The estimation closely follows the identification procedure in Proposition 5. However, additional difficulties come from the need to deal with auction heterogeneity. Controlling for observable and unobservable heterogeneity is important. The estimation procedure includes three steps and is described in the next section. Sections 6.2 reports the results and 6.3 performs out-of-sample validation using additional data and simulations.

6.1 Structural estimation

The estimation is based on comparing two types of auctions with different $q$. Figure 1 indicates that in most auctions the auctioneer donates either 85%, 78% or 10%. Clearly, as $q^A \rightarrow q^B$ the necessary rank condition fails to hold and the model is not identified (see the Monte Carlo experiments in Table 24 in Appendix H). Thus, the primitives are estimated using the sample of auctions with $q \in \{10\%, 85\\%\}$. The remaining auctions ($q = 78\%$) will be helpful to test the model.

Given that the samples are not random, an important empirical issue is whether auctions at 10% systematically differ from the others. This would complicate the comparison between the two sets of auctions. A logistic regression in Appendix E.2 explores this further by analyzing the probability that a listing is chosen to be at 10% or at 85%. Table 18 tabulates the results. The most important regressors to be accounted for are the reserve price, the number of bidders and some charity dummy variables. It will be therefore necessary to account for these observables in the estimation.

The first step of the estimation procedure deals with auction heterogeneity. A common approach to deal with observables is to perform a hedonic regression of the bids on covariates, and use the error terms as pseudo-winning bids (Haile et al., 2003). The advantage is that it pools all the data together by homogenizing bids into residuals ($\epsilon = b - x'\gamma$), ultimately offering a very tractable way to analyze the data.

One of the major shortcomings of this approach is that it fails to properly account for unobservables. However, if unobservables and reserve prices are affiliated, Roberts (2013) shows how a control function approach can be used to account for them. Section 2 shows that the reserve price is not set optimally. Thus, oscillations in the reserve price may reflect different market conditions (e.g., news, or charitable events) or object characteristics that affect bids but are not explicitly accounted for in the data (e.g., the jersey was worn and sold right after the player won a particular individual award which is not captured by the covariates).

First step. The first step performs an OLS regression of the logarithm of the reserve price on the covariates (i.e., $\log(\text{reserve price}_t) = \delta_0 + x_t \delta$). Denote the generated regressor by $\widehat{UH}$ (i.e., parameters to change across charity characteristics (each auction is linked to a given charity), or other observables, using a flexible functional form for the parameters $\alpha$ and $\beta$.)
\( \hat{U}H_t = \log(\text{reserve price}_t) - \delta_0 - x_t \delta \). Successively, the following hedonic regression yields the homogeneized pseudo-winning bids, \( \hat{b}_w \), as

\[
\log(\text{price}_t) = \gamma_0 + x_t \gamma + \gamma \hat{U}H_t + b_{w,t} \tag{6.1}
\]

where \( t \) indexes the auctions. (6.1) regresses the logarithm of the transaction price on the observed (\( x \)) and the unobserved heterogeneity (\( \hat{U}H \)). The error term in (6.1) are the homogenized pseudo-winning bids, \( b_w \), which will be used in the next two steps of the estimation.\(^{39}\) Table 17 in Appendix E.2 displays the estimated coefficients from (6.1).

In Section 3 it was entertained the possibility of asymmetry among bidders who won multiple auctions and the other bidders. As an additional check, the distributions of the pseudo-winning bids from (6.1) conditional on the winner being a collector should be more skewed to the right compared to that of the other bidders, as collectors bid higher values on average. The data fail to support this thesis. Instead, the Kolmogorov-Smirnov test does not reject equality of the distributions at 0.1422 in the large sample and 0.1520 in the small sample.\(^{40}\) This result further confirms that the alleged weak asymmetry vanishes once accounting for unobserved heterogeneity, either by using a more homogeneous sample (as in the last two columns of Table 16) or by a control function approach (as in equation 6.1).

**Second step.** To derive the distribution of bids in a “parallel second-price auction” as by Proposition 5, separate the pseudo-winning bids for the two auction types. Let the superscript \( a \in \{10\%, 85\%\} \) indicate whether the variable belongs to the 10\% auctions or the 85\% auctions. Following the proof, the distribution of bids, \( G^a(b) \), is obtained by observation of the winning bids \( b^a_w \). Denote the distribution of the winning bids by \( G^a_w(\cdot) \). Then \( G^a(b) \) is found as the solution in \([0, 1]\) of

\[
G^a_w(b) = nG^a(b)^n - (n - 1)G^a(b)^n \tag{6.2}
\]

where the second term in brackets is the distribution of the second-highest order statistic expressed in terms of the distribution \( G^a(b) \). Solving this equation gives the distribution of bids that would have been observed in the parallel sealed-bid auction. The density \( g^a(b) \) is found similarly. The derivative with respect to \( b^a \) of equation (6.2) is:

\[
g^a_w(b) = n(n - 1)G^a(b)^{n-2}[1 - G^a(b)]g^a(b) \tag{6.3}
\]

which uniquely pinpoints the density of the bids, \( g^a(b) \). Solution of (6.2) and (6.3) requires the computation of \( G^a_w(b) \) and \( g^a_w(b) \). This is done by a Gaussian kernel whose bandwidths are

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\(^{39}\)x includes all the variables labelled Main Variables in Appendix B. This set of covariates corresponds to column (II) in Table 2, which has the lowest BIC. Note that the percentage donated (and the reserve price which is used in the control function approach) is not used in either OLS regressions, as variation over \( q \) is key for identification.

\(^{40}\)Figure 19 in Appendix E.4 plots the pdfs. Adding more covariates further increase the p-value of the test statistics.
chosen according to Li et al. (2002).\textsuperscript{41}

**Third step.** In the last step, $G^a(b)$ and $g^a(b)$ form the inverse hazard rates, $\lambda^a(b)$, for each auction format $a$. The objective is to match the FOCs for the two types of auctions for each quantile of the bid distribution. According to the identification, at the true parameters the LHS of the FOC (5.1) computed for the 85% auctions at the $\tau$-quantile, $\hat{\sigma}^a_{85\%}$, is equal to the LHS of the FOC for the 10% auctions at the same $\tau$-quantile, $\hat{\sigma}^a_{10\%}$. This delivers the moment condition

$$\hat{\sigma}^a_{10\%} - \hat{\sigma}^a_{85\%} = 0$$

The number of possible moment conditions is theoretically infinite. This issue is solved by matching the quantiles of the bid data for the 10% auctions with the smoothed version of the 85% auction data, for which more observations are available (and therefore it can better approximate its bid distribution). Define $\Theta = \{\alpha, \beta\}$; the criterion function to be minimized is:

$$\Theta^* = \arg\min_{\Theta} \frac{1}{T} \sum_{\tau} (\hat{\sigma}^a_{10\%}(\Theta) - \hat{\sigma}^a_{85\%}(\Theta))^2$$

where $T$ is the number of 10% auctions. The minimization algorithm searches for the values of $\alpha$ and $\beta$ minimizing the criterion function in the admissible region ($\alpha, \beta \in [0, 1]$). Finally the distribution of private values $F(v)$ is found as the empirical distribution of the left-hand side of (5.1).

The performance of the estimator is good even in small samples as shown by the Monte Carlo simulations reported in Table 25 in Appendix H.\textsuperscript{42} The simulations suggest asymptotic normality of the estimator as the RMSE decreases at a rate close to $\sqrt{n}$ when the number of auctions increases.

### 6.2 Estimation results

As a requirement of the second step of the structural estimation, the number of potential bidders must be fixed in order to compute the distributions of the pseudo-bids from the pseudo-winning bids (as shown in equation 6.2). Setting the number of bidders at the 99th-quantile of the distribution of bidders is a good choice as it avoids concerns with outliers. In fact, extremely high number of bidders may depend more on the item characteristics rather than on the distribution of values and parameters. Additional estimations of $\alpha$ and $\beta$ for lower quantiles of $n$ are

\textsuperscript{41}The bandwidth of the kernel estimators are $h_{g}^a = c^a \cdot T_a^{-1/5}$ for each pdf and $h_{G}^a = c^a \cdot T_a^{-1/4}$ for each CDF, where $c^a = 1.06 \cdot \min\{\sigma^a, \text{IQR}^a / 1.349\}$, $T_a$ is the number of auctions of type $a \in \{l, h\}$, $\sigma^a$ is the standard deviation and IQR$^a$ is the interquantile range of the transaction prices of auction $a$. Trimming is used to account for the bias at the extreme of the support of the bids.

\textsuperscript{42}Comparing the columns referring to the median estimates shows that the Gaussian kernel outperforms the Triweight kernel in small samples, while the RMSE is very close in the two cases. Using a triweight kernel does not affect the estimates.
considered as robustness checks.

Table 6 reveals that $\beta > \alpha$, coherent with the warm glow model of altruism (Andreoni, 1989). The estimates hardly vary with the number of potential bidders and are always significant. The 5% confidence interval are also reported in square brackets and are obtained by bootstrap. Finally, the CI for $\alpha$ are smaller than those for $\beta$ across all rows.

Table 6: Estimation of $\alpha$ and $\beta$

<table>
<thead>
<tr>
<th>Number of bidders</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[5% CI]</td>
<td>[5% CI]</td>
</tr>
<tr>
<td>99% 16</td>
<td>19.5%</td>
<td>46.2%</td>
</tr>
<tr>
<td></td>
<td>[10.0%, 29.1%]</td>
<td>[25.5% 59.9%]</td>
</tr>
<tr>
<td>95% 14</td>
<td>19.3%</td>
<td>46.2%</td>
</tr>
<tr>
<td></td>
<td>[10.3%, 28.6%]</td>
<td>[26.4% 62.9%]</td>
</tr>
<tr>
<td>90% 12</td>
<td>19.0%</td>
<td>46.1%</td>
</tr>
<tr>
<td></td>
<td>[10.4%, 27.1%]</td>
<td>[28.1% 62.0%]</td>
</tr>
<tr>
<td>75% 10</td>
<td>18.6%</td>
<td>46.1%</td>
</tr>
<tr>
<td></td>
<td>[9.2%, 27.3%]</td>
<td>[27.4% 62.6%]</td>
</tr>
<tr>
<td>50% 7</td>
<td>17.4%</td>
<td>45.9%</td>
</tr>
<tr>
<td></td>
<td>[7.9%, 27.1%]</td>
<td>[26.1% 62.6%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). Dataset restricted to all auctions such that $q \in \{10\%, 85\\%\}$ and price between €100 and €1000. 731 observations in total.

The same exercise is reported in Table 19 in Appendix E.2, where the dataset is restricted to observations with prices greater than €100 and smaller than €400. The $\alpha$ parameter stays roughly unchanged, while its counterpart $\beta$ has slightly dropped (ca. 38% instead of 46%), but still greater than $\alpha$. The confidence intervals are much larger than in the previous estimates. This does not rule out a greater $\beta$ than that estimated in the smaller sample. This is probably due to the smaller number of observations which is only ca. 60% of those in the larger database (470 auctions in total). Overall, the smaller sample confirms that bidders take into account the charitable gifts according to warm glow theory.43

Finally, Tables 20 and 21 in Appendix E.2 report estimation results when the three steps of the estimation routine are applied to the auctions with $q = 10\%$ and $q = 78\%$. The estimated $\alpha$ and $\beta$ are close to those in Tables 6 and 19 and the confidence intervals largely overlap.

43Adding more covariates does not qualitatively affect these estimates.
6.3 Tests to estimation and identification

The model predicted expected gross revenues are close to the realized revenues. Table 7 compares the simulated and realized median and average revenues. These prices are computed as the expectation of the second-highest bid using the estimated primitives. To account for auction heterogeneity, these quantities are transformed in Euro by adding back either the average or the median fitted prices from the first step estimation in (6.1). The table displays a good fit of the estimates with values within 10% of the realized ones.44

Table 7: Estimated revenues vs realized revenues

<table>
<thead>
<tr>
<th></th>
<th>$q = 85%$</th>
<th>$q = 10%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median revenues</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimated</td>
<td>353.58</td>
<td>299.27</td>
</tr>
<tr>
<td>Realized</td>
<td>347.50</td>
<td>300.50</td>
</tr>
<tr>
<td>(+1.74%)</td>
<td>(-0.41%)</td>
<td></td>
</tr>
<tr>
<td>Average revenues</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimated</td>
<td>355.78</td>
<td>301.14</td>
</tr>
<tr>
<td>Realized</td>
<td>374.09</td>
<td>334.92</td>
</tr>
<tr>
<td>(-4.89%)</td>
<td>(-10.09%)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimated median and average unitary revenues for Charitystars. Revenues in Euro are computed in multiple steps. (i) Subtract the median number of bidders times its estimated coefficient in the OLS regression (6.1) from the fitted values of the same regression. (ii) Compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 6.1 ($F(\cdot), \alpha, \beta$). (iii) Sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. Realized revenues are determined at the median number of bidders for each auction type. The covariates used in (6.1) include the total number of bids as in Appendix E.3.1.

Another way to validate the estimates in Table 6 is to simulate bids and try to match them with some stylized facts highlighted in Section 3. A striking result from that section was the linearity between prices and donations. Figure 5b replicates this finding by plotting the simulated bids (y axis) for different percentages donated (x axis). Remarkably, the three curves (corresponding to the bids placed by bidders with values at the first three quartiles of the estimated $F(\cdot)$) are linear and increasing in $q$ (compare with Table 3 and Figure 16).

A direct test to identification is to apply the estimates to the 78% auctions and check whether the implied distribution of values differ from that estimated in Section 6.2. This can be done

44The computation of the expected revenues is standard. First, to find the expected (or median) revenues from the homogenized auctions I integrate the bidding function (equation 4.2) with respect to the distribution of the second highest-bid, yielding $p^e = \int b(t; \hat{\alpha}, \hat{\beta})dF^{(2)}(t)$. Second, I evaluate the entity of the cross-auctions heterogeneity as $\bar{X} = \log(\text{price}) - \hat{\gamma} \cdot \log(\text{bidders})$, where $\hat{\gamma}$ indicates the fitted (or estimated in case of $\gamma$) value of each covariate $x$ in (6.1). Finally, the expected (or median) revenues is the sum of $p^e$ and the average (or median) value of $\bar{X}$. In the second step, the effect of the number of bidders is subtracted from the fitted prices because this variable already affects the first step through the distribution of the second highest bid. Since the number of bidders may correlate with other unobservables, the results reported in Table 7 are obtained by including also the log(Total Number of Bids Placed) among the covariates in the first step of the estimation routine (equation 6.1). Additional robustness checks in the appendix show that adding this variable does not qualitatively change the estimated $\hat{\alpha}$ and $\hat{\beta}$ (see Appendix E.3.1).
in multiple ways. The most challenging one is to apply the full three-step procedure to the 78% data. This consists in (i) manipulating the winning bids and covariates with the coefficients estimated in the two regressions at the first-step of the estimation procedure (using the coefficient in equation 6.1). Then, (ii) the distribution of bids are computed following (6.2) (and (6.3) for the density). Finally, (iii) the private values are determined assuming the same $\hat{\alpha}$ and $\hat{\beta}$ exposed in Table 6.

Figure 5: Model fit

(a) Out-of-sample validation

\[ n = 16 \]

(b) Linearity of the bids

1st, 2nd, 3rd quartile of $F(v)$

<table>
<thead>
<tr>
<th>Bid (EUR)</th>
<th>0.1</th>
<th>0.25</th>
<th>0.4</th>
<th>0.55</th>
<th>0.7</th>
<th>0.85</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private value at Q(25%)</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Private value at Q(50%)</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Private value at Q(75%)</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>250</td>
</tr>
</tbody>
</table>

Note: Panel (a). Comparison of the density of the private values estimated from the structural model employing data from auctions with $q = \{10\%, 85\%\}$ and the density of the private values estimated by projecting the three-step estimation on the $q = 78\%$ auctions. The null hypothesis (equality) cannot be rejected at standard level. The computation assumes $n = 16$, but the same result can be replicated with different $n$. The plotted densities are computed using a Gaussian kernel and Silverman’s rule-of-thumb bandwidth (Silverman, 1986). Panel (b). Simulated bids computed at the first, second and third quartiles of the estimated value distribution. The density $f(v)$ is approximated using a cubic spline. Only auctions with price between €100 and €1000.

Figure 5a plots the densities from auctions at 10% (solid line) and those at 78% (dotted line) for $n = 16$. The two pdfs have similar shapes, displaying a slight bimodality at the same values. The Kolmogorov-Smirnov test does not reject the null hypothesis that the two distributions are equal at 0.1154 level.\footnote{An analogous plot is found for other values of $n$ and the KS test always fails to reject identity of the distributions.} The outcome would not change by replacing the first step with a regression on all observations with $q \in \{10\%, 78\%, 85\%\}$.

The same exercise can also be repeated on a different set of covariates. For example, Appendix E.3.1 report estimates of $\alpha$ and $\beta$ when the “Total Number Of Bids Placed” is added as a dependent variable. First, the estimates do not vary substantially from those in Table 6. Second, Figure 23 shows that the pdf from a similar out-of-sample analysis does not reject the null
hypothesis of equality of densities.  

This is a remarkable result: the distribution of private values computed using the auxiliary data cannot be distinguished from that found by matching the FOCs from other two auction datasets. This is a direct test of Assumption 2 because dependence between \( q \) and \( F(\cdot) \) would immediately return an incongruence in the estimates. A similar result would be expected in case of heterogeneous \( \alpha \) and \( \beta \). Furthermore, this test is clearly conservative as the way covariates impact prices in the auctions at 78% does not need to be the same as for the other auctions (\{10%, 85%\}). This is because auctions vary both in terms of timing (e.g., period of the year) and the object characteristics (both observable and unobservable).

These checks confirm the goodness of the estimates of \( \alpha \) and \( \beta \). These tests provide indirect support for the way nonpecuniary motives are modelled in the theoretical literature of auctions with externalities (e.g., Jehiel et al., 1999, Lu, 2012) and, more generally, revenue-dependent profits: constant marginal return to charity revenues.  

7 Counterfactual experiments on the demand side  

With the estimated primitives, the model can be used to gauge the effectiveness of CSR strategies at increasing consumers’ willingness to pay. The first question to ask is whether bids are greater than private values. To address this, I simulate private values (solid line) and bids (dotted line) and plot them in Figure 6a. The plot suggests stochastic dominance of bids on values. Thus all bidders increase their bids beyond their private values. This means that (for any given number of bidders) the expected second-highest bid from the second-price auction is larger than the second-highest private value. Hence, gross revenues to Charitystars are greater than those to a comparable non-charity auction.

The price premium is not constant. The distance between values and bids is quite large, especially for higher quantiles, as donations increase willingness to pay by a little less than \( \varepsilon 100 \) for a bidder at the 80th-percentile of the distribution (about 1/4 of her valuation). This distance widens further were consumers to receive an additional glow from winning the auction. This is evident from Figure 6b, which plots simulated bids with \( \beta = 0.8 \) instead of the estimated \( \hat{\beta} \).

The way the distribution of private values and the charitable parameters interact in shaping bids is further investigated in Appendix F. The discussion shows that bids in charity auctions would still dominate bids in standard auction even when sharply increasing the externality parameter. This is because the distribution of values determines the probability of winning the auction which control overbidding (see Lemma 2).

---

46 The p-value of the Kolmogorov-Smirnov test is 0.1874. Adding even more covariates further increases the p-value of the test statistics with no affect on the estimated \( \alpha \) and \( \beta \).

47 For example, Goeree et al. (2005) at page 906 state to “keep the constant marginal benefit assumption because it provides a tractable model [...]” despite believing that a model with diminishing returns would mirror reality better.
The charity premium is increasing in $q$ and is about 11.3% when the auctioneer donates 85% (see Figure 7a). This is close to the estimates in Elfenbein and McManus (2010) who estimated a premium of 12% when the auctioneer donates 100% using eBay’s Giving Works data. There’s almost no charity premium when the auctioneer donates little (the estimated premium when $q = 0.10$ in Elfenbein et al. (2012) is much larger at 6%). Obviously this analysis assumes that the number of bidders is constant across auctions (in this case 8). The premium increases slightly with the number of bidders, reaching about 14% when there are 16 bidders and the auctioneer donates 85%. Figure 7b shows that a greater glow would lead to much larger premiums at the right end of the distribution of the amount donated.

7.1 Net revenues in charity auctions

Despite the positive gross return to donating, the demand side analysis so far performed does not justify positive donations. In fact net revenues are decreasing almost linearly in the percentage donated (Figure 8a). If Charitystars were to sell its items without donating, it would make over €300 in net revenues, while it only brings home €53 when it donates 85%. Interestingly, while bidders are much more aggressive when the glow is greater, as indicated by Figure 7b, these higher revenues should not induce the auctioneer to donate (compare the solid and the dotted line in Figure 8a). Even increasing the number of bidders does not make charity auctions more appealing than non-charity auctions for any positive $q$ (Figure 8b).
Figure 7: Expected prices in charity vs non-charity auctions

(a) Estimated scenario

(b) Counterfactual scenario with \( \beta = 0.8 \)

Note: The two panels report the expected gross revenues to Charitystars at the estimated parameters (solid line), and from a standard auction with no donation (dotted line). The density \( f(v) \) and the distribution \( F(v) \) are approximated using a cubic spline. Only auctions with price between \( \text{€} 100 \) and \( \text{€} 1000 \). Revenues in Euro are computed in multiple steps. (i) Subtract the median number of bidders times its estimated coefficient in the OLS regression (6.1) from the fitted values of the same regression. (ii) Compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 6.1 \( (F(\cdot), \alpha, \beta) \). (iii) Sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. The covariates used in (6.1) include the total number of bids as in Appendix E.3.1.

The suboptimality of charity auctions at creating revenues is due to the rigidity of the elasticity of prices to giving, \( \eta \). Proposition 2 states that the revenue optimal percentage donated, \( q^* \), sets the elasticity equal to the donation-ratio \( q / (1 - q) \). Figure 9a shows that these two curves intersect only at the origin. The same is true for the \( \beta = 0.8 \) case (Figure 9b).

Charitystars faces a possible deviation of \( \text{€} 250 \) in terms of higher revenues by interrupting the donations (Appendix D discusses profits when the firm keep a fixed percentage in any auction). Moreover, as investigated in Appendix G, the firm has already adopted the best mechanism to maximize its revenues across standard charity auction mechanisms (aside for a standard noncharity auction). To understand why Charitystars donates at all, the next section explores possible decreasing marginal costs in the donation as a result of bargaining between celebrities and charities.

8 The supply side and the profit-maximizing donation

One reason for Charitystars to donate is that celebrities could be more willing to provide items. Celebrities could be interested in the “greater good”. Alternatively, their giving could create more positive awareness of their social responsibility and this form of advertisement could ben-
Figure 8: Expected net revenues decrease in the donation

(a) n = 7 bidders

(b) n = 16 bidders

Note: The two panels report the expected net revenues to Charitystars (solid line) at the estimated parameters, for different number of bidders (7 in Panel (a) and 16 in Panel (b)), and the marginal cost reduction due to a marginal increase in the percentage donated in terms of the expected bid (dotted line). The density $f(v)$ and the distribution $F(v)$ are approximated using a cubic spline. Only auctions with price between €100 and €1000. Revenues in Euro are computed in multiple steps. (i) Subtract the median number of bidders times its estimated coefficient in the OLS regression (6.1) from the fitted values of the same regression. (ii) Compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 6.1 ($F(\cdot), \alpha, \beta$). (iii) Sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. The covariates used in (6.1) include the total number of bids as in Appendix E.3.1.

Note: The two panels report the expected net revenues to Charitystars (solid line) at the estimated parameters, for different number of bidders (7 in Panel (a) and 16 in Panel (b)), and the marginal cost reduction due to a marginal increase in the percentage donated in terms of the expected bid (dotted line). The density $f(v)$ and the distribution $F(v)$ are approximated using a cubic spline. Only auctions with price between €100 and €1000. Revenues in Euro are computed in multiple steps. (i) Subtract the median number of bidders times its estimated coefficient in the OLS regression (6.1) from the fitted values of the same regression. (ii) Compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 6.1 ($F(\cdot), \alpha, \beta$). (iii) Sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. The covariates used in (6.1) include the total number of bids as in Appendix E.3.1.

On one side of the platform there is a charity auction, while on the other side an item is exchanged at a cost for the platform. The firm incurs transaction costs in terms of organization and dealing with charities, celebrities, and agents which may vary based on their relative bargaining powers. For example, in certain occasion Charitystars could negotiate a payment to collectors for some objects while asking the relevant footballers to sign them at a later time. Shareholders of the company confirmed that the observed $q$ are subject to bargaining between the platform and the celebrities resulting in the observed, and that the reserve price is set by the firm to cover costs.

To further confirm the presence of bargaining, I resort to reduce form evidence by exploiting a period of the year where the relevant bargaining power is tilted toward the celebrities. I compare the donations in the inactive Summer months of July and August with those in the other months. The hypothesis is that footballers have greater bargaining power in these months because due to the stop of the major European leagues there are less objects available at this time. In fact, Charitystars sold only 24.5 objects on average in these months (SD 13.3), versus an average of 50.6 objects in the other months (SD = 30.0). Thus, Charitystars should be more generous in
Figure 9: The revenue maximizing $q$ is zero

(a) $n = 7$ bidders
(b) $n = 16$ bidders

Note: The elasticity of the bid as the percentage donated increases. The number of bidders is assumed to be 7. The density $f(v)$ and the distribution $F(v)$ are approximated using a cubic spline. Only auctions with price between €100 and €1000. Revenues in Euro are computed in multiple steps. (i) Subtract the median number of bidders times its estimated coefficient in the OLS regression (6.1) from the fitted values of the same regression. (ii) Compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 6.1 ($F(\cdot), \alpha, \beta$). (iii) Sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. The covariates used in (6.1) include the total number of bids as in Appendix E.3.1.

this period on average. This hypothesis is first confirmed by testing the difference between the $q$ for August vs the other months in the sample (Welch test: $\bar{q}^{other} = 0.698, \bar{q}^{Aug} = 0.775$, one-sided p-value 0.030) and for July and August vs the other months in the sample (Welch test: $\bar{q}^{other} = 0.693, \bar{q}^{Jul\&Aug} = 0.770$, one-sided p-value < 0.001).

Arguably the same result could originate from Charitystars’ reacting to the expectation of less bidders in the Summer by increasing $q$ (to increase the number of bidders). To control for the number of bidders, the jersey’s value and a number of other variables I perform the following OLS regressions

$$q_t = \gamma_0 + x_t \gamma + \gamma_m \text{month}_t + \varepsilon_t$$

where “month” refers to either the month of August only, or July and August. Table 8 reports the results, confirming the presence of bargaining between parties, as Charitystars is more generous when it has less bargaining power. The results are robust to the inclusion of more covariates (even columns).

48 Although Table 4 shows that variation in $q$ does not affect the extensive margin, the number of bidders is significantly smaller in the Summer months. However, this difference is not economically significant, with only 1 bidder difference on average (Welch tests: $\#\text{bidders}^{other} = 7.862, \#\text{bidders}^{Aug} = 6.519$, one-sided p-value = 0.029 and $\#\text{bidders}^{other} = 7.917, \#\text{bidders}^{Jul\&Aug} = 6.929$, one-sided p-value = 0.002). Instead, competition does not changes in the Summer, as denoted by the small variation in the total number of bids (Welch tests: $\#\text{bids}^{other} = 24.785, \#\text{bids}^{Aug} = 24.296$, p-value = 0.9134 and $\#\text{bids}^{other} = 24.970, \#\text{bids}^{Jul\&Aug} = 22.735$, p-value = 0.2902)
Table 8: Evidence of bargaining

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>August</td>
<td>0.095**</td>
<td>0.088***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.033)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>July &amp; August</td>
<td></td>
<td>0.091***</td>
<td>0.086***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.487***</td>
<td>0.391***</td>
<td>0.476***</td>
<td>0.389***</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.108)</td>
<td>(0.040)</td>
<td>(0.107)</td>
</tr>
<tr>
<td>Number of Bidders</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Total Number of Bids</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Main Variables</td>
<td>Y</td>
<td></td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>3.09%</td>
<td>27.76%</td>
<td>3.70%</td>
<td>28.23%</td>
</tr>
<tr>
<td>N</td>
<td>1,109</td>
<td>1,109</td>
<td>1,109</td>
<td>1,109</td>
</tr>
</tbody>
</table>

Note: The table presents the OLS estimate for the dummy variables August (1 if the month is August, 0 otherwise) and July and August (1 if the month is either July or August, 0 otherwise). The reserve price is not included as a covariate because it depends on the equilibrium percentage donated from the bargaining stage. Including the reserve price would not qualitatively change the results. Only auctions with price between €100 and €1000. Robust standard errors in parenthesis.

* – p < 0.1; ** – p < 0.05; *** – p < 0.01.

Instead of exploring a full bargaining problem between Charitystars and the celebrity, I focus on a simpler monopolist problem: the firm must optimize its profits by choosing the optimal percentage donated considering that costs are a decreasing function of $q$. In this sense, the cost, $c(q)$, includes the bargaining costs sustained to procure the item as well as other costs (e.g., storing the item or publishing the listing online). Appendix D shows that this marginal cost can be micro founded in terms of a simple bargaining problem between firm and celebrity. The analysis shows that the marginal cost ($c' = \frac{\partial c(q)}{\partial q}$) is decreasing in $q$ under the mild condition that the net expected price is larger than costs.

Given knowledge of the number of bidders, $\alpha, \beta$ and $F(v)$, a profit maximizing platform would donate the profit maximizing $q^*$ defined as

$$q^* = \arg \max_{q \in [0,1]} \int_v (1 - q) b(v, \alpha, \beta, q) - c(q) dF^{(2)}(v)$$

where $F^{(2)}(v)$ is the distribution of the second highest private value out of the $n$ bidders. Denoting the expected price by $p^e$ and by the dominated convergence theorem (as in the proof of Proposition 2) the equation can be written in terms of the donation-elasticity, $\eta$

$$c'(q) = \frac{1-q}{q} \eta p^e - p^e \quad (8.1)$$
The expression on the RHS is the difference between the updated price after a marginal increase in \( q \) (i.e., \( \frac{1-q}{q} \eta p^e \)) and the original price \( (p^e) \). The optimal donation sets the marginal cost reduction from a marginal increase in \( q \) equal to the reduction in marginal benefits.

Equation 8.1 can be manipulated to show that if \( c' \) is negative, \( q^* > 0 \). Denote the revenue optimal donation by \( q_R \) as the solution of \( q_R / (1 - q_R) = \eta (q_R) \) (as in Proposition 2). The demand analysis in the previous section states that to maximize net revenues Charitystars should set \( q_R = 0 \). The profit optimal donation is such that \( \left( \frac{c'(q^*)}{p'(q^*)} + 1 \right) \frac{q^*}{1-q^*} = \eta (q^*) \). Comparing the last two equations, since that the ratio in parenthesis is negative, it must be that \( q^* > q_R^* \).

How much should the firm donate? Unfortunately, costs are not observable from the website, but can be inferred from variation in the net reserve price. In order to avoid losses, Charitystars’ managers ensure that the smallest net revenue covers costs. This is equivalent to setting the reserve price, \( R \), such that \( (1 - q)R = c(q) \). The fact that Charitystars does not set the reserve price to maximize revenues and the aforementioned zero correlation between \( R \) and \( q \) corroborate this assumption (Appendix C.3). In fact, if \( R = c(q) / (1 - q) \), the correlation between \( R \) and \( q \) may be zero because both numerator and denominator decrease in \( q \).

### Table 9: Cost estimation

<table>
<thead>
<tr>
<th></th>
<th>Quadratic Cost</th>
<th>Cubic Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
<td>(II)</td>
</tr>
<tr>
<td>( \pi_0 )</td>
<td>237.58***</td>
<td>249.01**</td>
</tr>
<tr>
<td></td>
<td>(15.10)</td>
<td>(90.73)</td>
</tr>
<tr>
<td>( \pi_1 )</td>
<td>-432.35***</td>
<td>-572.24</td>
</tr>
<tr>
<td></td>
<td>(80.31)</td>
<td>(1059.12)</td>
</tr>
<tr>
<td>( \pi_2 )</td>
<td>214.86***</td>
<td>521.43</td>
</tr>
<tr>
<td></td>
<td>(78.75)</td>
<td>(2235.97)</td>
</tr>
<tr>
<td>( \pi_3 )</td>
<td>-186.06</td>
<td>-1312.63</td>
</tr>
</tbody>
</table>

|          | Adjusted R-squared | 58.22% | 58.19% |
|          | BIC                | 11,966.43 | 11,973.14 |
|          | \( N \)            | 1,108 | 1,108 |

Note: OLS regression of the homogenized reserve price on a quadratic (Column I) and a cubic (Column II) polynomial expansion of \( q \). \( \pi_0 \) is the intercept, while \( \pi_1, \pi_2 \) and \( \pi_3 \) refers to the linear \( (q) \), quadratic \( (q^2) \) and cubic term \( (q^3) \). Only auctions with price between €100 and €1000. Control variables are defined in Appendix B. Robust standard errors in parenthesis.

* \(- p < 0.1; ** - p < 0.05; *** - p < 0.01."

The reserve price accounts for unobserved heterogeneity in the structural demand model because it reflects variation in the value of the item’s characteristics across auctions through the value that the auctioneer attaches to it. In this section, the variables “Number Of Bidders” and “Number Of Total Bids Placed” account for unobserved heterogeneity, as they are realized after
Note: Panel (a) shows the optimal percentage donated as the intersection of marginal costs (dotted and dashed lines) and marginal net benefit (solid line). Panel (b) displays how profits change with $q$. The vertical line at 85% indicate the median donation by Charitystars. The number of bidders is assumed to be 7. The density $f(v)$ and the distribution $F(v)$ are approximated using a cubic spline. Only auctions with price between €100 and €1000. Marginal revenues in Euro are computed in multiple steps. (i) Subtract the median number of bidders times its estimated coefficient in the OLS regression (6.1) from the fitted values of the same regression. (ii) Compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 6.1 ($F(\cdot), \alpha, \beta$). (iii) Sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. The covariates used in (6.1) and (8.2) include the total number of bids as in Appendix E.3.1.

setting the reserve price. For example, higher quality items will receive attention by a larger and more aggressive crowd. As long as additional unobservables (uncorrelated with the number of bidders or the number of bids) are uncorrelated with $q$, the marginal cost is consistently estimated. This means that unobservables should not affect the way $q$ is chosen.\footnote{To assess the relation between unobserved heterogeneity and $q$ in the data, I study how the residual from the control function approach in the first step of the estimation in Section 6.1 vary with $q$. The Spearman’s rank correlation coefficient between the estimated unobserved heterogeneity and $q$ is only -0.0666 (Pearson: -0.0714). Had the choice of $q$ depended on unobservables, a much larger correlation coefficient would have been expected. As a robustness check, running a similar analysis after including more regressors to better account for cross-auction heterogeneity does not affect the conclusions drawn from this analysis (see Figure 15 in Appendix D).}

To take the model to the data, the reserve price is homogenized by regressing it on the same covariates previously used in the estimation procedure

$$\log(\text{reserve price}_t) = \mathbf{x}_t \gamma + \epsilon_t$$  \hspace{1cm} (8.2)
are equal to the net homogenized reserve price \((nr_t = (1 - q)r_t)\). Marginal costs are derived by smoothing \(nr_t\) over \(q \in (0, 1)\) using a polynomial expansion for \(q\).

Table 9 gives the estimated coefficients using either a quadratic or a cubic functional form for costs. Comparing the two columns, the estimated intercept \((\pi_0)\) and linear coefficient \((\pi_1)\) are similar, while the introduction of the cubic term \((\pi_3)\) expands the size of the quadratic coefficient \((\pi_2)\) in the second column. Although the coefficients are similar across columns, adding the cubic term inflates the standard errors in column (II). This depends on a small number of auctions for \(q\) in a neighborhood of 50% (Figure 1a). Nevertheless, both specifications seem to fit the data fairly well as they explain a large portion of the variance of the net reserve price (Adjusted \(R^2 \sim 60\%\)).

Figure 10a plots marginal costs and marginal benefits, demonstrating that Charitystars should indeed donate a share of its revenues in order to maximize profits. The estimated marginal costs are negative across both specifications for all \(q\), empirically verifying the intuition of decreasing costs. In addition, a marginal increase in \(q\) seems to decrease costs less when \(q\) is high, so that the cost reduction follows the logic of decreasing returns. Thus, even accounting for costs large donations cannot be justified: \(q^*\) is 30\%, not 85\%. 50 By setting the donation optimally, Charitystars would almost quadruple its profits. The median amount pocketed by Charitystars is only 15\% for each object sold, yielding an expectation of €25 of profit for each auction, as displayed in Figure 10b. The figure shows that profits would jump to €95 per auction on average with the optimal policy. In addition, profits almost double when the optimal donation is compared to the average donation (70\%).

9 Discussion

This paper studies the case of Charitystars, an online platform selling celebrities’ belongings through charity auctions. Donations impact the firm’s business in two ways. On the demand side, the firm extracts surplus from warm glow bidders who raise their bids in proportion to the percentage of revenues that the firm pledges to give to charities. On the cost side, the underlying bargaining between celebrities and the auctioneer for the provision of the auctioned items results in cost reductions when the firm donates more.

The first finding is that there exists an interior solution for the optimal percentage donated. The optimal donation is found intersecting the marginal net revenues to the auctioneer with the marginal costs. The first variable is estimated through the structural auction model developed in Sections 4 and 5. In addition, recognizing that the reserve price is not set optimally allows costs to be inferred. The satisfactory performance of the model in terms of goodness of fit and

50To test the significance of this estimate I sample with replacement all the auctions in the dataset. I use the sampled data to estimate marginal costs and the 10\% and 85\% sampled auctions to estimate marginal revenues. This process is repeated 400 times. The 99\% confidence interval for \(q^*\) is [0.035, 0.45], while the median bootstrapped \(q^*\) is 0.295. The standard deviation is 0.102.
out-of sample testing suggests that marginal net revenues and costs are consistently estimated. Therefore, donating is a profit maximizing decision for Charity Stars.

The relation between social responsibility and financial return is hotly debated. The seminal work of Margolis et al. (2007) argues that the possible correlation between these two dimensions fails to indicate the direction of causality--successful firms are as likely to engage in CSR as socially responsible firms are to become financially successful. Adopting a somewhat nuanced view of CSR as long term sustainability, Eccles et al. (2014) show that highly sustainable companies outperform other similar but non-sustainable ones in the long run. However, this result comes at the cost of a vague definition of CSR, which could include firms that consumers do not deem socially responsible. This problem is so significant that it even plagues standard measures of CSR (Chatterji et al., 2009). Examples of this problem are firms like BP and Volkswagen who engaged in expensive CSR programs after being found guilty of environmental violations, or firms donating to charities linked to politicians as a form of lobbying (Bertrand et al., 2018).

Donating to charities is one of the most common types of CSR programs, across large conglomerates and start-ups. Thanks to Charitystars’ business, this paper isolates CSR from additional concerns generally affecting CSR studies and shows that Charitystars can quadruple its profits by donating the optimal amount (see Figure 10b). Thus, social impact is conducive of higher profits, a result that, with different magnitudes, could be extended to similar firms.

This large effect relies on the dovetailed role of giving, which impacts both willingness to pay and, importantly, costs. The observed rigidity of the elasticity of prices implies that willingness to pay does not react markedly to incremental changes in Charitystars’ giving. As a result, net revenues slip below non-charity ones. Keeping costs fixed, reverting to non-charity auctions would maximize profits. This could be the destiny of Toms Shoes, whose case was briefly discussed in the introduction. Recent news report that the firm’s strategy of gaining consumers by donating a pair of shoes for each pair sold is not paying off, as sales do not balance operating costs. Comparing Toms with Charitystars, both firms face a rigid elasticity of demand. However, unlike Toms, Charitystars’ production technology enjoys decreasing costs with each euro donated while Toms must produce two pairs of shoes for each pair sold.

Donations and money-making activities are clearly non-separable for Charitystars. If the firm were to interrupt its donations to charities the firm would either bear higher procurement costs,

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51 Similarly, Edmans (2011) finds higher financial returns for the 100 best firms to work for in America.

52 While reports show that in 2014, CEO and Founder Blake Mycoskie sold 50% of Toms to the venture capital firm Bain Capital for $625m, performance have not met the expectations (more information on the sale to Bain is available at https://tinyurl.com/ybthmpdm), as the company currently shoulders a huge debt and has troubles attracting new customers. For more information see Bloomberg at https://tinyurl.com/y8qubl47 and the Wall Street Journal at https://tinyurl.com/y8s737bb.

53 The elasticity of demand is not rigid in all situations. In a natural experiment at a famous amusement park, Gneezy et al. (2010) finds no significant impact of donations when the price is fixed. The adoption of a pay-what-you-want pricing policy in connection with 50% donation resulted in much greater revenues. In a way, pay-what-you-want policies are similar to auctions. However, as shown in Section 4 the competition inherent to charity auctions can negatively affect pricing, reducing the price elasticity.
or even be unable to obtain the celebrities’ belongings it would like to sell. According to Hart et al. (2017), this non-separability places shareholders’ welfare maximization before market value. CSR and money-making activities would be separable if instead Charitystars’ donations had no impact on the production technology (e.g., celebrities were indifferent to the donations). This is the case of Toms: if the firm were not to donate shoes, its shareholders could do so.54

This discussion leads directly to the second main finding: Charitystars is not maximizing profits. The firm donates beyond what is optimal, leaving 3/4 of the value of its optimal profits to charities and consumers. As Charitystars, a for-profit firm, aims to boost charities’ fundraising activities by means of charity auctions, the large donations observed in the data suggest that the firm may actually care for how much it donates. Thus, Charitystars’ objective function may include elements other than profits, like the amount it donates.

The case of Charitystars offers first evidence that firms can choose to not maximize profits (Romer, 2006, Massey and Thaler, 2013, Kolstad, 2013). This does not mean that profit maximization is a faulty assumption in the literature. Managers at Du Pont, a large chemical corporation, offer a fitting example. Shapira and Zingales (2017) studied the choice of the company to not invest in pollution abatement technologies for its Teflon production plants in West Virginia in the 80s. The authors judged such a non-socially responsible decision to be profit maximizing if the probability of a law suit was small enough and argue that this was indeed the case.

Yet, the revived interest in social responsibility disputes this point (Bénabou and Tirole, 2010). Recent trends across markets and business owners report increasing concerns for charitable donations and environmentally friendly technologies. This could ultimately sway some managers away from profit maximization. The recent introduction of new legal corporate forms designed for for-profit entities also wishing to serve a broader “social purpose” is a point in case (Talley, 2012). According to the status quo, shareholders can sue managers who fail to maximize profits. This new legal paradigm shelters the managers of responsible firms to achieve social impact. This novelty opens interesting scenarios on the economic limits of this type of fiduciary duty.

In addition, a greater social impact may be required by socially responsible shareholders who also demand the company they invest in to behave accordingly (Riedl and Smeets, 2017). Firms with large social impact could have an easier access to capital. Previous research documents this for certain private equity funds (Barber et al., 2018) and for firms dedicated to long term sustainability (Cheng et al., 2014). Easier access to funds could protect responsible firms from market forces. In this case, consideration of the optimal donation should also include an assessment of the cost of capital.

Still, there are also more classic explanations for Charitystars’s suboptimal decision. A limitation of the analysis is that it only considers a static environment. Accounting for dynamics, the

54Hart et al. (2017) argues in favor of shareholders’ value maximization if either the two activities are not separable, if governments do not fully internalize all externalities generated by the production or if shareholders are not prosocial.
firm could donate more than optimal in its early years to attract more bidders. In fact, a classic result in auctions is that revenues increase with the number of bidders (Klemperer and Bulow, 1996). Yet, as highlighted by the reduced form analyses, higher donations are not correlated with more bidders in Charitystars’ auctions, reducing the case for high donations. In addition, donating too much reduces cash flow, which are fundamental for start-ups to survive without resorting to external capital. Moreover, Charitystars is a de facto monopolist in the e-commerce of VIP’s belonging for charity because of the large number and variety of auctions it offers. This differentiates it from eBay’s Giving Works, the other largest charity auction player, where many sellers and consumers trade over a vast array of item categories. There are also large barriers to entry in Charitystars’ business due to the need for a large net of contacts with celebrities on one side, and for a great number of users on the other. Furthermore, consumers’ inertia in search markets may raise the barrier to entry even higher (Hortaçsu et al., 2017b).

Another possibility is that the firm fails to properly assess the elasticity of prices. This explanation is in disagreement with the presence of institutional shareholders such as venture capital funds and angel investors. These investors are interested in quickly setting the firm on a path for profitability in order to resell its shares for a capital gain, and are generally well-equipped in terms of data analysis tools (Bloom et al., 2015).

A more likely reason for Charitystars’ large donations could be that it is optimal for other areas of the business, such as selling a painting, or a dinner with a CEO. There could be a cost for the firm to set multiple donation percentages for different business areas (DellaVigna and Gentzkow, 2017). Nevertheless, soccer jerseys is the most active area of the Charitystars’ business and should probably have priority. While none of these hypothesis can be ruled out with the data at hand, the intuition that Charitystars cares also for how much it donates, as its bidders do, seems to be more plausible.

Methodologically, this paper performs an econometric analysis of a platform, as Charitystars’ profits depend on the number of agents on each side of the market (bidders and footballers). Similar analysis could be conducted on other firms facing a similar trade-off in oligopolistic markets. To estimate demand, the researcher would likely resort to a BLP approach, and employ prices and donations of competitors as instruments to infer consumers’ willingness to pay (Berry et al., 1995). In order to discuss profits, cost shifters are also needed to estimate marginal costs. In this regards, Charitystars’ auctions are a unique environment where prices are set endogenously by bidders and where procurement cost information is available through the reserve price.

This paper also contributes to the auction literature both theoretically and empirically. First,

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55 Charitybuzz, a large American charity auction website, is not present in Europe, which is Charitystars’ main market, and does not offer charity auctions of soccer jerseys.

56 For instance, the cement industry accounts for 5-7% of greenhouse emissions globally. The introduction and diffusion of new technologies capable of capturing CO₂ emissions from concrete production and infusing it back in the concrete as a substitute for a portion of cement help curbing emissions while reducing production costs (e.g., Hasanbeigi et al., 2012, Lord, 2018). Socially responsible construction firms may be solving a similar problem as that of Charitystars’ managers.

45
the paper extends a charity auction model to the case where the auctioneer donates only a percentage of the price (Engers and McManus, 2007). The comparative statics prove that charity auctions can result in lower prices than in standard auctions, which can account for the different estimates in the empirical literature (e.g., Carpenter et al., 2008, Elfenbein and McManus, 2010). Moreover, the analysis provides conditions under which it is optimal for the auctioneer to be charitable. Second, the paper introduces tools for the identification and estimation of structural auction models in the presence of cross-bidders externalities (e.g., Kuehn, 2016), by building on results from auctions with risk averse bidders (e.g., Guerre et al., 2009). The paper also performs a direct test of the restriction necessary for identification. Third, the structural estimation shows that bidders’ behavior is well represented by a warm glow model, and discusses how charitable motives affect bidding and outcomes to the auctioneer (Bénabou and Tirole, 2006).

Finally, the empirical analysis in this paper can be translated to other markets where there are subsidies paid to bidders (e.g., Athey et al., 2013) or where an agent’s utility is affected by the actions of the others (e.g., Bresnahan, 1982, Sullivan, 2017). These conditions hold in several environments. For instance, in corporate buyouts, Singh (1998) and Bulow et al. (1999) illustrate how owning shares of the target company pushes a bidder to bid more aggressively as a high sale price increases the value of her toehold. Similar externalities are also present in bidding rings and cartels, as bidders internalize the outcome of their coalition partners.

10 Conclusion

A common assumption in economics is that firms are profit maximizers. This assumption is important for several reasons, ranging from market efficiency to economic growth. Profit maximization is also a theoretical cornerstone that helps empirical researchers identifying unobserved quantities, such as costs and valuations. Nonetheless, this assumption is hardly discussed empirically. To fill this gap, this paper considers the operation of an international for-profit firm offering charity auctions. The analysis encompasses both the demand side, estimating an auction with externalities, and the supply side, which accounts for costs. Counterfactuals provide direct micro-level evidence showing that profits increase with CSR activities as it is optimal for the firm to donate a portion of its revenues. Yet, Charystars donates about 55% more than what is optimal suggesting that firms do not always maximize profits (e.g., DellaVigna and Gentzkow, 2017). This indicates that a firm’s strategy can encompass a combination of profitability and shareholders’ welfare (Romer, 2006, Massey and Thaler, 2013, Hart et al., 2017).
References


Appendix

Auction webpage

Figure 11: A screenshot of the webpage for a listing at the time of data collection (2016-2017)

Note: Screenshot of a webpage of a running auction on Charitystars.com for an AC Milan jersey worn and signed by the player Giacomo Bonaventura. The standing price is GBP 110: this bid was placed by an Italian bidder with username “Supermanfra”. A total of five bids are placed at this point. Although at the current highest bid the reserve price is not met, this can change by the end of the auction. The auction will be active for other 3 days and 18 hours and will expire on June 7th at 7AM. 85% of the proceeds will be donated to “Play for Change”.
B  Data description

The data was collected using a Python script that searched a list of keywords across the pages dedicated to soccer items on the company website, available at http://www.charitystars.com. The script would gather all the available information regarding the auction, the charity, the listing and the bid history. This information was augmented with data from other sources. For example, a similar python script was used to recover footballers’ quality scores from a renowned videogame (FIFA). Information on each charity’s mission was obtained from both Charitiesstars as well as each charity’s website.

The analysis considers only a subset of the available auctions (see Section 2), according to the following conditions: (i) transaction prices higher than the reserve price, (ii) reserve prices greater than zero, (iii) two or more bidders, (iv) minimum increment is within €25 and (v) maximum donation of 85% of the final price.

B.1  Description of the variables

The regressions in Section 2 and in the Appendix display only some of the actual variables used in the analyses due to space limitations. These variables were distinguished in four groups based on their relevance.

1. Main Variables: these are the variables used in all regression tables and in the structural model. They are listed in Table 10 and their meaning is described by their label. Some variables whose meaning is not immediately clear are described in the following list:

- The variable Length counts the number of days between the first bid and the closing date (the listing date of the auction is unknown);
- The dummy Extended time is 1 if two or more bidders placed a bid in the last minutes of the auction. In this case the time is extended until all but 1 bidders drop out
- Auctions within 3 weeks (same team) counts the number of auctions listing jerseys of the same team as the one of the auctioned item. It only includes auctions within a 3 week window from the end of the auction. It considers all auctions not only those with final price larger than €100 and smaller than €1000;
- Auctions up to 2 weeks ago (same player/team) counts the number of auctions for a jersey worn by the same player playing with the same team in the same year as the match of the jersey that is auctioned. It considers all the listings up to 2 weeks from the end of the auction (Charitystars’ auctions last between 1 and 2 weeks). It considers all auctions not only those with final price larger than €100 and smaller than €1000;
- Count auctions same charity is a progressive count of the number of listings for each charity;
- The dummy Player belongs to FIFA 100 list is 1 if the player is in the FIFA 100 list (the list of the best soccer players ever);
- The variable Number of goals scored is equal to the number of goals scored by the player with the auctioned jersey in a particular match if this number is mentioned in the content of the listing. It is zero otherwise.
- The dummy Player belongs to an important team is 1 if the player plays for one of the following teams (alphabetic order): AC Milan, Argentina, Arsenal FC, AS Roma, Atletico de Madrid, Barcelona FC, Bayern Munich, Belgium, Borussia Dortmund, Brazil, Chelsea FC, Colombia, England, FC Internazionale, France, Germany, Italia, Juventus FC, Liverpool FC, Manchester
City, Manchester United, Netherlands, PSG, Real Madrid, Sevilla FC, Spain, SS Lazio, SSC Napoli, Uruguay.  

2. **Add. Charity Dummies:** this group includes dummies for heterogeneity across charities based on the mission of each charity. These dummies are not exclusive bins as most charities do more than only one activity. There are 101 different charities in total. The dummy variables used

- *Helping disables* for charities involved in assistance to disable subjects;
- *Infrastructures in developing countries* for charities building infrastructures in developing countries;
- *Healthcare* for charities dealing with healthcare and health research;
- *Humanitarian scopes in developing countries* for charities helping people in situation of poverty and undernourishment;
- *Children’s wellbeing* for charities providing activities (e.g., education and sport) to the youth;
- *Neurodegenerative disorders* for charities helping those suffering from neurodegenerative disorders;
- *Charity belongs to the soccer team* are charities linked to a football team;
- *Improving access to sport* charities give opportunity of integration through sport activities;
- *Italian charity*
- *English charity*

3. **League/Match Dummies:** these are dummies for soccer league and match heterogeneity. They include:

- Dummies for each major competition (**Champions League**, **Europa League**, **Serie A**, **Italian Cup**, **Premier League**, **La Liga**, **Copa del Rey**, **European Supercup**, **Italian Supercup**, **Spanish Supercup**, **UEFA European Championship**, **Qualifications to UEFA European Championship**, **World Cup**, **Qualification to the World Cup**
- Dummies weather the listings mentions that the match was won (**Won**) and for unofficial/replica jerseys which are not worn but only signed (**Unofficial**).

4. **Time Dummies:** this group include **Month** dummies (12 variables) and **Year** dummies (2 variables).

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1In unreported analysis player quality was accounted also using the players’ evaluation from the videogame FIFA. These variables do not affect prices significantly and are dropped.

2All remaining competitions are treated as friendly matches.
Table 10: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Q(25%)</th>
<th>Q(50%)</th>
<th>Q(75%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Auction characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Percentage donated (q)</td>
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<td>0.27</td>
<td>0.78</td>
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<tr>
<td>Transaction price in €</td>
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<td>187.50</td>
<td>222.00</td>
<td>315.00</td>
<td>452.50</td>
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<td>Reserve price in €</td>
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<td>132.02</td>
<td>100.00</td>
<td>145.00</td>
<td>210.00</td>
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<tr>
<td>Minimum increment in €</td>
<td>1.71</td>
<td>3.15</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Number of bidders</td>
<td>7.83</td>
<td>3.27</td>
<td>5.00</td>
<td>7.00</td>
<td>10.00</td>
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<td>Sold at reserve price (d)</td>
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<td>0.20</td>
<td>0.00</td>
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</tr>
<tr>
<td>Length (in # days)</td>
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<td>3.07</td>
<td>7.00</td>
<td>7.00</td>
<td>7.00</td>
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<td>Extended time (d)</td>
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<td>0.50</td>
<td>0.00</td>
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<td>1.00</td>
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<td><strong>Web-listing details</strong></td>
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<tr>
<td>Length of description (in # words)</td>
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<td>42.11</td>
<td>123.00</td>
<td>140.00</td>
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<td>7.00</td>
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<td>Auctions within 3 weeks (same team)</td>
<td>1.46</td>
<td>4.08</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Auctions up to 2 weeks ago (same player/team)</td>
<td>5.06</td>
<td>7.92</td>
<td>0.00</td>
<td>1.00</td>
<td>7.00</td>
</tr>
<tr>
<td>Count auctions same charity</td>
<td>128.35</td>
<td>151.94</td>
<td>14.00</td>
<td>54.00</td>
<td>216.50</td>
</tr>
<tr>
<td><strong>Player and match characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player belongs to FIFA 100 list (d)</td>
<td>0.11</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Unwashed jersey (d)</td>
<td>0.09</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Jersey is signed (d)</td>
<td>0.52</td>
<td>0.50</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Jersey is signed by the team players/coach (d)</td>
<td>0.06</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Jersey worn during a final (d)</td>
<td>0.03</td>
<td>0.17</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Number of goals scored</td>
<td>0.03</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Player belongs to an important team (d)</td>
<td>0.88</td>
<td>0.32</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Charity provenience</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charity is Italian (d)</td>
<td>0.90</td>
<td>0.29</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Charity is English (d)</td>
<td>0.08</td>
<td>0.28</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Charity’s activity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helping disables (d)</td>
<td>0.35</td>
<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Infrastructures in developing countries (d)</td>
<td>0.09</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Healthcare (d)</td>
<td>0.23</td>
<td>0.42</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Humanitarian scopes in developing countries (d)</td>
<td>0.14</td>
<td>0.34</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Children’s wellbeing (d)</td>
<td>0.84</td>
<td>0.36</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Neurodegenerative disorders (d)</td>
<td>0.06</td>
<td>0.23</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Charity belongs to the soccer team (d)</td>
<td>0.10</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Improving access to sport (d)</td>
<td>0.63</td>
<td>0.48</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Overview of the main covariates used in all specifications in the reduced form analysis and in the structural model. (d) stands for dummies. Only auctions with price between €100 and €1000. Prices are in Euro. If the listing was in GBP the final price was converted in Euro using the exchange rate of the last auction day.
B.2 Prices and revenues

This section reports the density plots for transaction prices and reserve prices for the most common auction types.

Figure 12: Density of key variables by percentage donated

(a) Transaction Price  
\[ p \in (100, 1000) \]

(b) Reserve Price  
\[ p \in (100, 1000) \]

Note: Panel (a) and Panel (b) show the density of the transaction price and reserve price respectively for selected auctions. The plotted densities are computed using a Gaussian kernel and Silverman’s rule-of-thumb bandwidths (Silverman, 1986).
C Omitted proofs

This section outlines the proofs that are omitted in the text. Lemmas C1, C2 and C3 report ancillary results that are useful for the other proofs.

C.1 Lemma C1

Lemma C1. \( \lim_{x \to 0} x \log x = 0. \)

Proof: \( \lim_{x \to 0} x \log x = \lim_{x \to 0} \frac{\log x}{1/x}. \) Applying L’Hospital’s Rule \( \lim_{x \to 0} \frac{1/x}{-1/x^2} = \lim_{x \to 0} -x = 0. \)

C.2 Proof of Lemma 1

The equilibrium bid in a symmetric second-price charity auction where the auctioneer donates \( q \) is:

\[
b^*(v; \alpha, \beta, q) = \begin{cases} 
\frac{1}{1+q/(\alpha-\beta)} \left( v + \int_v^{\infty} \left( 1-F(x) \right)^{1+q/\beta} \frac{dF(x)}{dx} \right) dx & \text{if } \alpha > 0 \land q > 0 \\
\frac{v}{1-q/\beta} & \text{if } \alpha = 0 \lor q = 0
\end{cases}
\]

Proof: A similar proof can also be found in Engers and McManus (2007). This proof is only reported for completeness. All the following results hold for \( 0 < q \leq 1 \). Let \( F^{(k)}(v) \) be distribution of the \( k \)th highest element out of \( n \); the problem in (4.1) can be rewritten as

\[
\mathbb{E}[u(v; \alpha, \beta, q)] = \int_v^{\infty} \left[ v - (1-q\beta_i) b(u) \right] dF(u)^{n-1}
\]

\[
\begin{cases} 
i \text{ wins and pays } b^{II} \\
i \text{ loses and bids } b_i = b^{II} \\
i \text{ loses and price } p = b^{II} > b_i
\end{cases}
\]

Here \( F^{(2)}(u) = F(u)^{n-1} + (n-1)F(u)^{n-2}(1-F(u)) \) is the distribution of the second highest value, given that the number of agents is \( n-1 \) because bidder \( i \) is counted out as his bid is below \( b^{II} \). The equilibrium bidding function is found where \( \frac{d\mathbb{E}[u(v; \alpha, \beta, q)]}{ds} \bigg|_{s=v} = 0. \)

\[
\begin{align*}
\frac{d\mathbb{E}[u(v; \alpha, \beta, q)]}{ds} \bigg|_{s=v} &= 0 \\
&= v \frac{\partial F(s)^{n-1}}{\partial s} + b(q\beta - 1) \frac{\partial F(s)^{n-1}}{\partial s} + qab(n-2) \frac{\partial F(s)^{n-1}}{\partial s} \frac{1-F(s)}{F(s)} \\
&\quad + qab'(n-1)[1-F(s)]F(s)^{n-2} - qab \frac{\partial F(s)^{n-1}}{\partial s} \\
&\quad - qab(n-2) \frac{\partial F(s)^{n-1}}{\partial s} \frac{1-F(s)}{F(s)} = 0
\end{align*}
\]
After deleting and moving terms,

\[ vf(v) = (1 + q\alpha - q\beta)b(v)f(v) - qab'(v)[1 - F(v)] \]  \hspace{1cm} (C.1)

This differential equation can be finally solved by multiplying both sides of (C.1) by \(-\frac{1 - F(s)}{qa}\) and integrating.

\[-vf(v)^{\frac{1 - q\beta}{qa}} = -\frac{1 + q\alpha - q\beta}{qa}b(v)[1 - F(v)]^{\frac{1 - q\beta}{qa}} + b'[1 - F(v)]^{\frac{1 + q\alpha - q\beta}{qa}} \]

\[
\frac{1}{q\alpha} \int_0^\sigma t [1 - F(t)]^{\frac{1 - q\beta}{qa}} dF(t) = \int_0^\sigma \frac{\partial b}{\partial t}[1 - F(t)]^{\frac{1 + q\alpha - q\beta}{qa}} dt
\]

This leads to the symmetric bidding function:

\[
b^*(v; \alpha, \beta, q) = \begin{cases} 
\frac{1}{q\alpha} \int_0^\sigma t[1 - F(t)]^{\frac{1 - q\beta}{qa}} dF(t) & \text{if } qa > 0 \land q > 0 \\
\frac{1}{1 - q\beta} & \text{if } q = 0 \lor q = 0 \end{cases}
\]

(C.2)

Note that the constant of integration that arises after integrating the RHS of the FOC is 0 (to see this, check the bidding function when \(v = \sigma\)). Further integrating the integral in the top row by parts yields (4.2):

\[
b^*(v; \alpha, \beta, q) = \frac{1}{1 + q \cdot (\alpha - \beta)} \left\{ v + \int_v^\sigma \frac{1 - F(x)}{1 - F(v)}^{\frac{1 - q\beta}{qa} + 1} dx \right\}
\]

which collapses to \(\frac{v}{1 - q\beta}\) when \(\alpha \) or \(q\) are zero. Finally, this is a legitimate bidding function because it is increasing in \(v\):

\[
\frac{\partial b^*(v; \alpha, \beta, q)}{\partial v} = \begin{cases} 
\frac{1}{q\alpha} \int_v^\sigma \frac{1 - F(x)}{1 - F(v)}^{\frac{1 - q\beta}{qa} + 1} dx \frac{f(v)}{1 - F(v)} > 0 & \text{if } \alpha > 0 \land q > 0 \\
\frac{1}{1 - q\beta} > 0 & \text{if } \alpha = 0 \lor q = 0 \end{cases}
\]

(C.3)

\[\boxed{\blacksquare}\]

### C.3 Reserve price

Engers and McManus (2007) discuss the optimal reserve price and participation fee across the most common sealed-bid charity auctions. This section studies the optimal reserve price, \(r^*\), (with no participation fee) in second-price charity auctions, and shows that it is increasing in \(q\). Following Engers and McManus (2007), there exists a unique threshold type \(\hat{\upsilon} \in (\upsilon, \sigma)\), so that bidders do not bid for \(v < \hat{\upsilon}\) and bidders bid \(b^*(v; \alpha, \beta, q) > r\) in \(v \geq \hat{\upsilon}\). This \(\hat{\upsilon}\) is defined by (see Engers and McManus, 2007, page 968):

\[
[(1 - q\beta)\upsilon - \hat{\upsilon}]F(\hat{\upsilon})^{n-1} = q\alpha(n - 1)F(\hat{\upsilon})^{n-2}[1 - F(\hat{\upsilon})][b^*(\hat{\upsilon}; \alpha, \beta, q) - r]
\]

An increase in \(r\) or a decrease in \(q\) are associated with an increase in \(\hat{\upsilon}\) to keep the equation balanced. Thus \(\partial \hat{\upsilon} / \partial r > 0\) and \(\partial \hat{\upsilon} / \partial q < 0\). To simplify the analysis, I assume that the threshold bidder \(\hat{\upsilon}\) bids exactly \(r\) so that \(b(\hat{\upsilon}; \alpha, \beta, q) = r\). This is a fair assumption in an English auction, especially if the parallel with the button auction is maintained. This assumption implies that \(\partial^2 \hat{\upsilon} / \partial r \partial q < 0\), which means that in case of a simultaneous increase in \(q\) and \(r\), the sign of this derivative is dominated by the effect of \(q\).
If the auctioneer maximizes gross revenues, it will set \( r \) such that (conditional on \( q \)):

\[
r^* = \arg \max_r \left[ \int_{\hat{\theta}}^{\theta} F(\hat{\theta})^{-1}r + \int_{\hat{\theta}}^{\theta} b^*(x) dF_{\theta-1}(x)f(v) dv \right] = \arg \max_r \left[ (1 - F(\theta))F(\theta)^{-1}r + \int_{\hat{\theta}}^{\theta} b^*(v)[1 - F(v)](n - 1)F_{\theta-2}(v)f(v) dv \right]
\]

The first order conditions can be rearranged to yield

\[
r^* = \frac{1 - F(\theta)}{f(\theta)} \left( \frac{\partial \theta}{\partial r} \right)^{-1}
\]

In conclusion, \( r^* \) is increasing in \( q \) because of the increasing hazard rate assumption and because \( \partial \theta / \partial q < 0 \). The analysis would not have changed if the auctioneer were to set the reserve price to maximize net revenues (i.e., the net revenue factor \((1 - q)\) cancels out in the FOC as it does not depend on \( n \)).

### C.4 Lemma C2 and Lemma C3

The following lemmas show conditions for \( b(v) = v \) when \( \alpha \geq \beta \) and \( b(v) = v/(1 - q\beta) \) when \( \alpha < \beta \). These results are necessary to prove both Lemma 2 and Lemma 3.

**Lemma C2.** Assume that \( \alpha \geq \beta \). The bid function \( b(v) \) crosses the 45° line only once if \( b(v) \geq \tilde{v} \) and either never or two times if \( b(v) < \tilde{v} \).

**Proof:** First, I focus on the case \( b(\tilde{v}) \geq \tilde{v} \). The bid function evaluated at the upper bound, \( b(\bar{v}) = \frac{\bar{v}}{1 + q\alpha - q\beta} \) implies that \( b(\bar{v}) < \bar{v} \). Thus, given the requirement that \( b(\bar{v}) < \bar{v} \), one need to show that there exists only one \( \hat{\theta} \) such that \( b(\hat{\theta}) \leq \hat{\theta} \forall \hat{\theta} \in [v, \hat{\theta}] \) and \( b(\hat{\theta}) < \hat{\theta} \), then \( b(\hat{v}) < \hat{v} \forall \hat{v} \in (\hat{\theta}, \bar{v}] \).

The following condition holds at \( b(\hat{v}) = \hat{v} \):

\[
(\alpha - \beta)q = \frac{1}{\hat{v}} \int_{\hat{v}}^{\hat{\theta}} \left( \frac{1 - F(x)}{1 - F(\hat{v})} \right)^{\frac{1 - q\beta}{q\alpha} + 1} dx
\]

which is obtained substituting \( b(\hat{v}) = \hat{v} \) in the LHS of (4.2). Multiplying both sides of the equation by \( (1/q\alpha) \cdot f(\hat{v}) / [1 - F(\hat{v})] \) gives

\[
\frac{1}{q\alpha} \frac{f(\hat{v})}{1 - F(\hat{v})} (\alpha - \beta)q = \frac{1}{\hat{v}} \frac{\partial b(\hat{v})}{\partial \hat{v}}
\]

where the RHS includes (C.3).

Assume that there are three values \( v_1 < v_2 < v_3 \) such that the bid computed at each value is equal to the value itself. Given that \( b(v) \) is differentiable and \( b(v) \neq v \) for \( v \notin \{v_1, v_2, v_3\} \), it must be that \( b(v) < v \) for \( v \in (v_1, v_2) \) and \( v \in (v_3, \bar{v}] \) and \( b(v) > v \) for \( v \in (\tilde{v}, v_1) \) and \( v \in (v_2, v_3) \). The increasing hazard rate

---

3This result holds also if the auctioneer’s objective is to maximize profits. In this case, the cost of procuring an item, \( c \), is sunk; the auctioneer sustain \( c \) independently of the outcome of the auction. Therefore, \( c \) cannot affect an optimally set reserve price.

4This result is obtained by applying L’Hospital’s rule to \( \int_{\hat{v}}^{\hat{\theta}} \left( \frac{1 - F(x)}{1 - F(\hat{v})} \right)^{\frac{1 - q\beta}{q\alpha} + 1} dx \) and the increasing hazard rate assumption.
property gives
\[
\frac{1}{\alpha q} \frac{f(v_3)}{1 - F(v_3)} (\alpha - \beta) q > \frac{1}{\alpha q} \frac{f(v_2)}{1 - F(v_2)} (\alpha - \beta) q
\]
Because (C.4) must hold at \(v_1, v_2\) and \(v_3\),
\[
\frac{\partial b(v_2)}{\partial v_2} \frac{\partial b(v_3)}{\partial v_3} > \frac{\partial b(v_1)}{\partial v_1} \frac{\partial b(v_2)}{\partial v_2} > 0
\]
a contradiction. In fact, while the ratio of the derivative at \(v_2\) and \(v_1\) must be larger than 1, as the curve intersects the 45° line from below the line, this cannot happen at \(v_3\) and \(v_2\), because the intersection happens from above the line. The bid function crosses the 45° line at \(v_2\) from below, while it crosses the same line from above at \(v_3\), implying
\[
\frac{\partial b(v_3)}{\partial v_3} < \frac{\partial b(v_2)}{\partial v_2}
\]
Figure 13b provides a graphical representation. Because \(v_1, v_2\) and \(v_3\) are arbitrary values, the proof holds for all \(v\). Given that \(b(\bar{v}) \geq \underline{v}, b(\bar{v}) < \bar{v}\) and because \(b(v)\) cannot cross the 45° line more than twice without violating the increasing hazard rate property, it must be that \(b(v) = v\) at most once.

The second case \((b(\bar{v}) < \underline{v})\) follows immediately from the previous derivation, given that \(b(v)\) cannot cross the 45° line more than twice without violating the increasing hazard rate assumption. This implies that \(b(v) = v\) for either two values \(v_1\) and \(v_2\) or no value at all. In this case, in order to respect the increasing hazard rate property, the bid function meets the diagonal line from below at the first cutoff and from above at the second cutoff, making a cutoff like \(v_3\) infeasible.

Lemma C3. Assume that \(\alpha < \beta\). The bid function \(b(v)\) crosses the line \(v / (1 - q\beta)\) only once if \(b(\bar{v})(1 - q\beta) \geq \underline{v}\) and either never or two times if \(b(\bar{v})(1 - q\beta) < \underline{v}\).

Proof: This proof is similar to that in Lemma C2, the bid evaluated at the upper bound is \(b(\bar{v}) = \frac{\bar{v}}{1 + qa - q\beta}\) implying that \(b(\bar{v}) < \frac{\bar{v}}{1 - q\beta}\). The remaining derivations follow immediately from the Proof of Lemma C2 by replacing the points \(v_1 < v_2 < v_3\) with the points \(\hat{v}_1 < \hat{v}_2 < \hat{v}_3\) such that \(b(v) = \frac{v}{1 - q\beta}\) for \(v \in \{\hat{v}_1, \hat{v}_2, \hat{v}_3\}\) and \(b(v) \neq \frac{v}{1 - q\beta}\) otherwise.

C.5 Proof of Lemma 2

If \(\alpha > 0\), there exists a value \(v^*\) such that all bidders with private values in \((v^*, \bar{v})\) decrease their bids after a marginal change in \(\alpha\).

Proof: Assume \(q > 0\) and let \(\tilde{\alpha} = qa\) and \(\tilde{\beta} = q\beta\). Take derivative of (4.2) w.r.t. \(q\) for \(\alpha > 0\)
\[
\frac{\partial b^*(v; \alpha > 0, \beta, q)}{\partial q} = -q \int_v^\pi \left(1 - F(x)\right)^{\frac{1 + \tilde{\alpha} - \tilde{\beta}}{\tilde{\alpha}}\frac{1}{\tilde{\beta}}\log\frac{1 - F(x)}{1 - F(\bar{v})} dx + v
\]
(C.5)
The integral is continuous and finite everywhere with respect to \(x\) by Lemma C1. The derivative will
cross the x-axis at \( v = v^* \) where the numerator is zero,

\[
- \frac{1 - \bar{\beta}}{\bar{\alpha}^2} \int_{v^*}^{\sigma} \left( \frac{1 - F(x)}{1 - F(v^*)} \right)^{1 + \bar{\alpha} - \bar{\beta}} \log \frac{1 - F(x)}{1 - F(v^*)} dx = \frac{v^* + \int_{v^*}^{\sigma} \left( 1 - F(x) \right)^{1 + \bar{\alpha} - \bar{\beta}} \frac{1}{1 - F(v^*)} dx}{1 + \bar{\alpha} - \bar{\beta}} = b(v^*)
\]

(C.6)

At \( v \neq v^* \), (C.5) is positive (negative) if the LHS in the first line of (C.6) is greater (smaller) than the RHS. To show uniqueness of \( v^* \) I focus on the derivative of (C.5) at \( v^* \). To limit some cumbersome notation, denote \( \int_{v^*}^{\sigma} \left( 1 - F(x) \right)^{1 + \bar{\alpha} - \bar{\beta}} \frac{1}{1 - F(v^*)} dx \) by \( \phi \). The derivative of (C.5) w.r.t. \( v \) at \( v^* \) is negative if

\[
- \frac{1 - \bar{\beta}}{\bar{\alpha}^2} \phi - \frac{1 - \bar{\beta}}{\bar{\alpha}^2} \frac{1 + \bar{\alpha} - \bar{\beta}}{\bar{\alpha}} \int_{v^*}^{\sigma} \left( 1 - F(x) \right)^{1 + \bar{\alpha} - \bar{\beta}} \log \frac{1 - F(x)}{1 - F(v^*)} dx \leq \frac{1}{\bar{\alpha}} \phi
\]

(C.7)

At \( v^* \), the second term on the LHS can be replaced by the RHS of (C.6). Solving this inequality gives the following condition

\[
- \frac{1 - \bar{\beta}}{\bar{\alpha}^2} \phi + \frac{v^* + \phi}{\bar{\alpha}} \leq \frac{1}{\bar{\alpha}} \phi
\]

\[
\Rightarrow (1 + \bar{\alpha} - \bar{\beta})\phi \geq \bar{\alpha}(v^* + \phi)
\]

\[
\Rightarrow b(v^*)(1 + \bar{\alpha} - \bar{\beta}) - v^* \geq \bar{\alpha}b(v^*)
\]

\[
\Rightarrow b(v^*)(1 - \bar{\beta}) \geq v^*
\]

where in the third row both sides of the inequality are divided by \( (1 + \bar{\alpha} - \bar{\beta}) \) and expressed \( \phi \) in terms of bids and values using the optimal bid function (4.2). Therefore, if the condition in (C.8) is respected, (C.5) is decreasing at \( v^* \).

The following result will be used below. The limit of (C.5) is negative for \( v \to \sigma \). In fact, repeatedly applying L’Hospital’s rule to the the limit of the first term yields

\[
\lim_{v \to \sigma} \frac{\int_{v}^{\sigma} \left( 1 - F(x) \right)^{1 + \bar{\alpha} - \bar{\beta}} \log \frac{1 - F(x)}{1 - F(v)} dx}{(1 + \bar{\alpha} - \bar{\beta})^2 (1 - F(v)^{1 + \bar{\alpha} - \bar{\beta}}) x} = 0
\]

while the limit of the last term (at the numerator) is \( \lim_{v \to \sigma} -q^v = -q^\sigma < 0 \).

There are two cases: in Case 1 \( v^* \) is unique and in Case 2 either \( v^* \) does not exist or there are two cutoffs.

**CASE 1.** The first case assumes that, depending on the parameters \( \alpha \) and \( \beta \), either \( b(\bar{v}) \geq v \) if \( \alpha \geq \beta \) or \( b(\bar{v})(1 - \bar{\beta}) \geq v \) if \( \alpha \leq \beta \). To complete the proof of uniqueness I rely on two results: (i) the limit of (C.5) is negative at the upper bound of the support of \( v \), and (ii) there is no region on the support of \( v \) such that \( b(v)(1 - \bar{\beta}) < v \) and \( b(v)(1 - \bar{\beta}) > v \) to its left and right. Therefore, once (C.5) becomes negative, it cannot switch sign from negative to positive and back to negative again, implying that if \( v^* \) exists, it is unique.

For \( \alpha \geq \beta \) Lemma C2 states that there exists at most one \( v \in [\bar{v}, \sigma] \) such that \( b(v) = \bar{v} \). Because of monotonicity, there exists at most one value of \( v \) such that \( b(v)(1 - \bar{\beta}) = v \). Lemma C3 states a similar result for the complementary case (\( \alpha < \beta \)). Denote by \( v^{**} \) the value such that \( b(v^{**})(1 - \bar{\beta}) = v^{**} \). For \( v < v^{**} \), \( b(v)(1 - \bar{\beta}) > v \) and for \( v > v^{**} \), \( b(v)(1 - \bar{\beta}) < v \). Given the two Lemmas and that (C.5) is continuous and negative when evaluated at the upperbound, then \( v^* \leq v^{**} \). To the contrary, assume that \( v^* > v^{**} \), then (C.5) must be increasing at \( v^* \). This implies that (C.5) is positive at \( \sigma \), a contradiction. This also implies that if (C.5) is negative at \( v \) it will be negative for all \( v \). Therefore, if \( v^* \) exists, it is unique and separates those who increase their bid (low-value bidders) from those who decrease it (high-value bidders).
CASE 2. The second case analyzes the complement to Case 1 (i.e., that depending on \( v \) or \( \tilde{\alpha} \)). Due to Lemma C2 and Lemma C3 there exist either no cutoff or exactly two cutoffs \( v^* \in \{ v_1^*, v_2^* \} \) such that (C.6) is increasing at \( v_1^* \) and decreasing at \( v_2^* \), with \( v_1^* < v_2^* \) because the limit of (C.5) is negative at the upper bound. In fact, it must be that \( b(v_1^*)(1 - \tilde{\beta}) < v_1^* \) and \( b(v_2^*)(1 - \tilde{\beta}) > v_2^* \).

Hence, there exists a value \( v^* \) such that (at least some) bidders with values below \( v^* \) increase their bids, while all bidders with values above \( v^* \) decrease it after a marginal increase in \( \alpha \).

C.6 Proof of Lemma 3

If \( \beta \geq \alpha \), bids are increasing in \( q \) for all bidders. If \( \alpha > \beta \), there exists a private value \( \tilde{\alpha} \) such that bidders with private values in \((\tilde{\alpha}, \tilde{\alpha})\) decrease their bids after a marginal change in \( q \).

Proof: Assume \( q > 0 \) and let \( \tilde{\alpha} = q \cdot \alpha \) and \( \tilde{\beta} = q \cdot \beta \). First we analyze the derivative of \( b(v) \) w.r.t. \( q \) when \( \alpha = 0 \), which is:

\[
\frac{\partial b^*(v; \alpha = 0, \beta, q)}{\partial q} = \frac{b_v}{(1 - \beta)^2} > 0
\]

The derivative in this case is always positive as \( \beta < 1 \) and \( q < 1 \), meaning that bids increase in \( q \).

Turn now to the derivative with respect to \( q \) when \( \alpha > 0 \). This proof has multiple steps. (i) I establish that for \( \beta \geq \alpha > 0 \) the bid is increasing in \( q \) \forall v. (ii) I focus on the remaining case \((\beta < \alpha)\) and show that the derivative of the bid w.r.t. \( q \) can have both positive and negative values. (iii) I show that, at the value such that the derivative is zero (called \( \tilde{\alpha} \)), if the bid is larger than the value, then \( b(v) \) is always decreasing. (iv) In the final step, I show conditions for the uniqueness of \( \tilde{\alpha} \). Steps (ii) - (iv) are similar to the proof of Lemma 2.

Step 1. Take derivative of (4.2) w.r.t. \( q \) for \( \alpha > 0 \) is

\[
\frac{\partial b^*(v; \alpha > 0, \beta, q)}{\partial q} = \int_{v}^{\tilde{\alpha}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{\frac{1 + \alpha - \beta}{\alpha^2}} \frac{1}{1 + \tilde{\alpha} - \beta} \log \frac{1 - F(x)}{1 - F(v)} \, dx - (\alpha - \beta) \frac{b_v}{(1 - \beta)^2}
\]

The integral is continuous and finite everywhere with respect to \( x \) by Lemma C1. Inspection of this equation reveals that is positive if \( \beta \geq \alpha \) (these refers to warm glow or pure altruism models). Therefore, bids are increasing in \( q \) if \( \beta > \alpha \) for all \( v \).

Step 2. Turn to the remaining case \( 0 < \alpha < \beta \) or shill bidders model. (C.9) crosses the x-axis at \( \tilde{\alpha} \). By rewriting (C.9), at \( \tilde{\alpha} \)

\[
- \int_{v}^{\tilde{\alpha}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{\frac{1 + \alpha - \beta}{\alpha^2}} \frac{1}{1 + \tilde{\alpha} - \beta} \log \frac{1 - F(x)}{1 - F(v)} \, dx = (\alpha - \beta) \left( \tilde{\alpha} + \int_{v}^{\tilde{\alpha}} \left( \frac{1 - F(x)}{1 - F(v)} \right)^{\frac{1 + \alpha - \beta}{\alpha^2} - 1} \, dx \right)
\]

where the second line replaces the expression to the RHS with the optimal bid function in (4.2). Thus, (C.9) can be positive or negative.

Step 3. This step shows that the derivative is decreasing at \( \tilde{\alpha} \) if \( b(\tilde{\alpha}) \geq \tilde{\alpha} \). Given that the RHS of (C.10) is increasing in \( v \) everywhere, it will also be increasing in \( v \) at \( \tilde{\alpha} \). To show that there is at most one \( \tilde{\alpha} \), it suffices to show that the LHS is a decreasing function of \( v \), at \( \tilde{\alpha} \). The derivative of the LHS w.r.t. \( v \) is

\[
- \int_{\tilde{\alpha}}^{\tilde{\beta}} \left( \frac{1 - F(x)}{1 - F(\tilde{\alpha})} \right)^{\frac{1 + \alpha - \beta}{\alpha^2} + 1} \frac{1}{1 + \tilde{\alpha} - \beta} \log \frac{1 - F(x)}{1 - F(\tilde{\alpha})} \, dx \frac{f(\tilde{\alpha})}{1 - F(\tilde{\alpha})}
\]
while the derivative of the RHS can be rewritten as

\[
\frac{(\alpha - \beta)(1 + \bar{\alpha} - \bar{\beta})}{\bar{\alpha}} \left( b(\bar{\alpha})(1 + \bar{\alpha} - \bar{\beta}) - \bar{\delta} \right) f(\bar{\delta})
\]

Putting together (C.11) and (C.12) and using (C.10) to rewrite (C.11) in terms of bids and values, (C.9) is decreasing at \( \bar{\delta} \) if

\[- \left( b(\bar{\delta})(1 + \bar{\alpha} - \bar{\beta}) - \bar{\delta} \right) \frac{1 + \bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} + (\alpha - \beta) \frac{(1 + \bar{\alpha} - \bar{\beta})^2}{\bar{\alpha}} b(\bar{\delta}) \leq \frac{(\alpha - \beta)(1 + \bar{\alpha} - \bar{\beta})}{\bar{\alpha}} \left( b(\bar{\delta})(1 + \bar{\alpha} - \bar{\beta}) - \bar{\delta} \right)
\]

\[
\Rightarrow \alpha \frac{1 + \bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} b(\bar{\delta}) \geq \frac{\alpha}{\bar{\alpha}^2} \bar{\delta} + \frac{\alpha - \beta}{\bar{\alpha}} \bar{\delta} = \alpha \frac{1 + \bar{\alpha} - \bar{\beta}}{\bar{\alpha}^2} \bar{\delta}
\]

\[
\Rightarrow b(\bar{\delta}) \geq \bar{\delta}
\]

where \( \int_{\bar{\delta}}^{\tau} \left( \frac{1 - F(x)}{F(\bar{\tau})} \right)^{\frac{1 + \alpha - \beta}{\alpha}} \frac{1}{x} \) is rewritten in terms of bid minus values. This means that (C.9) is positive at the left of \( \bar{\delta} \) and negative to the right of \( \bar{\delta} \). Therefore, as long as the equilibrium bid at the cut-off value \( \bar{\delta} \) is greater than the cut-off itself, bids will be decreasing in \( q \) for all \( q > \bar{\delta} \), if \( \alpha > \beta \).

**Step 4** When \( \alpha > \beta \), the limit of (C.9) for \( v \to \bar{\delta} \) is negative. In fact, under the increasing hazard rate condition (Assumption 1.3) applying L’Hospital’s rule to the first term (in the numerator) of (C.9) yields

\[
\lim_{v \to \bar{\delta}} \int_{v}^{\tau} \left( 1 - F(x) \right)^{\frac{1 + \alpha - \beta}{\alpha}} \left( -\frac{\alpha(1 + \bar{\alpha} - \bar{\beta})}{\bar{\alpha}^2} \log \left( \frac{1 - F(x)}{F(\bar{\tau})} \right) - (\alpha - \beta) \right) \frac{1}{x} \, dx = 0
\]

while the limit of the remaining part is \( \lim_{v \to \bar{\delta}} (\alpha - \beta)v = -(\alpha - \beta)\bar{\delta} < 0 \).

There are two cases: in Case 1 \( \bar{\delta} \) is unique and in Case 2 either \( \bar{\delta} \) does not exist or there are two \( \bar{\delta} \).

**CASE 1**. The first case assumes \( b(\bar{v}) \geq \bar{v} \). To show that \( \bar{\delta} \) is unique I merge two results: (i) the limit of (C.9) is negative at the upper bound of the support of \( v \), and (ii) there is no region on the support of \( v \) such that \( b(\bar{v}) < v \) inside and \( b(\bar{v}) > v \) to its left and right. Therefore, it cannot switch sign from negative to positive and back to negative again, implying that if \( \bar{\delta} \) exists, it is unique.

Recall from Lemma C2 that when \( b(\bar{v}) \geq \bar{v} \), \( b(\bar{v}) \) intersects the 45° line only once, at \( \bar{\delta} \), such that \( b(\bar{v}) > v \) for \( v < \bar{\delta} \) and \( b(\bar{v}) < v \) for \( v > \bar{\delta} \). The result in step 3 coupled with the requirements (i) that (C.9) is negative when evaluated at the upper bound, (ii) that (C.9) is continuous on the support of \( v \), and (iii) Lemma C2, necessarily means that (C.9) cannot switch sign more than once, imply that \( \bar{\delta} < \bar{\delta} \) as otherwise \( \partial b(\bar{v}) / \partial q |_{v=\bar{\tau}} > 0 \). In fact, for \( \bar{\delta} \geq \bar{\delta} \) (C.9) is increasing at \( \bar{\delta} \), which implies that (C.9) is positive for \( v > \bar{\delta} \). Then, (C.9) must switch sign again to negative in order to satisfy the \( \partial b(\bar{v}) / \partial q |_{v=\bar{\tau}} < 0 \) condition, but this is not possible in this region because of \( \bar{\delta} \) is unique and \( b(\bar{v}) < v \) for \( v > \bar{\delta} \) (see Figure 13a).

In addition, the derivative of the bid w.r.t \( q \) will always be negative. Therefore, if \( \bar{\delta} \) exists, it is unique and separates those who increase their bid (low-value bidders) from those who decrease it (high-value bidders).

**CASE 2**. The second case assumes \( b(\bar{v}) < \bar{v} \). Lemma C2 proves that there are either no \( \bar{\delta} \) or that there are exactly two \( \bar{\delta} \) so that the bid function cuts the 45° line twice. It follows that there exist either no cutoff or exactly two \( \bar{\delta} = \{\bar{\delta}_1, \bar{\delta}_2\} \) such that (C.10) is increasing at \( \bar{\delta}_1 \) and decreasing at \( \bar{\delta}_2 \), with \( \bar{\delta}_1 < \bar{\delta}_2 \) because the limit of (C.9) is negative at the upper bound. In fact, it must be that \( b(\bar{\delta}_1) < \bar{\delta}_1 \) and \( b(\bar{\delta}_2) > \bar{\delta}_2 \).

Hence, when \( \alpha > \beta \) there exists a value \( \bar{\delta} \) such that (at least some) bidders with values below \( \bar{\delta} \) increase their bids, while all bidders with values above \( \bar{\delta} \) decrease it after a marginal increase in \( q \). When instead \( \alpha \leq \beta \) all bidders increase their bids after a marginal increase in \( q \).
Note: Panel (a) shows the effect of a marginal increase in $q$ when $\alpha > \beta$ assuming that $b(v) > \varpi$. The complementary case would show two $\varpi$, such that the derivative (dotted line) is negative for the lowest value bidders, positive for the bidders with values between $\varpi_1$ and $\varpi_2$ and negative for the highest value bidders. The plot for the proof of Lemma 2 would be similar to Panel (a) with the exception that the $45^\circ$ line is replaced by the $v/(1-q\beta)$ line. Panel (b) shows that the bid is steeper at $v_2$ than at $v_3$. Lemma C2 states that if $b(v)$ oscillates around the $45^\circ$ line it violates the increasing hazard rate assumption.

C.7 Proof of Proposition 1

When $\alpha = 0$, the expected consumer surplus in a charity auction is equal to the consumer surplus in a non-charity auction. It is greater when $\alpha > 0$.

Proof: In a second-price non-charity auction a bidder’s dominant strategy is to bid her private value (i.e. $b^{NC}(x) = x$). The ex-ante consumer surplus is therefore:

$$
CS^{NC} = \int_{v}^{\varpi} \int_{v}^{\varpi} v - b^{NC}(x) dF(x)^{n-1} dF(v)
= \int_{v}^{\varpi} v \cdot (1 - F(v)^{n-1}) + \int_{v}^{\varpi} F(x)^{n-1} dx dF(v)
$$

(C.13)

where the second equality is derived integrating by parts. Showing that $CS^{NC}$ is identical to the consumer surplus in an analogous charity auction where bidders do not gain from externalities (i.e., $\alpha = 0$) is trivial because the expected payment in the former is equal to the expected payment minus warm glow of donating in the latter.$^5$

For the consumer surplus of a charitable second-price auction when $\alpha > 0$,

$$
CS^{C} = \int_{v}^{\varpi} \int_{v}^{\varpi} v - (1 - \beta q) \cdot b^{C}(x) dF(x)^{n-1} dF(v) + q\alpha \cdot b^{C}(v) F(v)^{n-2}(1 - F(v)) + q\alpha \cdot \int_{v}^{\varpi} b^{C}(x) dF^{(2)}_{(n-1)}(x)
$$

(C.14)

where $b^{C}(x)$ is defined as in (4.2) and $F^{(2)}_{(n-1)}(x) = F(x)^{n-1} + (n-1)F(x)^{n-2}(1 - F(x))$. Plugging-in the

$^5$That is, if the expected payment is $x$, then $x = \frac{x}{1-pq}(1 - \beta q)$. 

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bidding function in (C.14), and after some algebra, one obtains:

\[
CS^C = \int_v^\sigma v \cdot (1 - F(v)^{n-1}) + \int_v^\sigma F(x)^{n-1}dx \\
- \int_v^\sigma \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - q\beta + qa \over \alpha} \cdot \frac{1 - q\beta}{1 - q\beta + qa} - F(x)^{n-1}dx \\
+ \left[ v + \int_v^\sigma (1 - F(x))dx \right] + \int_v^\sigma \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - q\beta + qa \over \alpha} F(x)^{n-1}dx \\
\int_v^\sigma \left[ 1 - q\beta + qa \right] \left[ v + \int_v^\sigma (1 - F(x))dx \right] dF(v)
\]

Importantly, the first line in this equation is \(CS^{NC}\) in (C.13). Therefore, in order to prove the theorem I only need to show that the remaining two lines are positive.

To simplify the computations, approximate \(CS^C\) with \(\tilde{CS}^C\) where the expression in the second line \(- \int_v^\sigma \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - q\beta + qa \over \alpha} \cdot \frac{1 - q\beta}{1 - q\beta + qa} - F(x)^{n-1}dx\) is substituted with \(- \int_v^\sigma \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - q\beta + qa \over \alpha} F(x)^{n-1}dx\) as follows

\[
\tilde{CS}^C = CS^{NC} \\
+ \int_v^\sigma \left\{ - \int_v^\sigma \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - q\beta + qa \over \alpha} F(x)^{n-1}dx \right. \\
- \left. \int_v^\sigma \left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - q\beta + qa \over \alpha} F(x)^{n-1}dx \right\} dF(v)
\]

Because \(F(x)^{n-1} > F(v)^{n-1}\) for \(x > v\), the last expression is smaller than the former. Hence \(\tilde{CS}^C\) can be viewed as a lower bound for \(CS^C\). Therefore if \(CS^C > CS^{NC}\) then also \(CS^C > CS^{NC}\). The remainder of this proof heavily relies on L'Hospital's rule and integration by parts to show that this ordering holds. In particular I will show that the third line (which involves sums of positive terms) is greater than the second line.

To save on notation, denote \(\left( \frac{1 - F(x)}{1 - F(v)} \right)^{1 - q\beta + qa \over \alpha} \) by \(\Phi\) and \(\frac{1 - q\beta + qa}{q\alpha}\) by \(\psi\). Integrating by parts the first term in the second line of (C.16) (i.e., \(\int_v^\sigma \frac{\Phi F(x)^{n-1}}{1 - F(v)} dF(v)\)) gives

\[
- \left[ \int_v^\sigma \Phi F(x)^{n-1}dx F(v) \right]_{\sigma}^{v} - \int_v^\sigma - F(x)^{n} + \int_v^\sigma \Phi \cdot \psi \cdot \frac{f(v)}{1 - F(v)} F(x)^{n-1}dx F(v)dv \tag{C.17}
\]

The first term in brackets is 0, while the second term (negative) and the third term (positive) will be cancelled out using a combination of the second expression in the second line and the term \(+ \int_v^\sigma \frac{f(v)}{1 - F(v)} F(x)^{n-1} + (n - 1)F(x)^{n-2}(1 - F(x))\right)dx\) in the middle of the third line of (C.16). Starting from the former, and integrating it by parts:

\[
\int_v^\sigma \frac{1 - q\beta}{1 - q\beta + qa} \cdot \int_v^\sigma \Phi F(x)^{n-1}dx dF(v) = - \left[ \frac{1 - q\beta}{1 - q\beta + qa} \int_v^\sigma \Phi F(x)^{n-1}dx F(v) \right]_{\sigma}^{v} \\
- \int_v^\sigma \frac{1 - q\beta}{1 - q\beta + qa} F(v)^{n} + \int_v^\sigma \frac{1 - q\beta}{1 - q\beta + qa} \Phi \cdot \psi \cdot \frac{f(v)}{1 - F(v)} F(x)^{n-1}F(v)dx dv \tag{C.18}
\]
The first term in the square brackets is zero because of the L'Hospital's rule. In addition, the second two terms in (C.17) and (C.18) are similar, with the only difference being the multiplicative constant and the integration regions. However, because:

\[
\int_{v}^\sigma A(x)dx = \int_{\tilde{v}}^\sigma A(x)dx - \int_{v}^{\tilde{v}} A(x)dx
\]

where \(A(x)\) is a continuous function. The last term in (C.18) can be rewritten as

\[
\int_{\Sigma} \int_{v}^{\sigma} \frac{1 - q\beta}{1 - q\beta + q\alpha} \Phi^\phi \cdot \psi \cdot \frac{f(v)}{1 - F(v)} F(x)^{n-1} dx F(v) dv = \\
= \int_{\Sigma} \int_{v}^{\sigma} \frac{1 - q\beta}{1 - q\beta + q\alpha} \Phi^\phi \cdot \psi \cdot \frac{f(v)}{1 - F(v)} F(x)^{n-1} dx F(v) dv \\
- \int_{\Sigma} \int_{v}^{\sigma} \frac{1 - q\beta}{1 - q\beta + q\alpha} \Phi^\phi \cdot \psi \cdot \frac{f(v)}{1 - F(v)} F(x)^{n-1} dx F(v) dv
\]

where the term in the second line is positive while the remaining term is negative. Given this algebra, I can cancel out the last term in (C.17) by summing the last term in (C.19) with a similar one but with \(1/\psi\) as a multiplicative factor (i.e. \(1/\psi + (1 - q\beta)/(1 - q\beta + q\alpha) = 1\)). This term is found by integrating by parts the term in the second line of (C.16) that was previously mentioned:

\[
\int_{\Sigma} \int_{v}^{\sigma} \frac{1}{\psi} \Phi^\phi [F(x)^{n-1} + (n - 1)F(x)^{n-2}(1 - F(x))] dx dF(v)
\]

\[
= \frac{1}{\psi} \left[ \int_{v}^{\sigma} \Phi^\phi [F(v)^{n-1} + (n - 1)F(v)^{n-2}(1 - F(v))]dx F(v) \right]_{\Sigma}^{\sigma} - \int_{\Sigma} \int_{v}^{\sigma} \Phi^\phi \cdot \psi \cdot \frac{f(v)}{1 - F(v)} [F(x)^{n-1} + (n - 1)F(x)^{n-2}(1 - F(x))] dx F(v) dv
\]

Once again the first term in brackets goes to zero. Moreover, a portion of the last term (i.e., \(- \int_{v}^{\sigma} \Phi^\phi \cdot \psi \cdot \frac{f(v)}{1 - F(v)} F(x)^{n-1} dx F(v) dv \)) can be summed with the last term in (C.19) to cancel out the last term in (C.17).

Similarly, the second term in (C.17) cancels out with the sum of the second term in (C.18) and a portion of the second term in (C.20) (i.e., \(\int_{\Sigma}^{\sigma} F(x)^n dx\)).

To conclude the proof, the sum of the remaining terms in (C.20) and (C.16) must be positive. To show that the remaining part of (C.20) is positive, rewrite it as:

\[
\int_{\Sigma} \left( (n - 1) \cdot F(v)^{n-2}(1 - F(v)) \right) dx (1 - F(v))_\psi (n - 1)dx \psi (1 - F(v))^{-\psi - 1} f(v) F(v) dv
\]

\[
\text{In fact the } \lim_{v \to \sigma} \frac{1 - q\beta}{1 - q\beta + q\alpha} \int_{\Sigma} \Phi^\phi F(x)^{n-1} dx = 0, \text{ while the expression is also zero for } v \to \sigma \text{ because } F(v) = 0.
\]
Integration by parts of the second line of (C.21) yields

\[
- \left[ \int_{v}^{\overline{v}} (1 - F(x))^\psi (n - 1)F(x)^{n-2}(1 - F(x))F(v)dx \cdot (1 - F(v))^{-\psi} \right]_{\overline{v}}^v \\
+ \int_{v}^{\overline{v}} (n - 1) \cdot F(v)^{n-2}(1 - F(v))F(v) dv \\
- \int_{v}^{\overline{v}} \int_{v}^{\overline{v}} (1 - F(x))^\psi (n - 1)F(x)^{n-2}(1 - F(x))f(x) dx \cdot (1 - F(v))^{-\psi} dv
\]

(C.22)

The first term evaluated at the limits of the support of \( v \) is zero.\(^7\) In addition, the term in the second line of (C.22) is equal to the first term in (C.21) but with opposite signs so they cancel out, while the last term is positive. Finally, the sum of the first and last term in the third line of (C.16) is positive.

Therefore, \( \overline{\text{CS}}^C > \text{CS}^NC \) implying also that \( \text{CS}^C > \text{CS}^NC \).

**C.8 Proof of Proposition 2**

When \( \alpha = 0 \), the auctioneer should not donate. When \( \alpha > 0 \), the optimal donation solves \( \eta = \frac{q}{1-q} \).

**Proof:** Assume that the marginal cost is zero for simplicity. Then, the producer surplus is equal to the net revenue to the auctioneer for each object sold:

\[
\text{PS}(\alpha, \beta, q) = \begin{cases} \\
\int_{v}^{\overline{v}} (1 - q) \cdot \frac{1}{\frac{1}{1+q} - \frac{\alpha}{\frac{1}{1+q} + q}} \{ v + \int_{v}^{\overline{v}} \left( \frac{1-F(x)}{1-F(v)} \right)^{1-\psi} f(\alpha, \beta) dx \} dF_{(n)}^2(v) & \text{if } \alpha > 0 \wedge q > 0 \\
\int_{v}^{\overline{v}} (1 - q) \cdot \frac{v}{\frac{1}{1+q} - \frac{\alpha}{\frac{1}{1+q} + q}} dF_{(n)}^2(v) & \text{if } \alpha = 0 \vee q = 0 \\
\end{cases} \tag{C.23}
\]

When \( \alpha = 0 \) the derivative of the producer surplus w.r.t. \( q \) is negative as \( \beta \in (0,1) \). Therefore, the auctioneer is always better off by setting \( q = 0 \).

Consider the other case (\( \alpha > 0 \)). First suppose that \( q = 1 \): this cannot be optimal because it would leave the auctioneer with zero profits. In this case, setting \( q = 0 \) would be a profitable deviation, proving that \( q = 1 \) cannot be a solution for a profit maximizer auctioneer. To simplify the notation, given \( \alpha \) and \( \beta \), denote the bid of a bidder with valuation \( v \) in an auction where \( \hat{q} \) is donated by \( b(v, \hat{q}) \). The expected net revenue in a second-price charity auction is

\[
\int_{v}^{\overline{v}} (1 - q)b(v, q)dF_{(n)}^2(v)
\]

Let \( \eta \) be the elasticity of the expected price \( \psi = \int_{v}^{\overline{v}} b(v, q)dF_{(n)}^2(v) \) with respect to a marginal change in \( q \): \( \eta = \frac{\partial \ln \psi}{\partial \ln q} = \frac{\partial \psi}{\partial q} \frac{q}{\psi} \). By the Dominated Convergence Theorem, I can interchange differentiation and integration, so that the derivative of the expected price with respect to \( q \) can be rewritten as \( \frac{\partial \psi}{\partial q} = \int_{v}^{\overline{v}} \frac{\partial b(v, q)}{\partial q} dF_{(n)}^2(v) \). Using the elasticity formula, \( \int_{v}^{\overline{v}} \frac{\partial b(v, q)}{\partial q} dF_{(n)}^2(v) = \eta \frac{\psi}{q} \).

\(^7\)This result follows from L’Hospital’s rule. In fact

\[
\lim_{v \to \overline{v}} \frac{(1 - F(v))^{\psi+1}(n - 1)F(v)^{n-1}f(v)}{\psi(1-F(v))^{\psi-1}f(v)} + \frac{\int_{v}^{\overline{v}} (1 - F(x))^{\psi+1}(n - 1)F(x)^{n-2}dxf(v)}{\psi(1-F(v))^{\psi-1}f(v)} = 0
\]

The first term goes to zero, while to prove that the second term goes to zero as well I divide numerator and denominator by \( f(v) \) and apply L’Hospital’s rule again.
The optimal amount donated is the $q$ that solves the FOCs of the auctioneer’s problem (the second-order conditions are satisfied):

$$
\int_{v}^{v} -b(v, q) + (1 - q) \frac{\partial b(v, q)}{\partial q} dF^{(2)}_{(v)}(v) = 0
$$

Substituting $\eta p^e$ for the derivative of the bid, the optimal $q$ is given by:

$$
\eta = \frac{q}{1 - q}
$$

To illustrate the result in this proof, Figure 14 shows that the auctioneer should not donate when $\alpha = 0$.

**Figure 14:** Illustration of the results in Propositions 1 and 2

(a) $\alpha = 0$  
(b) $\alpha > 0$

Note: The plots display a bidder’s willingness to pay (y axis) in relation to her probability of losing the auction (x axis). Let $v$ denote the private value of the winner in the non-charity auction. In the charity auction, the willingness to pay, $w$, is the sum of a bidder’s private value plus the expected benefit from winning the auction ($\beta p^e$) and the expected externality from losing the auction ($\alpha p^e$). The two panels show how producer surplus and consumer surplus change when the auctioneer passes from $q = 0$ to $q > 0$. In Panel (a) $\alpha = 0$, and the produced surplus in the charity auction is necessarily smaller than in the non-charity auction. The consumer surplus stays unchanged. In Panel (b) the positive externality increases bidder’s willingness to pay ($\beta > \alpha$). This is displayed by a shift and a rotation of the dashed demand line compared to Panel (a). This implies a greater consumer surplus in charity auctions (not drawn). The size of the new producer surplus (shaded rectangle) will depend on $q$. In conclusion, in Panel (a) the auctioneer should not donate while in Panel (b) the auctioneer may find optimal to donate if the shaded area is greater than the checked area. Both plots maintain $\beta \geq \alpha$.

### C.9 Proof of Proposition 3

$\alpha, \beta$ and $F(v)$ are not identified without additional restrictions.

**Proof:** The model is not identified without (i) auxiliary data, or (ii) additional distributional assumptions. Denote a second-price charity auction with primitives $F(v), \alpha, \beta$ and $q$ by $\Gamma_2(F, \alpha, \beta, q)$. For simplicity, assume that $q = 1$. It is easy to see that the model is not identified when $\alpha = 0$. In fact, it is not possible to tell apart the following two expected utility models:

$$
I would like to thank Jorge Balat for these examples.
is the fraction of the transaction price that will be donated, i.e. \( q \) match the FOCs from the two auctions (C.1) based on the quantiles of the bid distribution. This gives the inverse hazard rate is not decreasing and that therefore \( b \) is increasing in \( \tau \).

Proof: The researcher observes two types of auction, \( A \) and \( B \), and that the only difference among them is the fraction of the transaction price that will be donated, i.e. \( q^A \neq q^B \). Because the distribution of values \( F(\cdot) \) is the same across auctions \( A \) and \( B \), then for each value corresponding to the \( \tau \)-quantile of the value distribution, \( \nu^\tau \), the distribution of bids computed at that \( \tau \)-quantile must be equal, i.e. \( G(b^\tau | q^A) = G(b^\tau | q^B) \). Therefore, after rewriting the first order condition for each set of auctions (C.1) as in (5.1) I can match the FOCs from the two auctions (C.1) based on the quantiles of the bid distribution. This gives the following equation for each quantile of \( \nu^\tau \):

\[
u^A_x - \nu^B_x = b^A_x - b^B_x + (\alpha - \beta) \cdot (q^A \cdot b^A_x - q^B \cdot b^B_x) - \alpha \cdot (q^A \cdot \lambda(b^A_x) - q^B \cdot \lambda(b^B_x)) \tag{C.24}\]

where \( G(b^\tau) = G(b^\tau | q^A) = G(b^\tau | q^B) \). Since \( \nu^A_x - \nu^B_x = 0 \), eq. C.24 can be rewritten in matrix notation as

\[
\Delta(b_x) = B_x \times \begin{bmatrix} \alpha - \beta \\ -\alpha \end{bmatrix},
\]

where \( \Delta(b_x) = -(b^A_x - b^B_x) \) and \( B_x \) is the matrix \([q^A b^A_x - q^B b^B_x; q^A \cdot \lambda(b^A_x) - q^B \cdot \lambda(b^B_x)]\).

It can be shown that the matrix \( B_x \) has full rank (the two columns are linearly independent). To prove this assume that \( B_x \) is not invertible and therefore that its columns are linearly dependent (i.e. \( b_x = k \cdot \lambda(b_x) \) for both auctions \( A \) and \( B \)). Dependence leads to \( G(b_x) = 1 - b_x \cdot g(b_x)/k \) for a constant \( k > 0 \). Note that \( k \) must be positive because otherwise (i) \( G(b_x) > 1 \) as \( g(b_x) \geq 0, \forall b_x \) and (ii) the FOC equation \( \xi(b_x, \alpha, \beta, q) \), which was defined in (5.1), may not be monotonically increasing in \( b_x \). The previous differential equation admits a solution \( g(b_x) = c \cdot (b_x)^{-(k+1)} \), where \( c \) is an integration constant. Thus, \( G(b_x) = 1 - c \cdot (b_x)^{-k}/k \). Evaluating the CDF at \( b_x = 0 \) yields \( G(0) = -\infty, \forall k > 0 \). Moreover, the inverse hazard rate

\[
\lambda(b_x) = \frac{1 - 1 + c \cdot (b_x)^{-k}/k}{c \cdot (b_x)^{-(k+1)}} = \frac{b_x}{k} \text{ for } k > 0
\]

is increasing in \( b_x \). This implies that

\[
\frac{1 - F(v_x)}{f(v_x)} = \frac{1 - G(b(v_x))}{g(b(v_x)) \cdot b'(v_x)} = \frac{1}{k} \frac{b(v_x)}{b'(v_x)}
\]

which is an increasing function because (i) the optimal bidding function \( b(v) \) is increasing in the private value \( v_x \) and (ii) the bidding function \( b(v_x) \) maximizes a bidder’s utility \( b''(v_x) \leq 0 \). This means that the inverse hazard rate is not decreasing and that therefore \( b(v_x) \) is not a best response for \( v_x \). This is a

C.10 Proof of Proposition 4

In second-price auctions, under Assumptions 1 and 2 the parameters \( \alpha \) and \( \beta \) and the distribution of values \( F(v) \) are identified by variation in \( q \) across auctions.

Proof: The researcher observes two types of auction, \( A \) and \( B \), and that the only difference among them is the fraction of the transaction price that will be donated, i.e. \( q^A \neq q^B \). Because the distribution of values \( F(\cdot) \) is the same across auctions \( A \) and \( B \), then for each value corresponding to the \( \tau \)-quantile of the value distribution, \( \nu^\tau \), the distribution of bids computed at that \( \tau \)-quantile must be equal, i.e. \( G(b^\tau | q^A) = G(b^\tau | q^B) \). Therefore, after rewriting the first order condition for each set of auctions (C.1) as in (5.1) I can match the FOCs from the two auctions (C.1) based on the quantiles of the bid distribution. This gives the following equation for each quantile of \( \nu^\tau \):

\[
u^A_x - \nu^B_x = b^A_x - b^B_x + (\alpha - \beta) \cdot (q^A \cdot b^A_x - q^B \cdot b^B_x) - \alpha \cdot (q^A \cdot \lambda(b^A_x) - q^B \cdot \lambda(b^B_x)) \tag{C.24}\]

where \( G(b^\tau) = G(b^\tau | q^A) = G(b^\tau | q^B) \). Since \( \nu^A_x - \nu^B_x = 0 \), eq. C.24 can be rewritten in matrix notation as

\[
\Delta(b_x) = B_x \times \begin{bmatrix} \alpha - \beta \\ -\alpha \end{bmatrix},
\]

where \( \Delta(b_x) = -(b^A_x - b^B_x) \) and \( B_x \) is the matrix \([q^A b^A_x - q^B b^B_x; q^A \cdot \lambda(b^A_x) - q^B \cdot \lambda(b^B_x)]\).

It can be shown that the matrix \( B_x \) has full rank (the two columns are linearly independent). To prove this assume that \( B_x \) is not invertible and therefore that its columns are linearly dependent (i.e. \( b_x = k \cdot \lambda(b_x) \) for both auctions \( A \) and \( B \)). Dependence leads to \( G(b_x) = 1 - b_x \cdot g(b_x)/k \) for a constant \( k > 0 \). Note that \( k \) must be positive because otherwise (i) \( G(b_x) > 1 \) as \( g(b_x) \geq 0, \forall b_x \) and (ii) the FOC equation \( \xi(b_x, \alpha, \beta, q) \), which was defined in (5.1), may not be monotonically increasing in \( b_x \). The previous differential equation admits a solution \( g(b_x) = c \cdot (b_x)^{-(k+1)} \), where \( c \) is an integration constant. Thus, \( G(b_x) = 1 - c \cdot (b_x)^{-k}/k \). Evaluating the CDF at \( b_x = 0 \) yields \( G(0) = -\infty, \forall k > 0 \). Moreover, the inverse hazard rate

\[
\lambda(b_x) = \frac{1 - 1 + c \cdot (b_x)^{-k}/k}{c \cdot (b_x)^{-(k+1)}} = \frac{b_x}{k} \text{ for } k > 0
\]

is increasing in \( b_x \). This implies that

\[
\frac{1 - F(v_x)}{f(v_x)} = \frac{1 - G(b(v_x))}{g(b(v_x)) \cdot b'(v_x)} = \frac{1}{k} \frac{b(v_x)}{b'(v_x)}
\]

which is an increasing function because (i) the optimal bidding function \( b(v) \) is increasing in the private value \( v_x \) and (ii) the bidding function \( b(v_x) \) maximizes a bidder’s utility \( b''(v_x) \leq 0 \). This means that the inverse hazard rate is not decreasing and that therefore \( b(v_x) \) is not a best response for \( v_x \). This is a
C.11 Proof of Corollary 1

In second-price auctions, $\alpha$, $\beta$ and $F(v)$ are nonparametrically identified also when the dataset includes more than 2 types of auctions.

Proof: Assume that $q$ has dimension $K_C$ and that $K_x$ quantiles of the distribution of values are observed. The FOC (5.1) in matrix notation becomes

$$V_{k_x \times k_C} = (\alpha - \beta) \times B_{k_x \times k_C} \times Q_C^{k_C} + B_{k_x \times k_C} - \alpha \times \Lambda_{k_x \times k_C} \times Q_C^{k_C}$$

where $V$ is a matrix of dimension $K_x \times K_C$ displaying the value $v_x$ for the $x-$quantile (row) in auction of type $C$ (column), and 0 otherwise. Similarly, $Q_C$ is a diagonal matrix with entries equal to the percentage donated $q^C$ and 0. The other matrices are defined as:

$$B = \begin{bmatrix} b^1_0 & b^2_0 & \ldots & b^K_C \\ \vdots & \vdots & \ddots & \vdots \\ b^1_1 & b^2_1 & \ldots & b^K_C \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda^1(b^1_0) & \lambda^2(b^2_0) & \ldots & \lambda^K_C(b^K_C) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda^1(b^1_1) & \lambda^2(b^2_1) & \ldots & \lambda^K_C(b^K_1) \end{bmatrix}$$

where superscripts indicate that the amount donated is equal to $q_j$ for $j \in [1, K_C]$ and subscripts indicate the $x-$ quantiles of the distribution of values or bids. There exists a projection $M$ (with rank $K_C - 1$) such that $V \times M = 0$. Postmultiplying (C.26) by $M$ and moving terms, the following equation represents the FOC where the dependent variable is a known object

$$-B \times M = (\alpha - \beta) \times B \times Q_C \times M - \alpha \times \Lambda \times Q_C \times M$$

After stacking the matrices in vectors, the last equation can be represented by the system of equations

$$y = [b \quad l] \times \begin{bmatrix} \alpha - \beta \\ -\alpha \end{bmatrix}$$

where $y = \text{vec}(-B \times M), b = \text{vec}(B \times Q_C \times M), l = \text{vec}(\Lambda \times Q_C \times M)$, and $\text{vec}(\cdot)$ indicates the vectorization of the matrices in parenthesis.

Nonparametric identification requires showing that $b$ and $l$ are not proportional to each other (i.e., that $B$ and $\Lambda$ are linearly independent). Nonproportionality follows directly from the argument in the proof of Proposition 4 in Appendix C.10. Hence, $b$ and $l$ are not linearly dependent, establishing identification of $\alpha, \beta$ and $F(v)$.

C.12 Proof of Proposition 5

In English auctions, $\alpha, \beta$ and $F(v)$ are nonparametrically identified by first deriving the distribution of bids that would have been observed in parallel second-price auctions, and then by applying Proposition 4.

Proof: Assume that the researcher observes two kind of auctions $A$ and $B$, characterized by $q^A$ and $q^B$ respectively (with $q^A \neq q^B$). The starting point is to note that $G(b) = F(v)$, and that also in charity auctions the distribution of the winning bid is equal to the distribution of the second-highest bid: $G_w(b) = 0$. Given that $B_x$ has full rank, $\alpha, \beta$ and $F(v)$ are nonparametrically identified.  

\footnotesize
\begin{itemize}
  \item If $k = 1$, then $g(b_x) = 0$ for $b_x < 0$ and $g(b_x) > 0$ for $b_x > 0$. Therefore $g(\cdot)$ is the Dirac delta function, which is not differentiable and does not admit a decreasing inverse hazard rate. In turn, given that $F(v) = G(b(v))$, the non-differentiability of $G(\cdot)$ implies that also the distribution of values $F(\cdot)$ is not differentiable, a contradiction.
\end{itemize}

\normalsize
\[ G^2_n(b). \] Therefore, the distribution of bids that would have been realized in an equivalent second-price auction is found using the classic inversion of the latter relation. In particular, \( G(b) \) is found as the root (in \([0, 1]\)) of \( G_w(b) - nG(b)^{n-1} + (n - 1)G(b)^n \).

Now, the FOCs (5.1) can be written for each \( \alpha \) and \( \beta \). Therefore, one can apply the same logic in Proposition 4, and identify \( \alpha, \beta \) and \( F(v) \).

D Bargaining with celebrities

This section provides a micro foundation to the condition for the optimal \( q^* \) in (8.1). The surplus to the firm is the difference of the net revenues \( (1 - q) p \) and the fixed cost \( \kappa \), which is a unit cost accounting, for example, for procuring and storing the item, publishing the listing online as well as managerial cost related to an auction. \( \kappa \) does not vary with \( q \). Celebrities cares for the amount that is ultimately donated. For simplicity they incur no cost in giving the item. The bargaining weights are \( \omega \) for the firm and \( 1 - \omega \) for the celebrity. In this bargaining framework, \( q \) maximizes

\[
((1 - q)p^e - \kappa)^\omega \cdot (qp^e)^{1-\omega}
\]

where \( p^e \) denotes the expected highest price, and is a function of the distribution of values, the charity parameters (i.e., \( \alpha \) and \( \beta \)), the number of bidders and \( q \). The first-order condition with respect to \( q \) can be rearranged to obtain

\[
-\frac{p^e}{q} \left( \frac{(1 - q)p^e\omega}{(1 - q)p^e - (1 - \omega)\kappa} \right) = \frac{1 - q}{q} \eta p^e - p^e
\]

\( \eta \) is the elasticity of the expected price to a change in \( q \). The RHS is the same as in (8.1), while the LHS is a function of the primitives.

The LHS can be interpreted as a marginal cost \( c'(q) \). When \( \omega = 1 \), all bargaining power is in the hand of Charitystars. The LHS becomes 0, and Charitystars acts as a monopolist by choosing the \( q \) that solves \( \eta = \frac{q}{1 - q} \), as in the revenue maximization case (see Proposition 2). Because there is no bargaining, setting a greater \( q \) does not yield any cost savings (i.e., \( c'(q) = 0 \)). When \( \omega = 0 \), the celebrity holds all the bargaining power. The firm will donate as much as possible as the condition for the optimal \( q \) is not feasible (i.e., \( \eta = -1 \)) if the elasticity of supply is positive (as in the warm glow model).

Charitystars reaps the largest cost saving when the LHS is the smallest possible \((-p^e/q)\), which happens as \( \omega \to 0 \). For \( \omega \in (0, 1) \), the LHS is increasing in \( \omega \), implying that the stronger is the celebrity (small \( \omega \)), the greater is the marginal cost savings. Since the term in parenthesis is always in the unit interval, the LHS will always be negative if the net expected price is greater than the unit cost (i.e., \((1 - q)p^e > \kappa\)). This condition must hold for the firm to operate, and it is ensured by the way the reserve price is set in practice (to break-even). Thus, \( c'(q) < 0 \).

Finally, the LHS in (D.2) can be rearranged by adding and subtracting \( \omega \) in the term in parenthesis

\[
-\frac{p^e}{q} \left( \frac{(1 - q)p^e - \kappa + \omega^2\kappa}{(1 - q)p^e - \kappa + \omega\kappa} - \omega \right) \approx -\frac{p^e}{q} (1 - \omega)
\]

which gives a clean interpretation of the marginal cost in terms of the bargaining weight and primitives of the model. Therefore, the monopolistic approach taken in the main text can be extended to account for a simple bargaining problem between Charitystars and the celebrity.

Optimal percentage donated with more covariates. Figure 15 shows the optimal percentage donated by intersecting marginal revenues and marginal costs as in Section 8. Marginal costs and revenues are derived as in the main body with the addition of covariates belonging to the groups: “Additional Charity Dummies”, “League/Match Dummies” and “Time Dummies” (the same regressors used in the fourth
Control variables are defined in Appendix B.

Figure 15: Optimal $q$ - accounting for more covariates

Note: The plot shows the optimal percentage donated as the intersection of marginal costs and marginal benefits. The number of bidders is assumed to be 8. The density $f(v)$ and the distribution $F(v)$ are approximated using a cubic spline. Only auctions with price between €100 and €1000. Marginal revenues in Euro are computed as in Figure 10. The covariates used in (6.1) include the total number of bids as in Appendix E.3.1.

E Tables and figures

Table 11: Relation between log($Price$) and percentage donated (small dataset)

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Bidders)</td>
<td>0.158***</td>
<td>0.182***</td>
<td>0.182***</td>
<td>0.180***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>$q$</td>
<td>0.050</td>
<td>0.127***</td>
<td>0.146***</td>
<td>0.157***</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.046)</td>
<td>(0.052)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td>0.261***</td>
<td>0.254***</td>
<td>0.252***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.029)</td>
<td>(0.030)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>4.584***</td>
<td>3.159***</td>
<td>3.390***</td>
<td>3.330***</td>
</tr>
<tr>
<td></td>
<td>(0.207)</td>
<td>(0.222)</td>
<td>(0.225)</td>
<td>(0.267)</td>
</tr>
</tbody>
</table>


Note: OLS regression of log of the transaction price on covariates. Only auctions with price between €100 and €400. Control variables are defined in Appendix B.

* – $p < 0.1; ** – $p < 0.05; *** – $p < 0.01.
Table 12: Regression of log($Price$) on $q$ and $q^2$

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Reserve Price)</td>
<td>0.356***</td>
<td>0.344***</td>
<td>0.347***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>0.295***</td>
<td>0.283***</td>
<td>0.276***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.029)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>$q$</td>
<td>0.881</td>
<td>0.624</td>
<td>1.152**</td>
</tr>
<tr>
<td></td>
<td>(0.535)</td>
<td>(0.566)</td>
<td>(0.578)</td>
</tr>
<tr>
<td>$q^2$</td>
<td>–0.686</td>
<td>–0.419</td>
<td>–0.952</td>
</tr>
<tr>
<td></td>
<td>(0.569)</td>
<td>(0.599)</td>
<td>(0.616)</td>
</tr>
<tr>
<td>Main Variables</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Charity Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>League/Match Dummies</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Time Dummies</td>
<td></td>
<td></td>
<td>Y</td>
</tr>
<tr>
<td>$q + q^2$</td>
<td>0.195***</td>
<td>0.206***</td>
<td>0.201***</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.062)</td>
<td>(0.068)</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>45.91%</td>
<td>47.17%</td>
<td>49.28%</td>
</tr>
<tr>
<td>BIC</td>
<td>1,168</td>
<td>1,276</td>
<td>1,337</td>
</tr>
<tr>
<td>$N$</td>
<td>1,108</td>
<td>1,108</td>
<td>1,108</td>
</tr>
</tbody>
</table>

Note: OLS regression of log of the transaction price on covariates and $q^2$ to test the linearity of donation. Only auctions with price between €100 and €1000. Control variables are defined in Appendix B.

* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 
Figure 16: Plot of the coefficient for $q$ from a quantile regression

Note: Coefficients from a series of quantile regressions of the log of the transaction price on $q$ and covariates as in the first column of Table 12 in the main text. The shaded regions is the confidence interval. The dashed (dotted) line reports the coefficient of the OLS regression (5% confidence interval). Only auctions with price between €100 and €1000. Boostrapped standard errors with 400 repetitions.
Note: Coefficients from a series of quantile regressions of the log of the transaction price on $q$ and covariates as in the first column of Table 12 in the main text. The shaded regions is the confidence interval. The dashed (dotted) line reports the coefficient of the OLS regression (5% confidence interval). Only auctions with price between €100 and €400. Boostrapped standard errors with 400 repetitions.
Table 13: Linearity of the relation between log(\(Price\)) and percentage donated (small dataset)

<table>
<thead>
<tr>
<th></th>
<th>(I) OLS</th>
<th>(II) Q(0.25)</th>
<th>(III) Q(0.50)</th>
<th>(IV) Q(0.75)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Reserve Price)</td>
<td>0.261***</td>
<td>0.372***</td>
<td>0.288***</td>
<td>0.182***</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.049)</td>
<td>(0.044)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>0.182***</td>
<td>0.188***</td>
<td>0.171***</td>
<td>0.124***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.034)</td>
<td>(0.045)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>q</td>
<td>0.127***</td>
<td>0.164***</td>
<td>0.171**</td>
<td>0.146**</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.056)</td>
<td>(0.071)</td>
<td>(0.070)</td>
</tr>
<tr>
<td>Constant</td>
<td>3.159***</td>
<td>2.294***</td>
<td>2.881***</td>
<td>4.057***</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.311)</td>
<td>(0.350)</td>
<td>(0.346)</td>
</tr>
<tr>
<td>Main Variables</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Adjusted/Pseudo R-squared</td>
<td>29.31%</td>
<td>26.51%</td>
<td>18.11%</td>
<td>14.70%</td>
</tr>
<tr>
<td>N</td>
<td>713</td>
<td>713</td>
<td>713</td>
<td>713</td>
</tr>
</tbody>
</table>

Note: OLS Regression and quantile regressions of log of the transaction price on covariates to test the linearity of donation. Only auctions with price between €100 and €400. Bootstrapped standard errors with 400 repetitions. The null hypothesis that q is the same in column (II), (III) and (IV) is not rejected beyond 90% level. Control variables are defined in Appendix B.

* – \( p < 0.1 \); ** – \( p < 0.05 \); *** – \( p < 0.01 \).

E.1 Asymmetric behavior

E.1.1 Nationalities

Table 14: Nationalities of the bidders

<table>
<thead>
<tr>
<th></th>
<th>Italian</th>
<th>UK</th>
<th>France</th>
<th>Other EU</th>
<th>North Am</th>
<th>China</th>
<th>Asia</th>
<th>East Asia</th>
<th>Rest World</th>
<th>Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winner</td>
<td>560</td>
<td>74</td>
<td>41</td>
<td>119</td>
<td>62</td>
<td>130</td>
<td>35</td>
<td>28</td>
<td>59</td>
<td>1,108</td>
</tr>
<tr>
<td></td>
<td>50.54%</td>
<td>6.68%</td>
<td>3.70%</td>
<td>10.74%</td>
<td>5.60%</td>
<td>11.73%</td>
<td>3.16%</td>
<td>2.53%</td>
<td>5.32%</td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td>603</td>
<td>79</td>
<td>28</td>
<td>113</td>
<td>51</td>
<td>117</td>
<td>33</td>
<td>26</td>
<td>58</td>
<td>1,108</td>
</tr>
<tr>
<td></td>
<td>54.42%</td>
<td>7.13%</td>
<td>2.53%</td>
<td>10.20%</td>
<td>4.60%</td>
<td>10.56%</td>
<td>2.98%</td>
<td>2.35%</td>
<td>5.23%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,163</td>
<td>153</td>
<td>69</td>
<td>232</td>
<td>113</td>
<td>247</td>
<td>68</td>
<td>54</td>
<td>117</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: Nationalities of the bidders by geographic area. “Rest of the World” includes Eastern Europe, Middle East, Africa, Oceania, Latina America and Unknown nationalities (which comprises 12 winners and highest losers).
Table 15: Regression of log(\(\text{Price}\)) on bidder nationality

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q)</td>
<td>0.230***</td>
<td>0.230***</td>
<td>0.235***</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.048)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>Win: Italy</td>
<td>-0.043*</td>
<td>-0.038</td>
<td>-0.058</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Italian Team</td>
<td></td>
<td></td>
<td>-0.068**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.031)</td>
</tr>
<tr>
<td>Win: North America</td>
<td></td>
<td></td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.077)</td>
</tr>
<tr>
<td>Win: France</td>
<td></td>
<td>-0.095</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
<td></td>
</tr>
<tr>
<td>Win: European Union</td>
<td></td>
<td>-0.008</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.071)</td>
<td></td>
</tr>
<tr>
<td>Win: China</td>
<td></td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.070)</td>
<td></td>
</tr>
<tr>
<td>Win: UK</td>
<td></td>
<td>-0.020</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.076)</td>
<td></td>
</tr>
<tr>
<td>Win: Unknown</td>
<td></td>
<td>-0.154</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.134)</td>
<td></td>
</tr>
<tr>
<td>Win: Asia</td>
<td></td>
<td>-0.079</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.089)</td>
<td></td>
</tr>
<tr>
<td>Win: South-East Asia</td>
<td></td>
<td>-0.121</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.089)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.386***</td>
<td>2.413***</td>
<td>2.368***</td>
</tr>
<tr>
<td></td>
<td>(0.229)</td>
<td>(0.230)</td>
<td>(0.238)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Main Variables</th>
<th>Y</th>
<th>Y</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td>1,108</td>
<td>1,108</td>
<td>1,108</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>46.02%</td>
<td>46.24%</td>
<td>46.11%</td>
</tr>
</tbody>
</table>

Note: OLS regressions of \(\log(\text{Price})\) on covariates to test the symmetry assumption over different nationalities of the winner. The first column tests Italian vs Non-Italian winners. The coefficient shows that Italian winners bid less than others, however Column (II) reveals that this correlation vanishes when a dummy variable for whether the football jersey is from an Italian team is also present. No geographic dummy variable is significant in Column (III). Only auctions with price between \(\)100 and \(\)1000. Control variables are defined in Appendix B.

\(* - p < 0.1; ** - p < 0.05; *** - p < 0.01.\)
### E.1.2 Collectors

Table 16: Regression of log($\text{Price}$) on recurrent winners

<table>
<thead>
<tr>
<th></th>
<th>(I) 100 &lt; $p$ &lt; 1000</th>
<th>(II) 100 &lt; $p$ &lt; 400</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Reserve Price)</td>
<td>0.351*** (0.023)</td>
<td>0.262*** (0.027)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>0.295*** (0.029)</td>
<td>0.181*** (0.028)</td>
</tr>
<tr>
<td>$q$</td>
<td>0.227*** (0.048)</td>
<td>0.120*** (0.046)</td>
</tr>
<tr>
<td>Recurrent Winner</td>
<td>0.047** (0.023)</td>
<td>0.021 (0.022)</td>
</tr>
<tr>
<td>Total Number of Bids Placed</td>
<td>0.218*** (0.023)</td>
<td>0.143*** (0.022)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.324*** (0.225)</td>
<td>3.135*** (0.221)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Main Variables</th>
<th>Y</th>
<th>Y</th>
<th>Y</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted R-squared</td>
<td>0.4605</td>
<td>0.5025</td>
<td>0.2947</td>
<td>0.3342</td>
</tr>
<tr>
<td>$N$</td>
<td>1108</td>
<td>1108</td>
<td>713</td>
<td>713</td>
</tr>
</tbody>
</table>

Note: OLS regressions of the log of the transaction price on covariates to test the symmetry assumption. The dummy variable Recurrent Winner is 1 if the winner of the auction won more than 3 auctions (the median in the data). Control variables are defined in Appendix B.

* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 
### E.2 Tables from the structural model

#### Table 17: First step of the structural estimation

<table>
<thead>
<tr>
<th></th>
<th>(I) 100 &lt; $p &lt; 1000$</th>
<th>(II) 100 &lt; $p &lt; 1000$</th>
<th>(III) 100 &lt; $p &lt; 400$</th>
<th>(IV) 100 &lt; $p &lt; 400$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{UH}$</td>
<td>0.341*** (0.025)</td>
<td>0.267*** (0.028)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum Increment</td>
<td>-0.010 (0.026)</td>
<td>0.003 (0.015)</td>
<td>-0.036 (0.026)</td>
<td>-0.018 (0.014)</td>
</tr>
<tr>
<td>Minimum Increment$^2$</td>
<td>0.0001 (0.0005)</td>
<td>0.0004 (0.001)</td>
<td>0.002 (0.001)</td>
<td>0.001*** (0.0005)</td>
</tr>
<tr>
<td>Log(Bidders)</td>
<td>-0.014 (0.084)</td>
<td>0.312*** (0.047)</td>
<td>-0.136** (0.074)</td>
<td>0.150*** (0.041)</td>
</tr>
<tr>
<td>Sold at Reserve Price</td>
<td>0.555*** (0.055)</td>
<td>0.042 (0.033)</td>
<td>0.490*** (0.058)</td>
<td>0.056 (0.031)</td>
</tr>
<tr>
<td>Length</td>
<td>0.045 (0.084)</td>
<td>0.071*** (0.047)</td>
<td>0.005 (0.074)</td>
<td>0.055*** (0.041)</td>
</tr>
<tr>
<td>Length$^2$</td>
<td>-0.003 (0.002)</td>
<td>0.003*** (0.001)</td>
<td>0.001 (0.001)</td>
<td>0.002*** (0.001)</td>
</tr>
<tr>
<td>Extended Time</td>
<td>0.010 (0.002)</td>
<td>0.072** (0.001)</td>
<td>0.091** (0.001)</td>
<td>0.073*** (0.001)</td>
</tr>
<tr>
<td>Number of Pictures</td>
<td>-0.074 (0.063)</td>
<td>-0.609*** (0.045)</td>
<td>0.010 (0.024)</td>
<td>-0.002 (0.002)</td>
</tr>
<tr>
<td>Number of Pictures$^2$</td>
<td>0.009 (0.050)</td>
<td>0.006*** (0.051)</td>
<td>0.002 (0.096)</td>
<td>0.001 (0.054)</td>
</tr>
<tr>
<td>Auctions within 3 weeks</td>
<td>0.013 (0.009)</td>
<td>0.016*** (0.047)</td>
<td>-0.016*** (0.007)</td>
<td></td>
</tr>
<tr>
<td>Auctions up to 2 weeks ago</td>
<td>0.004 (0.004)</td>
<td>0.011*** (0.004)</td>
<td>-0.005 (0.004)</td>
<td>0.008*** (0.002)</td>
</tr>
<tr>
<td>Count Auctions Same Charity</td>
<td>0.0004 (0.002)</td>
<td>-0.001*** (0.003)</td>
<td>0.000 (0.003)</td>
<td>-0.001*** (0.002)</td>
</tr>
<tr>
<td>Player belongs to FIFA 100 list</td>
<td>0.216*** (0.082)</td>
<td>0.201*** (0.091)</td>
<td>0.168** (0.091)</td>
<td>0.098** (0.054)</td>
</tr>
<tr>
<td>Unwashed Jersey</td>
<td>0.143 (0.010)</td>
<td>0.282*** (0.075)</td>
<td>-0.029 (0.096)</td>
<td>0.140*** (0.054)</td>
</tr>
<tr>
<td>Jersey is Signed</td>
<td>-0.405*** (0.032)</td>
<td>-0.054 (0.032)</td>
<td>-0.320*** (0.032)</td>
<td>-0.042 (0.032)</td>
</tr>
<tr>
<td>Jersey is signed by the team players/coach</td>
<td>-0.592*** (0.011)</td>
<td>-0.172*** (0.075)</td>
<td>-0.493*** (0.096)</td>
<td>-0.041 (0.096)</td>
</tr>
<tr>
<td>Jersey Worn During a Final</td>
<td>0.348* (0.196)</td>
<td>0.411*** (0.095)</td>
<td>-0.280*** (0.099)</td>
<td>0.204*** (0.055)</td>
</tr>
<tr>
<td>Number of Goals Scored</td>
<td>0.192 (0.114)</td>
<td>0.247*** (0.092)</td>
<td>0.012 (0.266)</td>
<td>0.336*** (0.087)</td>
</tr>
<tr>
<td>Player Belongs to an Important Team</td>
<td>0.230*** (0.069)</td>
<td>0.291*** (0.048)</td>
<td>0.144*** (0.071)</td>
<td>0.169*** (0.041)</td>
</tr>
<tr>
<td>Charity is Italian</td>
<td>-0.086 (0.126)</td>
<td>0.143 (0.137)</td>
<td>-0.168 (0.269)</td>
<td>0.113 (0.080)</td>
</tr>
<tr>
<td>Charity is English</td>
<td>0.304 (0.175)</td>
<td>0.233* (0.133)</td>
<td>0.034 (0.292)</td>
<td>0.230*** (0.082)</td>
</tr>
<tr>
<td>Helping Disables</td>
<td>-0.142** (0.064)</td>
<td>0.019 (0.040)</td>
<td>-0.214*** -0.076 (0.074)</td>
<td>-0.036 (0.076)</td>
</tr>
<tr>
<td>Infrastructures in Developing Countries</td>
<td>-0.069 (0.119)</td>
<td>0.152** (0.097)</td>
<td>-0.482*** -0.138** (0.175) (0.066)</td>
<td></td>
</tr>
<tr>
<td>Healthcare</td>
<td>-0.152 (0.104)</td>
<td>-0.232*** (0.109)</td>
<td>-0.116 (0.096)</td>
<td>-0.050 (0.095)</td>
</tr>
<tr>
<td>Humanitarian Scopes in Developing Countries</td>
<td>0.159 (0.119)</td>
<td>0.165** (0.068)</td>
<td>0.037 (0.180) (0.066)</td>
<td></td>
</tr>
<tr>
<td>Children’s Wellbeing</td>
<td>0.117 (0.104)</td>
<td>0.121 (0.107)</td>
<td>-0.056 (0.109)</td>
<td>0.038 (0.105)</td>
</tr>
<tr>
<td>Neurodegenerative Disorders</td>
<td>0.489*** (0.145)</td>
<td>0.273*** (0.089)</td>
<td>0.020 (0.153) (0.094)</td>
<td></td>
</tr>
<tr>
<td>Charity Belongs to the Soccer Team</td>
<td>0.037 (0.153)</td>
<td>-0.239*** (0.089)</td>
<td>0.375*** (0.185) (0.094)</td>
<td></td>
</tr>
<tr>
<td>Improving Access to Sport</td>
<td>-0.041 (0.115)</td>
<td>-0.026 (0.073)</td>
<td>-0.124 (0.145) (0.095)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>4.699*** (0.380)</td>
<td>4.175*** (0.230)</td>
<td>5.333*** 4.538*** (0.357) (0.383)</td>
<td></td>
</tr>
</tbody>
</table>

Adjusted $R^2$: 28.0% 46.8% 31.3% 36.4%

Note: The table reports the two regressions in the first step of the structural model. Columns (I) and (II) regress log(Reserve Price) on covariates, while Columns (III) and (IV) regress log(Price) on the unobserved heterogeneity ($\hat{UH}$) and covariates. Control variables are defined in Appendix B. * – $p < 0.1$, ** – $p < 0.05$, *** – $p < 0.01$. 

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Table 18: Logit regression of $q$ on covariates.

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th></th>
<th>(II)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>SE</td>
<td>Coefficient</td>
<td>SE</td>
</tr>
<tr>
<td>log(Reserve Price)</td>
<td>0.846***</td>
<td>(0.254)</td>
<td>0.795</td>
<td>(0.486)</td>
</tr>
<tr>
<td>log(Bidders)</td>
<td>−0.973***</td>
<td>(0.353)</td>
<td>−1.384***</td>
<td>(0.464)</td>
</tr>
<tr>
<td>Length</td>
<td>−0.577***</td>
<td>(0.215)</td>
<td>−0.542*</td>
<td>(0.307)</td>
</tr>
<tr>
<td>Length$^2$</td>
<td>0.016**</td>
<td>(0.007)</td>
<td>0.013</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Minimum Increment</td>
<td>−0.669***</td>
<td>(0.203)</td>
<td>−0.820**</td>
<td>(0.339)</td>
</tr>
<tr>
<td>Minimum Increment$^2$</td>
<td>0.026***</td>
<td>(0.007)</td>
<td>0.031***</td>
<td>(0.012)</td>
</tr>
<tr>
<td>PriceEqualRes</td>
<td>0.905</td>
<td>(0.583)</td>
<td>1.174</td>
<td>(0.869)</td>
</tr>
<tr>
<td>Length of description</td>
<td>0.026***</td>
<td>(0.007)</td>
<td>0.031***</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Content in English</td>
<td>−0.446</td>
<td>(0.453)</td>
<td>−0.556</td>
<td>(0.666)</td>
</tr>
<tr>
<td>Content in Spanish</td>
<td>0.000</td>
<td>(.)</td>
<td>0.000</td>
<td>(.)</td>
</tr>
<tr>
<td>Length of Charity Desc.</td>
<td>0.006*</td>
<td>(0.003)</td>
<td>0.011***</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Number of Pictures</td>
<td>0.319</td>
<td>(0.356)</td>
<td>0.887</td>
<td>(0.578)</td>
</tr>
<tr>
<td>Number of Pictures$^2$</td>
<td>−0.043</td>
<td>(0.033)</td>
<td>−0.103*</td>
<td>(0.053)</td>
</tr>
<tr>
<td>Within 3 Weeks</td>
<td>−0.933***</td>
<td>(0.222)</td>
<td>−0.765***</td>
<td>(0.190)</td>
</tr>
<tr>
<td>Count 2 weeks</td>
<td>−0.011</td>
<td>(0.021)</td>
<td>−0.006</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Count Auctions Same Charity</td>
<td>0.025***</td>
<td>(0.003)</td>
<td>0.026**</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Extended Time</td>
<td>−0.339</td>
<td>(0.310)</td>
<td>−0.019</td>
<td>(0.421)</td>
</tr>
<tr>
<td>FIFA 100</td>
<td>0.408</td>
<td>(0.390)</td>
<td>0.958*</td>
<td>(0.534)</td>
</tr>
<tr>
<td>Jersey not Washed</td>
<td>−1.096</td>
<td>(0.834)</td>
<td>0.000</td>
<td>(.)</td>
</tr>
<tr>
<td>Signed</td>
<td>−0.854**</td>
<td>(0.396)</td>
<td>−0.579</td>
<td>(0.532)</td>
</tr>
<tr>
<td>Signed Team</td>
<td>−0.876</td>
<td>(0.746)</td>
<td>−1.215</td>
<td>(0.855)</td>
</tr>
<tr>
<td>Goals scored</td>
<td>0.447</td>
<td>(0.579)</td>
<td>0.446</td>
<td>(1.005)</td>
</tr>
<tr>
<td>Italian Charity</td>
<td>−1.539</td>
<td>(1.391)</td>
<td>−2.540**</td>
<td>(1.242)</td>
</tr>
<tr>
<td>English Charity</td>
<td>5.195***</td>
<td>(1.458)</td>
<td>5.503***</td>
<td>(1.759)</td>
</tr>
<tr>
<td>Disability</td>
<td>1.840***</td>
<td>(0.425)</td>
<td>1.507**</td>
<td>(0.633)</td>
</tr>
<tr>
<td>Development</td>
<td>−0.901</td>
<td>(0.914)</td>
<td>−0.899</td>
<td>(0.860)</td>
</tr>
<tr>
<td>Health</td>
<td>−1.624***</td>
<td>(0.621)</td>
<td>−1.257</td>
<td>(0.812)</td>
</tr>
<tr>
<td>Humanitarian</td>
<td>−0.387</td>
<td>(1.057)</td>
<td>0.613</td>
<td>(1.647)</td>
</tr>
<tr>
<td>Children and Youth</td>
<td>−2.270***</td>
<td>(0.470)</td>
<td>−2.959***</td>
<td>(0.679)</td>
</tr>
<tr>
<td>Neurodegenerative dis.</td>
<td>−1.681**</td>
<td>(0.747)</td>
<td>−1.960**</td>
<td>(0.907)</td>
</tr>
<tr>
<td>Team Related</td>
<td>−0.602</td>
<td>(1.071)</td>
<td>−1.509</td>
<td>(1.228)</td>
</tr>
<tr>
<td>Sport</td>
<td>−1.883***</td>
<td>(0.504)</td>
<td>−1.138</td>
<td>(0.744)</td>
</tr>
<tr>
<td>Constant</td>
<td>−1.948</td>
<td>(2.418)</td>
<td>−2.674</td>
<td>(3.629)</td>
</tr>
</tbody>
</table>

Pseudo R-squared 52.67% 54.64%

N 729 442

Note: Logit regression of $q$ on covariates. The dependent variable is 1 if $q = 0.1$. Only auctions with $q \in \{10\%,85\\%\}$. Column (I) refers to auctions whose transaction price is in the interval €(100,1000), while the interval is €(100,400) in Column (II). Control variables are defined in Appendix B.

* – $p < 0.1$; ** – $p < 0.05$; *** – $p < 0.01$. 

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E.3 Robustness checks for the structural estimations

This section reports structural estimates for different versions of the models. In particular, the following tables replicate the results in the main text (i) by using only data in the €100 - €400 interval, (ii) by comparing auctions with $q$ at 10% and 78% (instead of 85%),\(^{10}\) and (iii) by using a larger set of covariates. Overall, the results highlights the robustness of the estimates in the main text (Table 6).

Table 19: Structural estimation when $q \in \{10\%, \ 85\%\}$ and price €100 and €400

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Number of bidders</th>
<th>$\alpha$ [5% CI]</th>
<th>$\beta$ [5% CI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>16</td>
<td>17.2% [5.3%, 32.1%]</td>
<td>37.6% [8.0%, 61.8%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>17.0% [4.6%, 29.7%]</td>
<td>37.5% [8.5%, 60.2%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>16.7% [4.1%, 28.9%]</td>
<td>37.5% [12.8%, 60.9%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>16.2% [3.6%, 27.5%]</td>
<td>37.4% [5.3%, 61.4%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>15.2% [3.8%, 27.3%]</td>
<td>37.1% [8.9%, 61.2%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 85\%\}$ and the price between €100 and €400. 470 observations in total.

\(^{10}\)Employing the auctions with $q = \{78\%, 85\%\}$ do not yield consistent estimates because the full rank condition assumption break down as the two $q$ are almost identical. See the simulations in Appendix H for a numerical example.
Table 20: Structural estimation when $q \in \{10\%, 78\%\}$ and price €100 and €1000

<table>
<thead>
<tr>
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<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[5% CI]</td>
<td>[5% CI]</td>
</tr>
<tr>
<td>99%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95%</td>
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</tr>
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<tr>
<td>50%</td>
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<td>52.4%</td>
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</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 78\%\}$ and the price between €100 and €1000. 366 observations in total.

Table 21: Structural estimation when $q \in \{10\%, 78\%\}$ and price €100 and €400

<table>
<thead>
<tr>
<th>Quantile</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>[5% CI]</td>
<td>[5% CI]</td>
</tr>
<tr>
<td>99%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95%</td>
<td>24.2%</td>
<td>51.9%</td>
</tr>
<tr>
<td>90%</td>
<td>23.8%</td>
<td>51.8%</td>
</tr>
<tr>
<td>75%</td>
<td>23.3%</td>
<td>51.7%</td>
</tr>
<tr>
<td>50%</td>
<td>21.8%</td>
<td>51.3%</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 78\%\}$ and the price between €100 and €400. 258 observations in total.
E.3.1 Different set of covariates

In this section the log(Total Number of Bids Placed) is added to the covariates. This variable helped explaining differences in competition in Table 16. In fact, the more bids are submitted by a given number of bidders (a control for the number of unique bidders is also included), the more intense is the competition in the auction. The estimates of the parameters $\alpha$ and $\beta$ do not change substantially from those reported in the main text (Table 6).

In addition, Figure 18 reports the same out-of-sample validation exercise proposed in the main text (see Section 6.3). It is based on the comparison between the pdf of the density obtained from estimating the model using auctions with $q \in \{10\%, 85\%\}$ (the $\alpha$ and $\beta$ parameters are shown in Table 22) with that derived by projecting the full three-step estimation procedure onto the auctions with $q = 78\%$.

Table 22: Structural estimation when $q \in \{10\%, 85\%\}$ and price €100 and €1000

<table>
<thead>
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<th>$n$</th>
<th>$\alpha$ [5% CI]</th>
<th>$\beta$ [5% CI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>16</td>
<td>17.4% [7.6%, 26.8%]</td>
<td>42.7% [23.2%, 57.7%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>17.2% [7.5%, 25.2%]</td>
<td>42.7% [18.7%, 56.1%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>16.9% [7.3%, 25.5%]</td>
<td>42.7% [20.7%, 57.6%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>16.5% [6.8%, 25.7%]</td>
<td>42.6% [21.4%, 60.3%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>15.5% [6.0%, 23.5%]</td>
<td>42.4% [19.8%, 57.5%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 78\%\}$ and the price between €100 and €1000. 731 observations in total.
Table 23: Structural estimation when $q \in \{10\%, 85\%\}$ and price €100 and €400

<table>
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<th>$n$</th>
<th>$\alpha$ [5% CI]</th>
<th>$\beta$ [5% CI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>16</td>
<td>14.5% [3.0%, 29.4%]</td>
<td>33.8% [3.6%, 57.0%]</td>
</tr>
<tr>
<td>95%</td>
<td>14</td>
<td>14.3% [2.7%, 27.7%]</td>
<td>33.7% [8.0%, 57.1%]</td>
</tr>
<tr>
<td>90%</td>
<td>12</td>
<td>14.1% [3.9%, 26.8%]</td>
<td>33.7% [3.6%, 56.2%]</td>
</tr>
<tr>
<td>75%</td>
<td>10</td>
<td>13.7% [2.2%, 26.0%]</td>
<td>33.6% [2.4%, 56.6%]</td>
</tr>
<tr>
<td>50%</td>
<td>7</td>
<td>12.8% [2.1%, 22.5%]</td>
<td>33.4% [6.4%, 53.0%]</td>
</tr>
</tbody>
</table>

Note: Results from the structural estimation of $\alpha$ and $\beta$ for selected quantiles of the distribution of the number of bidders. The 2.5% and 97.5% confidence intervals are reported in square brackets. The CI are found by bootstrap with replacement (401 times). The dataset is restricted to all auctions such that $q \in \{10\%, 78\%\}$ and the price between €100 and €400. 470 observations in total.

Figure 18: Out-of-sample fit of the estimated density (10% and 78% auction)

Note: Replication of the out-of-sample validation in Section 6.3 adding the variable $\log$(Total Number of Bids) to the covariates in the first step. The plot shows the comparison of the density of the private values estimated from the structural model employing data from auctions with $q = \{10\%, 85\%\}$ and the density of the private values estimated by projecting the three-step estimation on the $q = 78\%$ auctions (as in Figure 5a). The null hypothesis (equality) cannot be rejected at 0.1874 level. The computation assumes $n = 16$, but the same result can be replicated with different $n$. The plotted densities are computed using a Gaussian kernel and Silverman’s rule-of-thumb bandwidth (Silverman, 1986).
E.4 Additional figures

E.4.1 Returning winners

Figure 19 compares the densities of the homogenized pseudo-winning bids from the first stage of the estimation procedure across bidders who won multiple objects (more than the median, 3) and the other bidders. The null hypothesis that these densities are equal cannot be rejected. The two plots differ based on whether the focus is on the large sample (Panel a) or the small sample (Panel b). Controlling for additional covariates further reduces the distance between the two densities.

Figure 19: Density of the homogenized transaction prices by returning winners

(a) Returning vs Non-Returning winners

(b) Returning vs Non-Returning winners

Note: Density of the pseudo-winning bids in the structural model by returning winners. Returning winners (Collectors) are auction winners who won at least 4 bids (the median is 3). The Kolmogorov-Smirnov test does not reject the null hypothesis (equality) at 0.1554 level in Panel (a) and at 0.1423 in Panel (b). Adding the “Total Number of Bids Placed” among the covariates in the first step of the estimation procedure yields a p-value greater than 40%. The plot is obtained using only auctions with \( q \in \{10\%, 85\%\} \). The plotted densities are computed using a Gaussian kernel and Silverman’s rule-of-thumb bandwidths (Silverman, 1986).

F Counterfactual scenario with different altruistic parameters

Given the estimated distribution of values, \( \hat{F}(v) \), one can experiment with different \( \alpha \) and \( \beta \) parameters to assess the elasticity of bids to changes in the charitable parameters. For example, Figure 3b showed that a harsh version of the volunteer-shill model of giving (\( \alpha = 0.5 \) and \( \beta = 0.1 \)) may result in lower gross revenues to the auctioneer in charity auctions than non-charity auctions. Figure 20b performs the same analysis with the estimated distribution of values. The CDF shows that all bidders below the 0.8 quantile of the value distribution increase their bids substantially, while the remaining bidders shade their bids below valuation. Additional computations show that even though 20% of the highest value bidders submit smaller bids, expected revenues are still above non-charity auctions. Similarly, when \( \beta > \alpha \),
revenues in charity auctions dominate those in non-charity auctions (Figure 20a). This analysis shows graphically that the outcome to the auctioneer does not only depend on \( \alpha \) and \( \beta \), but also on how these two parameters relate with the distribution of values. In fact, it is the latter to establish the likelihood of a bidder to win or lose the auction, which in turns triggers the willingness of the bidder to raise her bids based on the charitable motives and the percentage donated.

Figure 20: Counterfactual scenario with different charity parameters

(a) Estimated \( F(v); q = 1 \)
    Warm glow (\( \beta > \alpha > 0 \))

(b) Estimated \( F(v); q = 1 \)
    Volunteer shill (\( \alpha > \beta > 0 \))

Note: Bids are computed drawing 200 values from the estimated distribution of private values \( F(v) \) and the selected \( \alpha \) and \( \beta \); \( q = 1 \). Prices are converted in euro by summing the median of the fitted prices in the first step of the estimation (6.1). The first step of the estimation also includes \( \log(\text{Total Number of Bids Placed}) \) as a covariate.

Panel (a) assumes warm glow bidding while Panel (b) assumes volunteer shill bidding. Only auctions with price between €100 and €1000.

G Revenue comparison across auction formats

This section investigates revenues under different scenarios. Figure 21 compares the expected revenues under different auction formats: the second-price auction (dashed lines), the all-pay auction (dotted lines) and a non-charity second-price auction (solid line).\(^{11}\) Engers and McManus (2007) theoretically show that first-price charity auctions perform worst than the other two charitable formats studied in this section and are not reported in the figure to simplify the exposition.\(^{12}\)

In both figures, the second price format outperforms the all-pay auction for all number of bidders in the graph. Charitystars cannot raise more funds by switching to all-pay auctions, a method that was highlighted in the theoretical literature as the best mechanism from a revenue standpoint, especially when the pool of bidders is large (e.g., Goeree et al., 2005, Engers and McManus, 2007).\(^{13}\)

\(^{11}\)Revenues are computed as described in the note to Table 7. The optimal bid in an all-pay auction is \( b^A(v) = (vF(v))^{n-1} - \int_0^v F(x)^{n-1}dx / (1 - q \cdot \beta) \), similar to the equilibrium studied in Engers and McManus (2007) for the \( q = 1 \) case.
\(^{12}\)This is also true for Charitystars’ data. The intuition for this outcome is that in a first-price auction losing bidders do not have an incentive to increase their bids. This happens because these bidders cannot affect the transaction price as they do in second-price auctions.
\(^{13}\)In the charity all-pay format each bidder gains from the sum of the contributions of the others and can accept
Figure 21: Expected gross revenues per auction

(a) Expected gross revenues; \( q = 85\% \)

(b) Expected gross revenues; \( q = 10\% \)

Note: Revenue comparison across mechanisms for different number of bidders. The percentage donated is set to 85\% and 10\% in Panel (a) and Panel (b) respectively. The density \( f(v) \) and the distribution \( F(v) \) are approximated using a cubic spline. Only auctions with price between \( €100 \) and \( €1000 \). Revenues in Euro are computed in multiple steps. (i) Subtract the median number of bidders times its estimated coefficient in the OLS regression (6.1) from the fitted values of the same regression. (ii) Compute the expected revenues obtained as the expectation of the second-highest bid using the primitives estimated in Section 6.1 \((F(\cdot), \alpha, \beta)\). (iii) Sum the fitted values in (i) with the homogenized expected price in (ii) and apply the log-level transformation. Realized revenues are determined at the median number of bidders for each auction type. The covariates used in (6.1) include the total number of bids as in Appendix E.3.1.

An additional reason to prefer second-price auctions over all-pay auctions is the variance of the expected revenues. Figure 23 in Appendix E.4 shows that the variance of the all-pay auctions sharply increases with the number of bidders, while it decreases with \( n \) for sealed bid formats. Therefore, English auctions do not only maximize Charitystars’ expected revenues, but also reduce the volatility of its cash flows: a key objective for any start-up. This fact could explain the preference for English auctions among most online charity marketplaces (e.g., Charitystars.com, eBay for Charity, CharityBuzz.com, BiddingFor-Good.com and many others).

The comparison across charity and non-charity formats is also in Figure 21a. The revenue difference between the two type of auctions appear to be rather small, reaching little above $50 for high \( n \). Still, this implies a \( \sim 12\% \) premium over the non-charity auctions, that is twice as much as that estimated for eBay’s Giving Works (Elfenbein and McManus, 2010).

These across auction comparisons have some limitations. For example, differential bidder-entry across mechanisms is one of them. This may be problematic if Charitystars users had a stronger taste for certain auctions than others. For instance, according to Figure 21b when 17 bidders are expected in an all-pay auction but only 6 in a second-price auction, switching to the all-pay auction would grant Charitystars revenues in excess of \( €50 \) over the second-price model.

to bid a value equal to her own private profit. This cannot happen in winner-pay formats because such a bid would be suboptimal to surely losing the auction and earning the value of the externality. This reasoning does not apply when the number of bidders is low. In this case the total contribution in the all-pay format is not high enough and bidders shades their bids as a result, making the second-price auction the optimal choice for the auctioneer. It can be shown numerically that the difference in Figure 21 between all-pay and second-price auctions goes asymptotically to 0 with the number of bidders.
Figure 22: Expected revenues across different auction formats

Note: Revenue comparison across different auction mechanisms. The plot shows that the distribution of bids and private value as in Figure 3b, which result in lower gross revenues in second-price charity auction than in non-charity auctions. The primitives are $q = 85\%$, $\alpha = 50\%$, $\beta = 10\%$, $F(\cdot) \sim N(50, 25)$ on $[0, 100]$.

Figure 23: Variance of the revenues across different auction formats

(a) Variance gross revenues; $q = 85\%$

(b) Variance gross revenues; $q = 10\%$

Note: The figure compares the variance of the revenues across mechanisms for different number of bidders. The percentage donated is set to 85% and 10% in Panel (a) and Panel (b) respectively. The density $f(v)$ is approximated using a cubic spline. Only auctions with price between €100 and €1000. The variances are computed without accounting for covariates.
Monte Carlo simulations

Objectives. The Monte Carlo simulations in this section fulfill two goals: (i) to show that the estimation routine described in Section 6 return consistent estimates of the parameters, and (ii) to support with some empirical evidence the claim that the estimates are not consistent when the amount donated in the two auction types is very close.

Design of the experiments. There are two auction types (A and B) such that \( q^A = .10 \) and \( q^B = .85 \). Private values are generated for all bidders drawing from a uniform distribution in \([0, 1]\) in Tables 24 and 25 and in \([-1, 1]\) in Tables 26. There are 10 bidders in each auctions. They bid according to the bid function in (4.2). The true charitable parameters are \( \alpha_0 = .25 \) and \( \beta_0 = .75 \).

The steps of the estimation procedure are outlined below:

1. Draw values from the distribution \( F(v) \) for each bidder in the two auctions. In total 20 values.
2. Compute the bids for each bidder in the two auctions. Save the winning bid in each auction.
3. Nonparametrically estimate the density of the winning bids (either by Triweight, or Gaussian Kernel). The bandwidth is chosen using the rule-of-thumb. Trimming follows Guerre et al. (2000) who suggested trimming observations close to \( 0.5 \times \) bandwidth to the boundary.\(^{14}\)
4. Given the number of bidders \( (n = 10) \) invert the distribution of the winning bids to determine the distribution and density of the bids as in (6.2) and (6.3).
5. Compute the distribution and density of auctions of type A for each losing bids in the interval between the smallest winning bid and the largest winning bid of type A.
6. Compute the distribution and density of type B \( (q = .85) \) over 100,000 points.
7. Match the quantile of the distribution of type B with those of the distribution of type A through (5.1).
8. Find the couple \( (\alpha, \beta) \) that minimizes the objective function (6.4) starting from a random seed. The search algorithm constraints the parameters in the unit interval.
9. Save the estimates and restart from 1.

These steps are repeated 401 times. The tables below report the mean, median, quantiles and root mean squared errors for \( \alpha \) and \( \beta \) for each combination of parameters.

Results. First, let’s assess the consistency of the estimates. Different experiments are reported in Tables 25 and 26, showing that the estimates are close to the true parameters. In particular, even with a small number of observations (the first line in each panel), the mean and medians are always within 3% of the true parameters.

These tables are composed by different panels: each panel refers to a different kernel used to estimate the distributions (and densities) of the winning bids. The Gaussian and Triweight kernels give similar results. Within each panel, the rows differ on the number of auctions used to estimate the primitives. The number of bidders in each auction is always constant and equal to 10. Since for each auction only the winning bid is used, I empirically consider the asymptotic properties of the estimator by looking at the rate

\(^{14}\)For the Gaussian case \( h_{pdf} = 1.06\sigma n^{-1/5} \) and \( h_{CDF} = 1.06\sigma n^{-1/3} \) where \( \sigma = \min\{\text{s.d.}(b^k_w), \text{IQR}/1.349\} \), where \( b^k_w \) is the vector of winning bid for auction of type \( k \), and \( h_{CDF} = 1.587\sigma n^{-1/3} \). For the triweight case \( h_{pdf} = h_{CDF} = 2.978\sigma n^{-1/5} \) (Härdle, 1991, Li et al., 2002, Li and Racine, 2007, Lu and Perrigne, 2008).
at which the root mean squared error (RMSE) decreases as the number of auctions grows (i.e., comparing RMSE across columns).\textsuperscript{15} Comfortingly, this rate is close to $\sqrt{n}$ for all experiments.

To study the consistency of the estimates when there is only limited variation over $q$ across auctions I run similar experiments varying $q$ instead of the nonparametric kernel. From Table 24 it is clear that $\alpha$ and $\beta$ cannot be estimated consistently when $q^A \approx q^B$ as the mean and median of the estimated parameters are about 0 and .50 instead of .25 and .75 for $\alpha$ and $\beta$ respectively.

<table>
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<th>$T^B$</th>
<th>$\mu_\alpha$</th>
<th>$\mu_\beta$</th>
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</table>

Note: Monte Carlo simulations of the second and third step of the estimation process. Auction types are denoted by $A$ and $B$. Each panel shows the estimated parameters for different percentage donated. The bandwidths in step 2 are computed with a Gaussian Kernel. The data is generated according to $\alpha = 25\%, \beta = 75\%$ and $F(v)$ is assumed uniform in $[0, 1]$. Each auction has 10 bidders. 401 repetitions.

\textsuperscript{15}I am analyzing the asymptotic properties of this class of estimators theoretically in another project, which is still a work in progress.
Table 25: Monte Carlo simulation 1

<table>
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<tr>
<th>$T^A$</th>
<th>$T^B$</th>
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<th>$\mu_\beta$</th>
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Note: Monte Carlo simulations of the second and third step of the estimation process. Auction types are denoted by $A$ and $B$. The bandwidths in step 2 are computed either with a Gaussian Kernel (top panel) or with a Triweigh kernel (bottom panel). The data is generated according to $\alpha = 25\%$, $\beta = 75\%$, $q^A = 10\%$, $q^B = 85\%$ and $F(v)$ is assumed uniform in $[0, 1]$. Each auction has 10 bidders. 401 repetitions.
Table 26: Monte Carlo simulation 2

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Gaussian kernel

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Triweigh kernel

Note: Monte Carlo simulations of the second and third step of the estimation process. Auction types are denoted by $A$ and $B$. The bandwidths in step 2 are computed either with a Gaussian Kernel (top panel) or with a Triweigh kernel (bottom panel). The data is generated according to $\alpha = 25\%$, $\beta = 75\%$, $q^A = 10\%$, $q^B = 85\%$ and $F(v)$ is assumed uniform in $[-1, 1]$. Each auction has 10 bidders. 401 repetitions.
References in Appendix


